Complex network from time series based on phase space reconstruction

Zhongke Gao and Ningde Jin
School of Electrical Engineering and Automation, Tianjin University, Tianjin 300072,
People’s Republic of China

(Received 23 April 2009; accepted 24 August 2009; published online 17 September 2009)

We propose in this paper a reliable method for constructing complex networks from a time series with each vector point of the reconstructed phase space represented by a single node and edge determined by the phase space distance. Through investigating an extensive range of network topology statistics, we find that the constructed network inherits the main properties of the time series in its structure. Specifically, periodic series and noisy series convert into regular networks and random networks, respectively, and networks generated from chaotic series typically exhibit small-world and scale-free features. Furthermore, we associate different aspects of the dynamics of the time series with the topological indices of the network and demonstrate how such statistics can be used to distinguish different dynamical regimes. Through analyzing the chaotic time series corrupted by measurement noise, we also indicate the good antinoise ability of our method.

Characterizing complicated dynamics from time series (or measured signals) is a fundamental problem of continuing interest in a wide variety of fields. Different measures have been proposed to analyze these dynamics, for example, Lyapunov exponent, entropies, and fractal scale. Complex networks have established themselves in recent years as being particularly suitable and flexible for representing and modeling many complex natural and artificial systems. Investigation of nonlinear time series in terms of complex networks has begun only recently. In this paper, we focus on one basic question: how to investigate the nonlinear dynamics of time series through complex network. First, we propose a reliable method for constructing complex network from time series with the purpose of encoding the underlying time series dynamics in the network topology. Then we associate different aspects of the dynamics of the time series with the topological indices of the network and demonstrate how such statistics can be used to distinguish different dynamical regimes. Our result can provide useful clues as how to properly construct the network to effectively investigate the complicated dynamics of different time series.

I. INTRODUCTION

The past few years have witnessed dramatic advances in the field of complex networks1–9 since the publication of the seminal works of Watts and Strogatz10 as well as Barabási and Albert.11 Complex networks, which provide us with a new viewpoint for understanding a complex system from the relations between the elements in a global way, may be a powerful tool for investigating nonlinear time series in practice. But how to construct a network from a time series (or measured signals) is still an essential problem to be solved. Zhang and Small12 first introduced a transformation from pseudoperiodic time series to complex network by representing each cycle of pseudoperiodic series as a basic node. Lacasa et al.13 proposed an algorithm to characterize fractal time series using the visibility graph. Yang and Yang14 built network from the correlation matrix of a time series. Although the network construction algorithms mentioned above have the advantage of being simple, which means that the embedding step is avoided, they may have difficulties in getting enough information from a high dimensional system. An important advantage of utilizing a phase space reconstruction is that if the embedding is chosen appropriately, the topological distribution of the set of vector points in phase space will accurately reflect the underlying dynamics of the original system. Therefore, the network inferred from that phase space reconstruction can be related directly back to the evolution operator of the underlying dynamical system.

In this paper, we introduce a reliable and effective method for constructing complex network from time series based on phase space reconstruction. That is, we use false nearest neighbors (FNNs) method15 and C-C method16 reconstruct phase space from time series and construct network with each vector point of the reconstructed phase space represented by a single node and connection determined by the phase space distance. Due to the fact that this method can encode the underlying time series dynamics in the network topology, we characterize the complicated dynamics of different time series through exploring a number of network topology statistics and demonstrate that some statistics not only can reflect different aspects of specific dynamics but also can be used to distinguish different dynamical regimes. In addition, the antinoise ability of our method is also investigated by analyzing the chaotic time series corrupted by measurement noise.

II. CONSTRUCTION OF NETWORK FROM TIME SERIES

We start from the phase space reconstruction. Takens’ embedding theorem,17 which is very often invoked as the motivation for applying a time delay embedding to reconstruct phase space from a time series, can be described as following, for arbitrary time series \( z(t) \), \( i=1,2,\ldots,M \) ( \( t \) is
sampling interval, \( M \) is the sample size), if the embedding delay time is selected as \( \tau \), and the embedding dimension as \( m \), the vector point in phase space can be represented as follows:

\[
\bar{X}_t = \{x_t(1), x_t(2), \ldots, x_t(m)\} = \{z(kt), z(kt + \tau), \ldots, z(kt + (m - 1)\tau)\}, \tag{1}
\]

where \( k = 1, 2, \ldots, N \) and \( N = M - (m - 1)^\tau / t \) denotes the total vector points of the reconstructed phase space. C-C method is the frequent method that can be used to determine the delay time \( \tau \). The general idea of C-C method is that we can subdivide a single time series \( \{z_1, z_2, \ldots, z_M\} \) into \( t \) disjoint time series, where \( t \) is the index lag, and then calculate \( S(m, M, r, t) \) from these time series as follows:

\[
S(m, M, r, t) = \frac{1}{t} \sum_{t=1}^{t} \left[ C_r(m, M/t, r, t) - C_r^m(1, M/t, r, t) \right], \tag{2}
\]

\[
\Delta S(m, r, t) = \max(S(m, M, r, t)) - \min(S(m, r, t, M)), \tag{3}
\]

where \( C_r(m, M, r, t) = 2/N(N-1)\sum_{1 \leq i < j \leq N} \theta(r - |x_i - x_j|), N = M - (m - 1)\tau, r > 0, \) and \( \theta(a) \) is the Heaviside function. The general range of \( m, r \) is \( 2 \leq m \leq 5, \sigma / 2 \leq r \leq 2\sigma, \sigma \) is the standard deviation of data sets,

\[
\bar{S}(t) = \frac{1}{16} \sum_{m=2}^{4} \sum_{j=1}^{5} S(m, r, j, t), \tag{4}
\]

\[
\Delta \bar{S}(t) = \frac{1}{4} \sum_{m=2}^{5} \Delta S(m, r, t). \tag{5}
\]

The corresponding time of the first local minimum of \( \Delta \bar{S}(t) \) is the optimal delay time \( \tau \). More details about C-C method can be found in Ref. 16. Through studying four algorithms that can be used to select \( \tau \), i.e., mutual information I,\(^{18}\) mutual information II,\(^{19}\) autocorrelation function method, and C-C method, we have argued in Ref. 20 that C-C method shows good antinoise ability. Hence, in this paper we use C-C method to determine the delay time \( \tau \) and use FNN method to determine the embedding dimension \( m \), that is, we use C-C method and FNN method reconstruct phase space from time series.

After reconstructing phase space from a given time series, denoted as \( \{z(1), z(2), \ldots, z(M)\} \), we proceed to construct complex network by considering each vector point in reconstructed phase space as a basic node and using the phase space distance to determine network connection. The phase space distance between vector points \( \bar{X}_i \) and \( \bar{X}_j \) in this study is defined as

\[
d_{ij} = \sum_{n=1}^{m} \|X_i(n) - X_j(n)\|, \tag{6}
\]

where \( X_i(n) = z(i + (n - 1)\tau) \) and \( X_j(n) = z(j + (n - 1)\tau) \) is the \( n \)th element of \( \bar{X}_i \) and \( \bar{X}_j \), \( m \) and \( \tau \) are the embedding dimension and delay time, respectively. The constructed network containing \( N = M - (m - 1)^\tau / t \) nodes can be studied either as a weighted network or as an unweighted one. We first construct weighted network by choosing the distance between each pair of vector points in reconstructed phase space as the weight between corresponding nodes and define the node strength \( S \) of weighted network as

\[
S_i = \frac{1}{N} \sum_{j=1}^{N} w_{ij}, \tag{7}
\]

where \( w_{ij} \) is the weight between nodes \( i \) and \( j \) and \( w_{ij} = d_{ij} \). We use a chaotic time series from Lorenz system and a white noise time series [see Figs. 1(a) and 1(d) for details] investigates the \( S \) distribution and find that, as shown in Fig. 2, the \( S \) distribution of the chaotic time series has significant difference from that of noisy time series. As can been seen, since the \( S \) distribution of the chaotic time series is rather random when \( S \) is over 25, compared to a more clear power law distribution, which can be seen in Figs. 7 and 9 of our previous paper,\(^{7}\) the overall \( S \) distribution of the chaotic time series is not a power law distribution. By contrast, the overall \( S \) distribution of noisy time series is bell-shaped distribution.

Since the weighted network is fully connected and may contain redundant information, we now convert the weighted network to its unweighted counterpart by setting a threshold on the weight. That is, choosing a critical threshold \( r_c \), the distance matrix \( D = (d_{ij}) \) can be converted into adjacent matrix \( A = (a_{ij}) \), the rules of which read \( a_{ij} = 1 \) if \( d_{ij} \leq r_c \) and \( a_{ij} = 0 \) if \( d_{ij} > r_c \). The topological structure of the network can be described with the adjacent matrix \( A \), and the conditions \( a_{ij} = 1 \) and \( a_{ij} = 0 \) correspond to connection and disconnection, respectively. An appropriate threshold, therefore, should be chosen to fully preserve the main property of the network, but not to be too large that may obscure or conceal the local property by overconnecting the nodes. In this paper, we employ the network density\(^{21}\) to study the threshold, which is defined as the number of edges divided by the largest number of edges possible. In order to show how to select a suitable threshold, a chaotic time series from Lorenz system [see Fig. 1(a)] is studied by our method, and Fig. 3 shows the density of the constructed network versus the threshold \( r \). As can be seen in Fig. 3(b), the increase in the whole network degree reaches the maximum rate at about \( r_c = 11.2 \), where we set the critical threshold. We here explain why we set the threshold \( r \) at this critical point. Since the nodes inside a cluster are adjacent to each other, the degree will increase very rapidly as the threshold changes within the cluster radius. For a network from a chaotic system that has many clusters differing in sizes, as shown in Figs. 4(a) and 4(b), the edge increase will reach the maximum rate when the threshold approaches the mean radius of all the clusters, i.e., the critical point \( r_c \). The unweighted network obtained at \( r_c \) will maintain the clustering property, and thresholds beyond this value will result in a much slower edge increase, causing redundant connections among nodes. Consequently we choose \( r_c \) to derive the unweighted network. Furthermore, it should be noted that the size of the network (i.e., the length of the time series) will influence \( r_c \), and that is why we choose \( r_c = 11.2 \) to derive network of 200 nodes, in contrast to \( r_c = 10 \) to derive network of 600 nodes from the Lorenz system.
FIG. 1. Four different scalar time series that are used to construct complex networks: (a) chaotic time series from Lorenz system ($\sigma=16, \rho=45.92, \beta=4$), (b) chaotic time series from Rossler system ($a=0.2, b=0.2, c=5.7$), (c) periodic time series, and (d) white noise time series.

FIG. 2. Node strength distribution for weighted network of 1000 nodes from (a) Lorenz system (log-log plot) and (b) white noise time series (regular plot).

FIG. 3. (Color online) Complex network from the Lorenz system with 200 nodes. (a) Density vs threshold. (b) Derivative of density vs threshold with $m=3, \tau=10, r_c=11.2$. 

Downloaded 18 Sep 2009 to 158.132.149.98. Redistribution subject to AIP license or copyright; see http://chaos.aip.org/chaos/copyright.jsp
We use the method presented in this paper to generate networks from Lorenz system, Rossler system, periodic time series, and white noise time series, respectively (details of the four different time series can be seen in Fig. 1), and find that the networks constructed from different kinds of time series demonstrate fundamentally different structures. The chaotic time series is the data from the $x$ component of Lorenz system

$$\frac{dx}{dt} = \sigma(y - x),$$

FIG. 4. (Color online) Complex network of 200 nodes from (a) Lorenz system with $m=3$, $\tau=10$, $r_c=11.2$, (b) Rossler system with $m=3$, $\tau=13$, $r_c=6.2$, (c) periodic time series with $m=4$, $\tau=3$, $r_c=0.15$, and (d) white noise time series with $m=4$, $\tau=5$, $r_c=0.41$. The structure of network drawn by software UCINET and NETDRAW is shown in the left panel and the corresponding network connectivity (adjacency) matrix is shown in the right panel.
\[
\frac{dy}{dt} = x(p-z) - y,
\]

\[
\frac{dz}{dt} = xy - \beta z,
\]

where \(\sigma = 16\), \(\rho = 45.92\), \(\beta = 4\), as shown in Fig. 1(a), and the data from the \(x\) component of Rossler system

\[
\frac{dx}{dt} = -y - z,
\]

\[
\frac{dy}{dt} = x + ay,
\]

FIG. 5. Small-world characteristics of the networks from chaotic time series.

FIG. 6. The regular plots of the nearest neighbor average connectivity of nodes with respect to connectivity of the networks containing 600 nodes from (a) Lorenz system with \(m=3\), \(r=23\), \(r_c=10\) and (b) white noise time series with \(m=4\), \(r=3\), \(r_c=0.8\).

FIG. 7. The bitmap of sorted adjacent matrix of network containing 600 nodes from (a) Lorenz system with \(m=3\), \(r=23\), \(r_c=10\) and (b) white noise time series with \(m=4\), \(r=3\), \(r_c=0.8\).
\[ \frac{dz}{dt} = b + z(x - c), \]

where \( a = 0.2, \; b = 0.2, \; c = 5.7 \), as shown in Fig. 1(b). It should be mentioned here that the structure of constructed network, i.e., Fig. 4, is drawn by the software NETDRAW, and the algorithm used is the Kamada–Kawai \(^22\) spring embedding algorithm. The general idea of Kamada–Kawai spring embedding algorithm is as follows. First, the algorithm calculates the energy for each node. It then loops over all the nodes to find the one with the highest energy and begins iterating the Newton–Raphson stage to compute new positions for the node until its energy is below epsilon. At this point, it again looks for the node with the highest energy and begins moving it (for more details, see Ref. 22). This process continues until there is no node with energy above epsilon, and then the algorithm is completed. As can be seen in Figs. 4(a) and 4(b), the network structure corresponding to chaotic time series has nodes congregating at different locations, and some regions are highly clustered with nodes and others are rather sparse. The network generated from periodic time series in Fig. 4(c) is obviously a regular network. In comparison, the network from the white noise time series looks like a random network with the nodes entangled with each other and their edges intersecting, as shown in Fig. 4(d).

### III. NETWORK STATISTICS VERSUS DYNAMICAL REGIMES

Unstable periodic orbits (UPOs)\(^{21,23,24}\) embedded in the chaotic attractor are fundamental to the understanding of the chaotic dynamics. For a chaotic attractor, its trajectory will typically switch or hop among different UPOs. Specifically, the trajectory will approach an UPO along its stable manifold. This approach can last for several loops during which the orbit remains close to the UPO. Eventually, the orbit is ejected along the unstable manifold and proceeds until it is captured by the stable manifold of another UPO. A UPO of order \( n \) contains \( n \) loops lying in different locations in phase space. Each loop that belongs to a certain UPO-\( n \) has many other loops in its vicinity due to the attraction of the stable manifold associated with the UPO-\( n \). It then becomes a center of a cluster and the density of the neighbors is related to its stability decided by the vector field along its trajectory. Since the stability of each center loop may vary, we can see sparse as well as dense regions in the structure of constructed complex network, as shown in Figs. 4(a) and 4(b). Due to the fact that the loops in phase space are spatially clustered around the UPOs and we use phase space to determine the network connection, the complex network generated from chaotic time series shows the small-world characteristics, i.e., small shortest path and large clustering coefficient, which can be seen in Fig. 5.
We now demonstrate how to discriminate different dynamical regimes of the time series through investigating the degree correlations, Pearson coefficient, betweenness distribution, and clustering coefficient-betweenness correlations of two distinct complex networks from the chaotic time series (Lorenz system) and white noise time series. An important way of capturing the degree correlations is to examine the average degree of the nearest neighbors of nodes with degree \( k \), which is defined as

\[
k_{nn}(k) = \sum_{k'} k' P(k'|k),
\]

where \( P(k'|k) \) denotes the conditional probability that an edge of degree \( k \) connects a node with degree \( k' \). If there are no degree correlations, Eq. (10) gives \( k_{nn}(k) = \langle k^2 \rangle / \langle k \rangle \), i.e., \( k_{nn}(k) \) is independent of \( k \). Correlated networks are classified as assortative mixing if \( k_{nn}(k) \) is an increasing function of \( k \), whereas they are referred to as disassortative mixing when \( k_{nn}(k) \) is a decreasing function of \( k \). In other words, in assortative networks the nodes tend to connect to their connectivity peers, while in disassortative networks nodes with low degree are more likely connected with highly connected ones. As can be seen in Fig. 6, \( k_{nn}(k) \) increases with \( k \) for chaotic Lorenz system, while it decreases with \( k \) for noisy time series. In order to quantify such a correlation, we calculate the Pearson coefficient, which is defined as follows. Let \( e_{kl} = e_{lk} \) be the joint probability distribution for an edge to be with a node with connectivity \( k \) at one end and a node with connectivity \( l \) at the other. Its marginal \( q_k = \sum_l e_{kl} \) obeys the normalization condition \( \sum_k q_k = 1 \). Then, the Pearson correlation coefficient is given by

\[
r = \frac{1}{\sigma_q^2} \sum_{k,l} e_{kl}(e_{kl} - q_k q_l),
\]

where \( \sigma_q^2 = \sum_k q_k^2 - (\sum_k q_k)^2 \) is the variance of \( q_k \). \( r \in [-1,1] \) and \( r \) is positive (negative) for assortative (disassortative)
sortative) mixing. The Pearson coefficient of the network containing 600 nodes from chaotic Lorenz system and noisy time series is 0.1009 and −0.0037, respectively. Consistent with the one obtained from the analysis of the nearest neighbor average connectivity, the Pearson correlation coefficient is positive, confirming that the network from chaotic time series has assortative mixing. On the other hand, the network from noisy time series is of no assortative mixing. We can explain why the network corresponding to chaotic time series possesses the property of assortative mixing in terms of UPOs. For a complex network generated from chaotic time series, there are multiple clusters and most nodes connected to each other within the same cluster have a roughly similar number of connected neighbors or degrees. Because of the different stability of the center loop of UPOs, the common degree shared by the nodes from one cluster may differ from that of another. Due to the fact that the nodes within the same cluster usually have similar degrees and such nodes are always interconnected to each other, chaotic time series possesses the property of assortative mixing.

Another way of capturing the clustering property of chaotic time series is the sorted adjacent matrix proposed in this paper. That is, for a given complex network, we first sort nodes in ascending order of the degree. If two or more than two nodes have the same degree, we then arrange them in ascending order of the sum of neighbor degree, which is the sum of degree of a node’s neighbor. Finally, the above node sorting rules produce a sorted adjacent matrix, which can be represented as a black-and-white bitmap, i.e., if nodes $i$ and $j$ are connected, a black pixel is placed at the coordinate of $(i,j)$; otherwise a white pixel is placed there. Figure 7 depicts the bitmap of sorted adjacent matrix of two networks from different time series, i.e., chaotic series and noisy series. As can be seen, multiple black square blocks could be clearly found along the principal diagonal in the sorted adjacent matrix bitmap of the network from chaotic time series, which implies the fact that there are multiple clusters in the network and the nodes with similar degrees are interconnected to each other (i.e., assortative mixing). On the contrary, no obvious black square block could be detected along the principal diagonal in the sorted adjacent matrix bitmap corresponding to the noisy time series.
From the above analysis, we can see that the clustering property of chaotic time series can be well characterized by the degree correlations, Pearson coefficient, as well as the sorted adjacent matrix, and for networks from the noisy time series, no assortative mixing is found at different thresholds. Assortative mixing provides the information about the interactions of all node pairs. Sometimes we may also want to know how important or how central a single node is in a network. A concept that fulfills this requirement is betweenness or betweenness centrality. Together with the degree and the closeness of a node, the betweenness is one of the standard measures of node centrality, originally introduced to quantify the importance of an individual in a social network.27,28 More precisely, the betweenness \( b_i \) of a node \( i \), sometimes referred to also as load, is defined as

\[
b_i = \sum_{j,k \in N, j \neq k} \frac{n_{jk}(i)}{n_{jk}}, \tag{12}
\]

where \( n_{jk} \) is the number of shortest paths connecting \( j \) and \( k \), while \( n_{jk}(i) \) is the number of shortest paths connecting \( j \) and \( k \) and passing through \( i \). Through exploring the betweenness distribution and clustering coefficient-betweenness correlations, we not only show in Fig. 8 that the overall betweenness of the network constructed from chaotic time series is different from that of the network generated from noisy time series, but also demonstrate in Fig. 9 that a large number of nodes with small clustering coefficient have high betweenness in the network from chaotic time series, in contrast to the high betweenness only for the nodes with large clustering coefficient in the network from noisy time series. The distinct distributions show that the vector points are structured with different mechanisms in the phase space. As can be seen in Fig. 8, compared to a more clear power law distribution which can be found in Figs. 7 and 9 of our previous paper,9 the overall betweenness distribution of the chaotic time series is not an exactly power law distribution. But the overall betweenness distribution of the network constructed from noisy time series is an exponential distribution. From Figs. 8 and 9, we find that for the chaotic time series, the number of nodes with high betweenness greatly exceeds that from the noisy time series. This is essentially a reflection of the clustering property associated with the UPOs embedded in the chaotic attractor. The high betweenness nodes correspond to the nodes in between adjacent clusters that act as bridges, and such nodes usually with small clustering coefficient compared to the nodes within the clusters. Since a chaotic attractor contains infinitely UPOs, there will be many clusters in the corresponding network, which implies the existence of numerous high betweenness nodes.
IV. ANALYSIS OF THE ANTINOISE ABILITY

Since most real data are corrupted by noise, in this section we will discuss the antinoise ability of our method by analyzing two chaotic time series that are corrupted by measurement noise. The time series we used are generated from the following equation:

\[ z_i = L_i + \eta \sigma e_i, \]

where \( L_i \) is the noise-free time series from chaotic Lorenz system [see Fig. 1(a)], \( \eta \) is the standard deviation, \( e_i \) is the Gaussian independent identically distributed (iid) random variable with zero mean and a standard deviation of 1, and \( \sigma \) is the strength of noise. Noise levels of 0.1 and 0.3 were added to the Lorenz time series to generate two time series whose signal-to-noise ratio (SNR) is 10 and 5 dB [see Figs. 10(a) and 10(d) for details]. We construct networks from these two time series and find that when SNR > 5 dB, the topological structure of constructed network attractor is very similar to the one constructed from deterministic chaotic Lorenz system, as shown in Figs. 11(a) and 4(a); when the noise intensity becomes very large, i.e., SNR \( \leq 5 \) dB, because of the strong influence of Gaussian white noise, the corresponding topological structure distorts to a certain extent, as shown in Fig. 11(b). To cast light into an important question of whether the network statistics mentioned in Sec. III can be used to distinguish chaotic time series corrupted by Gaussian white noise from white noise time series, we investigate the degree correlations, betweenness distribution, and clustering coefficient-betweenness correlations of the two series presented in Fig. 10. The increase in \( k_{\text{avg}}(k) \) with \( k \) in Fig. 12 and the multiple black square blocks along principal diagonal in Fig. 13 both indicate the fact that the chaotic time series corrupted by Gaussian white noise also possesses the property of assortative mixing. As can be seen in Figs. 14 and 15, the clustering property associated with the UPOs of the chaotic time series corrupted by Gaussian white noise also can be reflected by the betweenness distribution and clustering.

![Four different time series from logistic function.](image)

![The NLAM for networks from logistic function with different bifurcation parameters.](image)
coefficient-betweenness correlations. From the above analysis, we could see that our method can also be applied to distinguish chaotic time series corrupted by Gaussian white noise from white noise time series. Considering the fact that the SNR of most real data is greater than 5 dB, we can argue that the method proposed in this paper has good antinoise ability.

V. Network Laminarity

As is well known, for the period-doubling route to chaos, the transition to chaos is preceded by infinite levels of bifurcation. In order to effectively study the bifurcation in complex network, we propose a new network topology statistic, named network laminarity (NLAM), which is defined as follows:

$$NLAM = \frac{\sum_{l=1}^{N} \sum_{j=1}^{N-l} P(l)}{\sum_{i,j} a_{ij}}$$

where $P(l) = \sum_{i=1}^{N} \sum_{j=1}^{N-l} (1-a_{ij})(1-a_{i,j+1}) \prod_{k=1}^{l} a_{i,j+k}$, $a_{ij}$ is the element of network adjacent matrix $A$, and $N$ is the number of nodes. From the definition of $P(l)$, we can see that only when $a_{ij}=1$, $a_{i,j+1}=0$, and $a_{i,j+k}=1$ ($k=1, 2, \ldots, l$), $(1-a_{ij})(1-a_{i,j+1}) \prod_{k=1}^{l} a_{i,j+k}$ will be 1, which implies that there exists a horizontal (vertical) line that consists of $l$ successive connections in the adjacent matrix $A$. $P(l)$ denotes the number of such horizontal (vertical) lines that consist of $l$ successive connections in the adjacent matrix $A$. Since the network is constructed from time series based on phase space reconstruction, the stronger the intermittency strength of time series is, the larger the $P(l)$ will be. Hence, the NLAM can reflect the intermittency strength of time series. Take the logistic function

$$x_{n+1} = ax_n(1-x_n)$$

as an example. We extract different time series by controlling the bifurcation parameter $a$, as shown in Fig. 16. Through investigating the NLAM for the networks generated from those different time series, we present the results in Fig. 17 and find that when the bifurcation parameter $a=3.5$, the corresponding time series [see Fig. 16(a)] possesses the periodic property and its NLAM is zero. With the increase in bifurcation parameter $a$, the time series, which is in the transition from periodicity to chaos, shows the feature of intermittency [see Figs. 16(b) and 16(c)], and the corresponding NLAM that can reflect the strength of intermittency also becomes large. When the bifurcation parameter $a=4$, logistic function turns into a chaotic system and the corresponding time series shows very weak intermittency [see Fig. 16(d)], which results in the decrease in the NLAM. Hence, the distinction in NLAM, which is sensitive to the bifurcation parameter and strength of intermittency, can really reflect the different dynamic characteristics embedded in time series.

VI. Conclusions and Discussions

In summary, we have proposed an effective method for constructing complex networks from a time series. Through studying the complex networks generated from four different time series, useful and interesting results are found: Periodic time series and noisy time series convert into regular networks and random networks, respectively, and the networks built from chaotic time series typically exhibit small-world and scale-free features. By exploring an extensive range of network topology statistics, we not only indicate that the degree correlations and Pearson coefficient can fundamentally reflect the clustering property of the nodes induced by the UPOs, but also demonstrate that the betweenness distribution, combined with the clustering coefficient-betweenness correlations, can well distinguish different dynamical regimes in time series. In addition, we have also tested our method by analyzing the chaotic time series corrupted by measurement noise and indicated that the method proposed in this paper has good antinoise ability. Furthermore, we have introduced a new network topology statistic to characterize the bifurcation and have shown that the distinction in NLAM, which is sensitive to the bifurcation parameter as well as strength of intermittency, can really reflect the different dynamic characteristics embedded in time series.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (Grant Nos. 50674070 and 60374041) and the National High Technology Research and Development Program of China (Grant No. 2007AA06Z231).