Abstract  Magnetic storms are the most prominent global manifestations of out-of-equilibrium magnetospheric dynamics. Investigating the dynamical complexity exhibited by geomagnetic observables can provide valuable insights into relevant physical processes as well as temporal scales associated with this phenomenon. In this work, we utilize several innovative data analysis techniques enabling a quantitative nonlinear analysis of the nonstationary behavior of the disturbance storm time (Dst) index together with some of the main drivers of its temporal variability, the $V_{B,\text{South}}$ electric field component, the vertical component of the interplanetary magnetic field, $B_z$, and the dynamic pressure of the solar wind, $P_{\text{dyn}}$. Using recurrence quantification analysis and recurrence network analysis, we obtain several complementary complexity measures that serve as markers of different physical processes underlying quiet and storm time magnetospheric dynamics. Our approach discriminates the magnetospheric activity in response to external (solar wind) forcing from primarily internal variability and highlights the case-specific nature of interdependencies between the Dst index and its potential drivers that need to be accounted for in future improved space weather forecasting models.

1. Introduction

Geospace magnetic storms are major perturbations of the Earth's magnetic field that are initiated by enormous bursts of plasma erupting from the solar corona. In addition to coronal mass ejections (CMEs), high-speed solar streams emanating from coronal holes provide solar wind structures that create favorable conditions for the development of magnetic storms. The ejection of highly energetic charged particles onto a trajectory intersecting with the Earth's orbit can have severe impacts on the Earth's magnetosphere (Bothmer & Z Malikov, 2007; Richardson & Cane, 2012). Similar to other extreme events in nature, the resulting perturbations of the geomagnetic field can vary remarkably in both magnitude and duration. However, unlike many other natural hazards, they commonly manifest themselves in simultaneous effects across vast parts of the globe.

The mechanism underlying these large-scale perturbations of the Earth's magnetic field is closely related with mass, momentum, and energy input provided by the solar wind that is stored in the magnetotail—if not dissipated. Due to this continuous input by the highly dynamic solar wind, the magnetosphere is always far from equilibrium (Consolino et al., 2008). When a critical threshold is reached, the magnetospheric system may be reconfigured through a sequence of energy and stress accumulating processes (Baker et al., 2007; Klimas et al., 1997, 1998, 2005). During major magnetic storms, charged particles confined in the Earth's radiation belts are accelerated to high energies and the intensification of electric current systems results in characteristic disturbances of the geomagnetic field (Baker, 2005; Daglis et al., 2009). The response of the magnetosphere
to the external forcing by the solar wind is in general not simply proportional to the input, and changes are episodic and abrupt rather than slow and gradual.

This distinct behavior has motivated the description of the Earth's magnetosphere as a complex system composed of several nonlinearly coupled subsystems, within which multiple interconnected processes act on a wide range of spatial and temporal scales (Chang, 1992; Consolini, 2002; Klimas et al., 1996; Valdivia et al., 2005; Watkins et al., 2001, and references therein). Vassiliadis et al. (1990) provided evidence of large-scale coherence in magnetosphere dynamics manifested as low-dimensional chaos in time series of auroral electrojet (AE) index measurements. Following upon these initial findings, subsequent studies have been based on a variety of complementary concepts from nonlinear dynamics and complex systems science to derive in-depth knowledge on the magnetosphere's response to the solar wind forcing. Among other approaches, the nonlinearity of magnetospheric dynamics has been studied using nonlinear filters (Vassiliadis et al., 1995; Weigel et al., 2003), explicitly accounting for the magnetosphere being a nonautonomous, driven system and contributing to a more accurate and efficient prediction of imminent magnetic storms (Valdivia et al., 1996). Accordingly, recent attempts to developing empirical forecast models of geomagnetic activity have mostly been based on nonlinear approaches like nonlinear autoregressive moving average models with exogenous inputs (NARMAX), neural networks, fuzzy methods combined with singular spectrum analysis, or Gaussian process regression (e.g., Boynton et al., 2011; Chandorkar et al., 2017).

Building upon the current understanding of magnetic field fluctuations at the Earth's surface and in the surrounding space, recent studies have drawn the picture of the magnetosphere as a hierarchically organized multiscale system based on power law scaling identified in time series of geomagnetic activity indices (Consolini, 1997; Chapman et al., 1998; Takalo et al., 1994; Uritsky & Pudovkin, 1998; Watkins et al., 2001; Wanliss et al., 2004; Wei et al., 2004). For instance, the adoption of a phase transition approach (Shao et al., 2003; Sharma, 2006) revealed a close connection between global coherence and scale invariance of the magnetosphere's behavior. Specifically, Sitnov et al. (2001) suggested that while the multiscale activity during substorms resembles second-order phase transitions, the largest substorm avalanches exhibit common features of first-order nonequilibrium transitions. Moreover, Balasis et al. (2006, 2018) demonstrated the existence of two different regimes in the magnetospheric dynamics associated with the prestorm activity and magnetic storms, respectively, a picture that is compatible with the occurrence of a phase transition. Another intense point of research is the inherent separation of timescales between internal and externally driven/triggered processes (e.g., Alberti et al., 2017; Consolini & De Michelis, 2005; Kamide & Kokobun, 1996; Tsurutani et al., 1990). Low-dimensional dynamics, self-organized criticality (Consolini, 1997; Chapman et al., 1998; Uritsky & Pudovkin, 1998; Uritsky et al., 2006) and phase transitions offer different perspectives on geomagnetic activity, all of which need to be taken into account to obtain a coherent global picture of the underlying dynamical processes. However, when considered individually, each of these approaches has its intrinsic limitations that are inherent to the specific methodology and respectively taken viewpoint on specific aspects of nonlinear dynamics.

This paper aims to offer an additional viewpoint on nonlinear magnetospheric variability based on empirical observations of the system and its potential drivers that has rarely been addressed in previous studies. Specifically, we utilize three complementary measures characterizing the dynamical complexity of time series provided by the powerful tools of recurrence analysis (Marwan et al., 2007). The main idea behind this approach is that most physical processes lead to recurrences of previous states or sequences thereof, meaning that a system's current dynamical state has some close analog in its past dynamics, both of which are separated by some period with different system properties. Knowledge on such past analogs can thus be employed for studying the predictability of the investigated system or even provide model-free probabilistic forecasts. Recent developments in dynamical systems theory have provided evidence that such a recurrent behavior is a generic property of both deterministic and stochastic dynamics (Kané, 1987; Webber Jr. & Marwan, 2014). Among the existing nonlinear time series analysis approaches based on the evaluation of such recurrences, recurrence quantification analysis (RQA; Marwan et al., 2007) and recurrence network analysis (RNA; Donner et al., 2011) have already proven their potential for tracing time-varying dynamical complexity in a wide variety of different fields (Bastos & Caiado, 2011; Donges, Donner, Rehfeld, et al., 2011; Donges, Donner, Trauth, et al., 2011; Schinkel et al., 2009; Zbilut et al., 2002). In particular, a vast amount of publications has provided ample evidence that recurrence-based characteristics are well suited for quantitatively characterizing different expressions of chaotic dynamics, as well as transitions between regular and chaotic motion (Marwan et al., 2007; Webber Jr. & Marwan, 2014, and references therein).
In this work, we apply ROA and RNA to trace dynamical complexity variations in the Earth’s magnetic field that have distinct signatures in geomagnetic activity indices. We start by investigating hourly recordings of the disturbance storm time (Dst) index, one of the most intensively studied indicators for magnetospheric variability on timescales between hours and weeks (which integrates over any higher-frequency magnetic field fluctuations and is therefore less prone to exhibit bursty short-term dynamics than other conceptually related geomagnetic indices like Sym-H; Wanliss & Showalter, 2006). Specifically, the Dst index is defined as the average change of the horizontal component of the Earth’s magnetic field recorded at four low-latitude magnetic observatories and therefore provides an integral picture of the overall state of the magnetosphere. Dst alone cannot represent the full complexity associated with geomagnetic storms (i.e., phenomena with very specific electromagnetic and particle signatures in different regions of the inner magnetosphere — from ionospheric to thermospheric disturbances to the dynamics of particle populations in radiation belts, which can be directly observed by in situ measurements like those provided by the Van Allen probes; Fox & Burch, 2014) and thus cannot be used as an appropriate indicator of all magnetic storms (Borovsky & Shprits, 2017). Its continuous availability, however, makes it a useful proxy for studying long-term variations of the Earth’s magnetosphere.

Notably, the geomagnetic field component represented by the Dst index is determined by both solar wind forcing and internal magnetospheric processes. By comparing temporal changes in the dynamical complexity of Dst index fluctuations with those of variables associated with the driving processes, in this work we provide a possible strategy to disentangle the dynamical signatures originating from internal magnetospheric complexity from the additional complexity enforced by the solar wind. This is performed at timescales that are typical for geomagnetic storms, that is, from 1 hr to a few days. Specifically, we present the results of recurrence analysis for time variations of the VB_South electric field component acting as a coupling function between external (solar wind) and geomagnetic field, as well as for the vertical component of the interplanetary magnetic field (IMF) Bz and the solar wind dynamic pressure P_{dyn}. While the relationship of VB_South and Bz with Dst as a proxy of the Earth’s magnetic field is easy to understand, an intensification of the solar wind P_{dyn} can significantly compress the Earth’s magnetosphere and thus lead to global changes in the magnetospheric and ionospheric currents. Furthermore, Shi et al. (2006) concluded that pressure enhancements also cause further intensification of the storm time preexisting partial ring current, provided that the IMF Bz component has been southward for a while before the onset of the pressure enhancements. In this regard, anomalies in the dynamical behavior of P_{dyn} can be crucial for the development of intense magnetic storms. Taken together, the recurrence analysis of all four variables together potentially allows us to differentiate the level of magnetospheric variability due to solar wind forcing alone from the level of magnetospheric variability imposed both from external driving and internal magnetospheric processes. By means of recurrence analysis, we resolve subtle aspects of magnetospheric dynamics resulting from potentially nonlinear interactions between subsystems (Tsurutani et al., 1990; Valdivia et al., 1996), which have been hidden to previously employed methods. The properties explored here have not yet been captured by any other linear or nonlinear method of time series analysis previously applied for studying these four specific variables.

This paper is structured as follows: in section 2, we describe in detail the observational data used in this study, while the recurrence analysis methods employed to trace different aspects of the time-dependent dynamical complexity of the coupled solar wind-magnetosphere system are discussed in section 3. The resulting mean dynamical complexity and its associated temporal variability as revealed by different recurrence-based measures for the Dst index and solar wind parameters are detailed in section 4, while implications of our results are addressed in section 5. A summary of our main findings is provided in the concluding section 6.

### 2. Description of the Data

In this work, we study the dynamical complexity variations of the hourly Dst index together with three key variables of the solar wind: the VB_South component of the electric field, the Bz magnetic field component, and the dynamic pressure P_{dyn}. The Dst index measurements have been obtained from the World Data Center for Geomagnetism of the Kyoto University at http://wdc.kugi.kyoto-u.ac.jp/index.html, whereas the interplanetary data have been retrieved through the National Aeronautics and Space Administration space physics data facility OMNIWeb at http://omniweb.gsfc.nasa.gov/. While the corresponding analysis for the Dst index has already been largely provided and discussed elsewhere (Donner et al., 2018), we focus in this work on the intercomparison with the complexity variations of the different potential input variables associated with the solar wind. In this context, it is notable that there exist different versions of the Dst time series. While the one used by Donner...
Figure 1. Time series of four variables characterizing the states of the Earth’s magnetosphere and the solar wind: (a) Dst index, (b) VBSouth, (c) Bz, and (d) Pdyn. All data have hourly time resolution and cover the period from 1 January 2001 to 31 December 2001. Gray-shaded areas indicate two periods (Julian days 65–110 and 285–330 of the year 2001) characterized by intense magnetic storm activity, whereas the other intervals are considered as periods of relative magnetospheric quiescence. The horizontal line in the upper panel corresponds to a value of $\text{Dst} = -50 \text{nT}$, which is commonly considered as a threshold for defining a magnetic storm.

et al. (2018) and also throughout this paper has been recently reported to exhibit some bias (Love & Gannon, 2009), an alternative data set provided by the U.S. Geological Survey (https://geomag.usgs.gov/plots/dst.php) did not lead to markedly different results (for details, see the supporting information accompanying this paper, Figure S2). In the case of $P_{\text{dyn}}$ a few missing values in the considered time series have been filled by employing a gap filling procedure based on singular spectrum analysis (von Buttlar et al., 2014; Kondrashov & Ghil, 2006). This approach provides reasonable results as long as existing gaps in the data are sufficiently sparse and short.

As a period of interest, we focus on 1 year of observations during this solar activity maximum from 1 January 2001 to 31 December 2001 (corresponding results for another year of data — 2003 — can be found in the supporting information, Figures S3–S6). This period belongs to a prolonged activity maximum between 2000 and 2003 associated with the solar cycle 23 (May 1996 to January 2008), which was characterized by numerous strong solar eruptions followed by enhanced Earth’s magnetospheric activity. As it can be seen from Figure 1, the year 2001 has been of particular interest in the context of the present analysis, because it exhibited two unusually large CMEs on 29 March 2001 and 4 November 2001 that were followed by intense magnetic storms on 31 March 2001 and 6 November 2001, when the Dst index reached minimum values of $-387$ and $-292 \text{nT}$, respectively. These two magnetic storms marked the respective peaks of two time periods of intense magnetospheric disturbances that were separated by a phase of relatively low geomagnetic activity. Note that in general, a seasonal variation observed in the geomagnetic activity under extreme solar wind conditions during the solar cycle maximum can be explained by the Russell-McPherron effect (Russell & McPherron, 1973; Zhao & Zong, 2012): Geomagnetic activity is much more intense around the spring equinox (when the IMF is directed toward the Sun) and around the fall equinox (when the direction of IMF points away of the Sun) than during the rest of the year.
According to the overall mean state of the coupled solar wind-magnetosphere system during time intervals of the order of weeks, previous work has identified five distinct segments in these time series data based on the general geomagnetic activity level together with various associated dynamical characteristics (Balasis et al., 2006, 2008, 2009; Balasis, Daglis, Papadimitriou, et al., 2011; Donner & Balasis, 2013). The second and fourth segments (gray-shaded areas in Figure 1) have been characterized by enhanced solar and magnetospheric activity and include the two aforementioned intense magnetic storms of March and November 2001. The remaining three time intervals correspond to a rather quiescent Earth magnetosphere.

More specifically, the aforementioned studies—mostly focusing on the $Dst$ index alone—have shown that the variability of the Earth's magnetosphere during these periods of geomagnetic activity and quiescence was characterized by two distinct patterns of dynamical organization in the $Dst$ index: (i) periods with intense magnetic storms exhibit a markedly elevated degree of organization, representative of states of a “disturbed” magnetosphere and (ii) typical nonstorm periods, when the magnetosphere remained at “normal” states, with a lower degree of organization. It should be noted that this differentiation between distinct magnetospheric states has been based on the persistent versus antipersistent character of the $Dst$ index fluctuations (Balasis et al., 2006) rather than the actual hourly $Dst$ index values recorded within periods of different levels of geomagnetic activity.

3. Recurrence Analysis

The nonlinear time series analysis methods employed in this study make use of the concept of phase space, an abstract metric space in which each possible system state is represented as a unique point. For deterministic dynamical systems, the state vectors at each point in time are fully determined by a minimal set of variables that govern the system's dynamical equations of motion (which are commonly not explicitly known).

3.1. Phase Space Reconstruction by Time Delay Embedding

In the case of univariate time series \{x(t)\}$_{i=1}^{N}$—presumably originating from a dissipative and (at least partially) deterministic dynamical system, as implicitly considered in the present work—only one possible coordinate of the phase space is known explicitly. In such a case, phase space reconstruction by means of time delay embedding (Mañé, 1981; Packard et al., 1980; Takens, 1981) provides a widely applicable approach to qualitatively estimate the action of unobserved variables. A multivariate representation $X(t)$ in a new space, known as the reconstructed phase space or embedding space, is obtained from time-shifted replications of the original data:

$$X(t) = (x(t), x(t - \tau), x(t - 2\tau), \ldots, x(t - (m - 1)\tau)), \quad (1)$$

where the different coordinates of the reconstructed state vector $X(t)$ need to be sufficiently independent of each other (e.g., linearly decorrelated). Accordingly, in order to properly select the embedding delay $\tau$, estimates of the decorrelation time can be used. As it is discussed in full detail in the supporting information (Text S1 and Figure S1), in what follows we will use $\tau = 100$ hr for $Dst$ (consistent with the results of Donner et al., 2018) and $\tau = 24$ hr for the other three variables. Note that durations of severe magnetic storms or corresponding anomalies in the solar wind can markedly exceed the selected embedding delay, which can make the different components of the embedding vector \(1\) become insufficiently decorrelated. However, since such events are exceptional and “normal” (quickly decaying) dynamics is rather the rule, the possible error made by this setting is within tolerable limits. In general, as it is further discussed in the supporting information Text S1, we argue that the corresponding choices do not contradict results of other studies highlighting the presence of different characteristic timescales of relevant correlations in different variables or associated with different overall conditions (e.g., Borovsky, 2012; Johnson & Wing, 2005; Mourenas et al., 2018; Owens et al., 2017).

For the embedding dimension $m$, we use $m = 3$ for all four variables as a trade-off between the possibly larger dimensionality of the observed fluctuations and the increased requirements for the length of the time series that needs to be considered when operating in higher embedding dimensions. Note that the latter are incompatible with the demand for the highest possible temporal resolution when studying dynamical complexity within running windows in time (see below for details).

3.2. Recurrence Plots

Recurrence plots allow us to visualize the timing of observations of dynamically similar states of a system based on their mutual closeness in phase space (Eckmann et al., 1987; Marwan et al., 2007). In the case of the
coupled solar wind-magnetosphere system, we use the embedded time series \( \{X(t_i)\}_{i=1}^N \) as described above to define a binary recurrence matrix as

\[
R^t_j(\varepsilon) = \Theta(\varepsilon - \|X(t_i) - X(t_j)\|),
\]

(2)

where \( \Theta(\cdot) \) denotes the Heaviside function, \( \varepsilon \) the threshold distance used to define whether or not two embedded state vectors are close to each other, and \( \|\cdot\| \) some norm in the phase space. In our case, we use the maximum norm, also known as \( L_\infty \) or Chebyshev norm, which is defined as \( \|D\|_{\infty} = \max_k D^{k} \) with \( D^{k} \) being the \( k \)th component of the vector \( D \). The graphical visualization of this matrix is known as the recurrence plot.

Different from the three solar wind-related variables, the Dst index is provided in discretized form, that is, it takes only integer values, which would cause problems when evaluating the recurrence matrix with \( \varepsilon \) chosen such that a dedicated recurrence rate \( RR \) (i.e., the fraction of pairs of state vectors that are considered to be mutually close) is obtained. Specifically, since Dst is discrete, all possible distances between the three-dimensional embedding vectors constructed from Dst will be discrete, too. As a result, there may be a finite range of values of \( \varepsilon \), say, \( [\varepsilon_c, \varepsilon_+], \) resulting in a constant recurrence rate \( RR_c \) that is smaller than the desired value \( RR^* \), while there may be another finite range \( [\varepsilon_c, \varepsilon_-] \) for which the observed recurrence rate \( RR_c \) is larger than \( RR^* \). In other words, as we vary \( \varepsilon \) gradually, the recurrence rate \( RR \) commonly changes discontinuously and possibly never takes the desired value. In order to avoid this effect, we add artificial Gaussian white noise with a standard deviation of \( 10^{-5} \) times the respective standard deviation to all considered time series, which makes Dst a continuous variable. Note that this addition of noise is not intended to represent the effect of the original discretization of Dst and would also work for other symmetric distributions of the noise with zero mean. Different realizations of this noise have been found to have only negligible effects on the results described in the remainder of this paper (not shown). Note that the actual variance of the artificial noise will not affect the obtained results as long as it is small compared to the intrinsic discretization step of Dst.

In the following, we will exclusively consider the case of \( RR = 0.05 \), which has been found to be a reasonable value for recurrence analysis in many examples of geoscientific time series (Donges, Donner, Trauth, et al., 2011; Donges, Heitzig, et al., 2015).

We emphasize that recurrence plots have already been used in the context of geomagnetic activity indices as well as related observables by (Dendy & Chapman, 2006; Ponyavin, 2004; March et al., 2005a; Oludehinwa et al., 2018; Unnikrishnan, 2010), however, mostly for visualization purposes. March et al. (2005b) used two index time series as examples to illustrate how to infer time-localized information on the mutual information from time series. Recently, Mendes et al. (2017) presented a first study aiming at obtaining quantitative information on high-intensity and long-duration continuous auroral activity from recurrence plots. In turn, regarding different solar activity indicators, a number of studies have utilized recurrence plots to characterize the underlying nonlinear dynamics (see Donner, 2008; Stangalini et al., 2017, and references therein).

### 3.3. RQA and RNA

Beyond simple visual inspection of the recurrence matrix with the associated recurrence plots, a multitude of quantitative measures based on the pattern of recurrences can be used to reveal different aspects of a system’s underlying dynamical complexity. In this study, we employ three selected measures that have been found particularly suitable for this purpose when they were applied to time series from different fields of sciences (see Donner et al., 2018, for further details on their applications to the Dst index):

1. In a recurrence plot, noninterrupted diagonal line structures formed by recurrent pairs of state vectors indicate that similar states tend to evolve similarly over a certain period of time. This property is captured by the mean diagonal line length that can also be interpreted as a measure of predictability (Zbilut & Webber, 1992; Webber Jr. & Zbilut, 1994). In the case of white noise or other short-term correlated stochastic processes, diagonals only occur by chance and are most commonly short in length (Marwan et al., 2007). To distinguish between deterministic and stochastic dynamics, one can thus consider the so-called degree of determinism

\[
DET = \frac{\sum_{d=d_{\text{min}}}^{d_{\text{max}}} d p(d)}{\sum_{d=1}^{d_{\text{max}}} d p(d)}
\]

(3)

as one of the most standard RQA measures, where \( d \) denotes the length of a diagonal line, \( p(d) \) is the associated probability density function, \( d_{\text{max}} \) is the length of the longest diagonal (except for the main diagonal...
in the plot), and \( d_{\text{min}} \geq 2 \) (we use \( d_{\text{min}} = 2 \) in this study to cover also cases where the maximum line length is relatively small). \( \text{DET} \) gives the fraction of recurrences confined in diagonal structures and as such provides a heuristic measure that takes values close to one in the case of deterministic (predictable) dynamics but lower values for stochastic (less predictable) behavior. However, note that values of \( \text{DET} \) alone do not allow for identifying a possibly deterministic nature of a signal.

2. In a similar way as diagonal line structures, noninterrupted vertical line structures formed by recurrent state pairs in a recurrence plot indicate that a system’s state changes slowly with time (Marwan et al., 2002). With \( p(v) \) being the probability density function of the vertical line length \( v \), one convenient measure to quantify this aspect is the trapping time

\[
TT = \frac{\sum_{v = v_{\text{min}}}^{v_{\text{max}}} v p(v)}{\sum_{v = v_{\text{min}}}^{v_{\text{max}}} p(v)}.
\]

Low \( TT \) values generally indicate fast changes of the system’s state, whereas high values correspond to slow changes. Unlike \( \text{DET} \), \( TT \) is not normalized and can take any nonnegative value between 0 (when there are no vertical structures in the recurrence plot) and the length \( N \) of the considered time series (constant time series). Consistent with \( \text{DET} \), we will consider a minimum line length of \( v_{\text{min}} = 2 \) in this study. In general, the use of (diagonal and vertical) line-based recurrence measures is referred to as RQA (Marwan et al., 2007; Webber Jr. & Marwan, 2014).

3. Making use of the formal equivalence between the binary recurrence matrix \( R_{ij}(\varepsilon) \) and the adjacency matrix \( A_{ij}(\varepsilon) = R_{ij}(\varepsilon) - \delta_{ij} \) (with \( \delta_{ij} \) being the Kronecker delta) of an undirected and unweighted network, it is possible to exploit the toolbox of complex network analysis to characterize different geometric properties of the system’s organization in phase space (Donner, Small, et al., 2011; Donner et al., 2010; Marwan et al., 2009). In recent applications of RQA to artificial as well as real-world time series from various fields (Donges, Donner, et al., 2015; Donges, Donner, Rehfeld, et al., 2011; Donges, Donner, Trauth, et al., 2011; Marwan et al., 2009; Zou et al., 2010), it has been found that the recurrence network transitivity

\[
T = \frac{\sum_{i,j,k} A_{ij} A_{ik} A_{jk}}{\sum_{i,j,k} A_{ij} A_{ik}}
\]

provides a particularly useful measure for discriminating between qualitatively different types of dynamics. Specifically, this measure is closely related with the effective degrees of freedom of the system’s dynamics and can be used to obtain an easily calculable generalization of a fractal dimension (Donner et al., 2011; Donges et al., 2012). Specifically, high (low) transitivity values indicate a low (high) dimensionality of the observed dynamics.

In addition to the aforementioned three recurrence measures, both RQA and RNA provide a variety of further characteristics that can be used for tracing different aspects of the dynamical complexity of the system under study. Here we focus on just these three specific characteristics that have been demonstrated to be particularly useful in previous applications to different geoscientific time series (Donges, Donner, Rehfeld, et al., 2011; Donges, Donner, Trauth, et al., 2011).

### 3.4. Sliding Windows Analysis

In order to trace temporal changes in the dynamical complexity of the Earth’s magnetosphere coupled to the solar wind, we do not solely attempt a global characterization of the system’s state but consider sliding windows in time. For this purpose, we construct recurrence plots of all four variables of interest with a fixed recurrence rate \( RR = 0.05 \) and windows with a width of \( w = 168 \) hr (7 days, covering the typical durations of geospace storms) and a mutual offset of \( \Delta w = 1 \) hr. By conserving the recurrence rate \( RR = 0.05 \), we ensure that the results obtained for all four variables and three recurrence measures are quantitatively comparable over time.

As a reference time coordinate, we choose the second embedding coordinate, implying that information from past and future conditions is considered in a balanced way in the different recurrence characteristics (Donner et al., 2018). While such an approach is not particularly suited to the search for possible precursory structures in solar wind parameters and geomagnetic activity indices related to the initiation phase of magnetic storms, it allows the best possible differentiation between the dynamical characteristics of storm and...
Figure 2. Recurrence plots for the time series of (A) the geomagnetic activity index $D_{st}$ and the solar wind parameters (B) $V_{B\text{north}}$, (C) $B_z$, and (D) $P_{\text{dyn}}$ (see also Figure 1), obtained with a global recurrence rate of $RR = 0.05$. Gray shaded areas indicate the two time intervals considered as storm periods, whereas the time intervals in between are considered as epochs of quiescence (see section 2 for details).

nonstorm periods (Donner et al., 2018). It should also be noted that due to the finite embedding delay $\tau$, all running windows effectively include information from time intervals $[t_c - w/2 - \tau, t_c + w/2 + \tau]$, with $t_c$ denoting the window midpoint. In other words, the effective time window of data utilized by our analysis has a width of $w + 2\tau$, which is considerably larger than $w$, especially in the case of the $D_{st}$ index ($\tau = 100$ hr). This detail needs to be kept in mind when interpreting the results of our recurrence analysis, as well as when employing the proposed analysis approach to identify dynamical structures in the solar wind corresponding to possible precursory phenomena of magnetic storms. Specifically, considering such relatively large window sizes is necessary to obtain reliable statistics but may at the same time lead to a mixing of information from quite different dynamical regimes, since they clearly exceed the common duration of individual magnetic storms.

The aforementioned considerations imply that we employ sliding windows over the three-dimensional state vectors of the reconstructed phase spaces of all four variables of interest instead of the original univariate time series. Thereby, we explicitly disregard the time dependence of the temporal correlation structure of the variables of interest during different overall conditions of the system (which are particularly well known for the $D_{st}$ index; Balasis et al., 2006; Donner & Balasis, 2013) that might otherwise require using different embedding parameters $(\tau, m)$ for different time windows. In turn, choosing the latter parameters adaptively for each window would, on the one hand, require much larger windows than the chosen width $w$ and, on the other hand, hamper the comparability of dynamical characteristics obtained for different windows due to the different timescales covered by the respective embeddings.

4. Results
4.1. Recurrence Plots of All Variables
Figure 2 shows the recurrence plots for the full 1-year time series of the $D_{st}$ index and the three solar wind parameters analyzed in this work. We find that the global patterns exhibited by these plots markedly differ among the four studied variables. Notably, there is no clear distinction between storm and nonstorm periods.
as one could have expected since the magnitude range of fluctuations of the Dst index, but also the three solar wind variables, is much larger during storm periods, so that such high-amplitude variations should not find many recurrences within the considered time series interval. The reason why this does not have any remarkable effect here is the relatively short duration of individual storms and solar wind disruptions, respectively, as compared to the time interval covered by each individual state vector after time delay embedding (see above). Accordingly, even during storm periods, we have only few embedded data points that differ markedly from the distribution observed for periods of magnetospheric quiescence.

Regarding the qualitatively different structures in the obtained recurrence plots, we recall that the magnetosphere acts as a nonlinear filter to the temporal variations in the solar wind forcing, which can lead to similar magnetospheric dynamics during storm periods (as illustrated by similar Dst values during the two considered time windows with intense magnetic storms) even though the dynamical characteristics of the input variables (solar wind) can be distinctively different, as suggested by the recurrence plot for VB_South. Even more, the temporal recurrence patterns of solar wind variables (input) and Dst index (output) can be qualitatively different (see below). This finding is compatible with the hypothesis that during phases of an externally perturbed magnetosphere, additional internal processes are triggered and take place in the magnetosphere that lead to different dynamical complexity levels in input and output variables.

The qualitatively different appearance of the recurrence plots of the three considered quantities could serve as a starting point for additional quantitative characterizations of these global recurrence plots, including the distribution of durations of laminar phases or the properties of curved line segments, which resemble recurrence patterns known from Brownian motion. To this end, we leave corresponding in-depth analyses as a subject of future work while resorting next to the time-dependent recurrence properties obtained from sliding windows in time.

4.2. Time Dependence of Recurrence Characteristics

The results of our recurrence analysis for sliding windows in time are presented in Figure 3. We clearly recognize that as expected from the qualitative recurrence plot characterization above, all three recurrence measures (DET, TT, and T) exhibit marked variations with time, which are interpreted as changes in the dynamical complexity of different components of the coupled solar wind-magnetosphere system. Since both the solar wind and the magnetosphere exhibit strong fluctuations on a wide variety of timescales, there is no distinct baseline state for neither of the recurrence measures considered. Instead, they display variations on different timescales. In the following, we will discuss in some more detail (i) what information can be obtained from the typical ranges that the values of each recurrence measure take during the specific 1 year of observations and (ii) how the respective recurrence characteristics differ between storm and quiescence periods as well as between the two studied storm periods.

4.3. Typical Complexity Levels for All Variables

As a first step toward a more detailed interpretation of our recurrence analysis results, we compare the ranges of values that all three considered recurrence measures take for the four variables of interest. Here we aim to characterize the typical complexity levels of the latter from different perspectives.

Regarding DET, Figure 3 demonstrates that for the Dst index, the estimated values range from about 0.53 to 0.91 and maxima appear during storm periods. These relatively high values indicate a moderate level of statistical predictability of the Dst index variations. In other words, if the temporal evolution of Dst index fluctuations in the past is known, it is possible to anticipate at least short-term trends in future variations for similar starting conditions—a feature that is expressed in terms of diagonal line structures in the recurrence plot. Specifically, long diagonal line structures indicate that two segments of the (embedded) time series behave dynamically similar over a certain period of time (Marwan et al., 2007), which is essentially the definition of statistical predictability and the foundation of model-free forecasting based on dynamical analogs. Notably, the longer the diagonal lines, the larger the corresponding prediction horizon. Similar DET values are observed for P_dyn (with however much larger variance), whereas the two other solar wind parameters VB_South and B_z exhibit distinctively lower values with the calculated mean over all windows being approximately 0.3. This last observation suggests that (irregular) high-frequency variability is more pronounced in these two solar wind variables than in P_dyn and the Dst index. This finding is consistent with the fact that the power spectral scaling exponent for the Dst index can take values both above 2 (during storm periods, indicating persistent behavior) and below 2 (during periods of quiescence, indicating antipersistent dynamics), whereas it has been found to
Figure 3. Temporal variations of the four variables of interest along with the recurrence characteristics $DET$, $TT$, and $T$ (from top to bottom) calculated over sliding windows with a width $w$ of 168 hr (1 week). (left to right) Geomagnetic activity index $Dst$ and solar wind parameters $VB_{South}$, $B_z$, and $P_{dyn}$. Gray-shaded areas indicate the two time intervals considered as storm periods, whereas the time intervals in between are considered as epochs of quiescence (see section 2 for details).

always stay below 2 for $VB_{South}$ even during periods of intense geomagnetic activity triggered by enhanced solar activity (Balasis et al., 2006).

The trapping time $TT$ of the $Dst$ index variations takes a wide range of window-wise values between 3.0 and 6.8, with maximum values clearly coinciding with storm periods. For $P_{dyn}$, the obtained values of $TT$ cover an even slightly larger range. For the two other solar wind variables ($VB_{South}$ and $B_z$), both the minimum and maximum values of $TT$ are clearly smaller than for $Dst$ and $P_{dyn}$, indicating again more (irregular) high-frequency variability and the absence of time periods during which the respective observable varies only weakly. This general distinction between the $TT$ ranges for the $Dst$ index and $P_{dyn}$ versus $VB_{South}$ and $B_z$ is in agreement with the findings reported above for $DET$, indicating that the observed differences in the two RQA measures reflect dissimilar short-term fluctuations of the individual observables.

Finally, $T$ provides a measure for the dimensionality of the time series (Donner et al., 2011; i.e., the redundancies among components of the embedding vectors). Notably, for $VB_{South}$, we observe numerous time intervals with $T$ approaching this measure’s limit value of 1 (indicating a zero-dimensional object in the underlying dynamical system’s phase space, i.e., a fixed point). Because of this very specific behavior, we will not use $T$ for further interpretation of the $VB_{South}$ records. In turn, the variability of $T$ for $Dst$ and the two other solar wind variables exhibits better interpretable features. Specifically, the $Dst$ index and $P_{dyn}$ have larger average values of $T$ than $B_z$, indicating again the presence of lower-dimensional dynamical structures, while $B_z$ appears to be “more stochastic.” In general, $P_{dyn}$ shows the largest overall values of $T$ (with the exception of $VB_{South}$ with its distinct behavior as described above) during a few distinct time intervals corresponding to both storm and nonstorm periods. We observe that these periods correspond to situations where $P_{dyn}$ drops and remains at relatively low values for a certain period of time (see also Figure 1). This is consistent with the temporal profile of $T$ previously observed for other geoscientific data sets (Donges, Donner, Trauth, et al., 2011) of fluctuations in comparison with the “typical” values.

4.4. Differences Between Storm and Nonstorm Periods

For the $Dst$ index, the three considered recurrence measures clearly differentiate between storm and nonstorm periods. In particular, $DET$, $TT$, and $T$ reach higher values during periods of increased geomagnetic
activity than in periods of quiescence. This finding is in agreement with the general observation that during periods of enhanced geomagnetic activity, the magnetosphere exhibits a larger degree of dynamical organization, which is also reflected in longer timescales of variability becoming more relevant (e.g., the initiation and recovery phases for sequences of magnetic storms). As mentioned above, it has been demonstrated previously that this behavior is expressed in terms of persistent dynamics, reduced dynamical disorder characterized by lower values of several entropy measures, and stronger autocorrelations (Balasis et al., 2006, 2008, 2009, 2013; Balasis, Daglis, Papadimitriou, et al., 2011; Balasis, Daglis, Anastasiadis, et al., 2011; Donner & Balasis, 2013). Mourenas et al. (2018) recently reported a stronger organization of the magnetosphere and persistent cumulative effects over days during extreme time-integrated Dst events, which appears compatible with the observations of this as well as the aforementioned studies. In particular, Figure 4 (first row) demonstrates that the distributions of values of DET, TT, and T differ from each other during storm and nonstorm periods. As can be inferred from Figure 3 (first column), this difference visually appears most pronounced for TT. Note that while the difference in the distribution of recurrence characteristics is in most cases apparent from visual inspection, appropriate statistical testing against the null hypothesis of identical distributions (e.g., using Kolmogorov-Smirnov or similar statistics) is not straightforward since due to the strong overlap between successive time windows, the different values exhibit strong serial correlations and, thus, violate the common independence condition.
In turn, regarding the three solar wind observables, we find no similarly clear difference between the recurrence characteristics observed during storm and nonstorm periods (Figure 4, second to fourth rows). In general, the three considered measures cover similar ranges of values during periods of increased geomagnetic activity and quiescence; however, all three recurrence characteristics for $B_z$ differ in the maximum values reached during storm periods. A similar conclusion cannot be drawn for $V_{B,\text{South}}$ and $P_{\text{dyn}}$, where we observe an absence of marked differences in the recurrence characteristics during storm versus quiescence phases of magnetospheric dynamics. This points to a nonlinear and highly context-specific response of the magnetosphere to temporal changes in the dynamical characteristics of the solar wind variables, where similar dynamical complexity in the interplanetary medium may correspond to different overall conditions resulting in different levels of magnetospheric activity. For instance, the interplanetary driving through southward-oriented magnetic fields is not always by itself sufficient to drive magnetic storms, because it is subject to modulation by internal magnetospheric conditions (Daglis et al., 2003).

Specifically, a fast moving magnetic cloud with organized internal magnetic field is expected to cause an intense magnetic storm if it engulfs Earth, but this condition is not sufficient. A magnetic cloud (or, likewise, a CME triggering a magnetic storm) should have a magnetic field with a significantly negative $B_z$ component for reconnection to occur at the dayside magnetopause. Vörös, Jankovičová, et al. (2005) have shown that, in addition to the geoeffective southward component of IMF, intermittency, small-scale rapid changes, singularities and non-Gaussian statistics of IMF fluctuations play an important role in the solar wind-magnetosphere interaction. Figure 3 shows that for $B_z$, DET and $T$ exhibit a generally similar time evolution as the corresponding recurrence characteristics for the $Dst$ index (but at a clearly lower level of the respective measures). This underlines that geomagnetic fluctuations at the typical timescales of geospace storms result from the simultaneous action of multiple magnetospheric processes, including the component represented by intermittent solar wind fluctuations.

Generally, CMEs behave as nonlinearly interacting dynamical systems characterized by the presence of large-scale coherent structures of different sizes that decrease the degree of multifractality in the IMF’s $B_z$ component because they privilege only a few scales (Bolzan & Rosa, 2012). After the passage of a CME, an increase in the complexity is necessary to promote the dissipation of energy. However, some CMEs, like the first event in March 2001 studied in this work, are characterized by slow solar wind (approximately 550 km/s), which is more intermittent than fast solar wind and contributes to the intermittency of the interplanetary medium (Bruno et al., 2003). This is reflected in the lower maximum values of the three recurrence measures during this specific time period as compared to the second event in November 2001. However, even in the case of fast solar wind, intermittency increases with the heliocentric distance until it reaches the Earth’s orbit, and IMF fluctuations tend to be more intermittent than velocity fluctuations (Bruno et al., 2003).

In turn, fluctuations of the interplanetary electric field as reflected by $V_{B,\text{South}}$ are considered responsible for initiating magnetospheric substorms, while its quasi-steady component plays the central role in the enhancement of the ring current that is monitored by the $Dst$ index (Kamide, 2001). This is also corroborated by previous findings of Balasis et al. (2006), who have observed this behavior in geomagnetic time series where the degree of dynamical complexity of the $Dst$ index is reduced in the presence of powerful oscillations. The lack of similarity between the recurrence characteristics of $V_{B,\text{South}}$ and the $Dst$ index in Figure 3 also suggests that only a part of the $Dst$ index variation can be explained as a direct response to $V_{B,\text{South}}$. In the past, Price and Prichard (1993) focused on an interval when the IMF had a nearly constant $B_z$ component to be able to find some evidence for a deterministic nonlinear coupling between the solar wind forcing, expressed by the $V_{B,\text{South}}$ electric field component, and the terrestrial magnetosphere. Besides the IMF and electric fields, a whole set of variables, including solar wind velocity, density, $P_{\text{dyn}}$, and plasma $\beta$, affect to different degrees the energy input to the magnetosphere. This is reflected by the recurrence-based characteristics of the selected solar wind parameters considered in this study.

### 4.5. Differences between the Two Storm Periods

The magnetic storms of 31 March and 6 November 2001 were among the 11 superintense storms ($Dst \leq -250$ nT) that occurred during solar cycle 23 (Echer et al., 2008). The first event was caused by a combination of sheath and magnetic cloud fields, while the second one by sheath fields alone (Echer et al., 2008). This is reflected in the $TT$ values of the solar wind variables and in the response of the magnetosphere as expressed by the $Dst$ index.
In the gray-shaded area of Figure 3 that is related to the second magnetic storm period, $TT$ exhibits a pronounced maximum for both solar wind parameters $B_z$ and $P_{\text{dyn}}$, while there are no peaks of similar magnitude during the first magnetic storm period. Specifically, the peak values of all three recurrence measures for $P_{\text{dyn}}$ are observed between days 285 and 332.5 of the year 2001, and their timing of occurrence corresponds to the onset phase of the 6 November 2001 magnetic storm. This finding is in accordance with Wang et al. (2003), who argued that the ring current injection increases when the magnetosphere is compressed by a particularly strong solar wind forcing and that the injection rate is proportional to $P_{\text{dyn}}$. We note that the $Dst$ variations of the 6 November 2001 magnetic storm were also found to obey a power law with log-periodic oscillations (Balasis, Papadimitriou, et al., 2011), which is a sign for the emergence of discrete scale invariance in the magnetosphere.

One possible interpretation of our results on the values of $TT$ is as follows. During the first storm phase, $VB_{\text{South}}$ and $B_z$ exhibit faster variations (lower $TT$) than $Dst$ and $P_{\text{dyn}}$ that both show comparable maximum values (around 6) reflecting similar timescales at which the two variables change. During the second storm phase, we find a general tendency toward higher $TT$ values than during the first storm phase (and, hence, slower changes) for all three solar wind variables. In order to explain this observation, we suggest that in the first case, the magnetosphere (and in particular the ring current) has absorbed or screened the faster changes of the two solar wind variables $VB_{\text{South}}$ and $B_z$ and follows more closely the (somewhat slower) variations of $P_{\text{dyn}}$. This situation is compatible with the scenario proposed by Wang et al. (2003), who argued that the ring current injection increases when the magnetosphere is more compressed by a particularly strong solar wind forcing and that the injection rate is proportional to $P_{\text{dyn}}$. In contrast, during the second storm phase, the ring current seems to have more closely followed the variations in $VB_{\text{South}}$ and $B_z$.

5. Discussion

Based on the results described in the previous section, we suggest that recurrence-based complexity measures have a great potential to trace temporal variations in the dynamical complexity of geomagnetic and solar wind dynamics but also other nonstationary geophysical observables. In particular, the dynamical complexity profile of magnetospheric fluctuations during storm and nonstorm conditions (which we have studied in terms of $DET$, $TT$, and $T$, capturing the regularity of fluctuations in the $Dst$ index from different perspectives) is in good agreement with the existing body of literature on this subject (cf. Balasis et al., 2009), as will be discussed in the following.

Consolini et al. (2008) investigated long-term variations in the dynamical state of the Earth’s magnetosphere in terms of the $Dst$ index. Their results clearly demonstrated the nonequilibrium nature of magnetospheric dynamics. Specifically, the Earth’s magnetosphere behaves like expected for a system far from equilibrium due to the continuous interaction with the time-dependent solar wind forcing. The presence of two different dynamical regimes—near and away from a nonequilibrium stationary state—has been independently confirmed by other studies. Specifically, Sitnov et al. (2001) provided evidence that substorms exhibit dynamical characteristics that are typical for phase transitions. This picture is consistent with the findings of Balasis et al. (2006) who reported the transition from antipersistent to persistent behavior when an intense magnetic storm is imminent. Moreover, the metastability and topological complexity of the geomagnetic variations established with the model of Chang (1999) are in good agreement with the transitions from the observed prestorm activity to magnetic storms that have been found in our study. Chang et al. (2003, 2004) and Vörös, Baumjohann, et al. (2005) provided indications for the presence of intermittent turbulence in space plasmas, which further supports our results. Furthermore, the statistical properties of magnetic fluctuations in the Venusian magnetosphere determined by Vörös et al. (2008) point to multiscale turbulence at the magnetosheath boundary layer and near the quasi-parallel bow shock.

In addition, a reduction of multiscale complexity was observed in the geomagnetic activity at high latitudes before strong substorms. With the use of cellular automata models, Uritsky and Podvokin (1998) and Uritsky et al. (2001) demonstrated transitions between subcritical, critical, and supercritical states. A similar behavior was found in the spatial scaling of the auroral brightness (Uritsky et al., 2006; Uritsky et al., 2008). The multiscale complexity of geomagnetic substorms was explored in a series of studies (Chang, 1992; Consolini, 1997; Chapman et al., 1998; Klimas et al., 2000; Lui et al., 2000), while the first discussions on the critical nature (self-organized criticality) of geomagnetic storms were initiated by Consolini (1997), Uritsky and Podvokin (1998), and Chapman et al. (1998). Wanliss et al. (2004) and Wanliss and Uritsky (2010) reported evidence for intermittency...
and non-Gaussianity associated with large magnetic storms using symbolic dynamics analysis of the $Dst$ time series. These findings strengthen the hypothesis of a constantly out-of-equilibrium ring current that undergoes state changes in terms of multiplicative cascades. In general, the results obtained in this study further support the existence of two distinct dynamical regimes of the magnetospheric variability corresponding to storm and nonstorm conditions.

Attempts to determine the nonlinear properties of the magnetospheric system from just its response — no matter how robust and complete the diagnostic means are — would not be meaningful without considering the solar wind forcing. Following the observed power law behavior of the $AE$ index and the southward component of the IMF, the multiscale properties of magnetospheric dynamics have been interpreted in terms of intermittent turbulence and self-organized criticality (Consolini, 1997; Consolini et al., 1996; Chang, 1999). In the seminal paper by Tsurutani et al. (1990), the magnetosphere was considered as an input-output system to find a typical timescale of 5 hr allowing to disentangle the internal fast and bursty dynamics of the magnetosphere from the directly driven one. In this study, we analyzed the response of the magnetosphere as expressed by the $Dst$ index, while the solar wind input has been represented by the IMF’s $B_z$ and the dawn-dusk component of the electric field $VB_{South}$ together with $P_{dyn}$ to provide evidence that (at least) a part of the $Dst$ index variations can be explained as a direct nonlinear response of the magnetosphere to the solar wind.

Several previous studies focused on the overall characteristics of fractal properties of (a) longer $Dst$ time series (e.g., 20 years of $Dst$ data in Wei et al., 2004) or (b) solar wind data from 1 solar maximum year and 1 solar minimum year (Hnat et al., 2007). In particular, Hnat et al. (2007) applied a novel sensitive discriminator of fractality to the magnetic energy density time series from WIND and ACE, showing that at a year of solar maximum activity (2000), the time series is found to be fractal, whereas there is a weak but clearly discernible departure from fractality at solar minimum activity (1996). The difference with respect to the current study is that while we have largely focused on specific intense magnetic storm events that occurred within a year close to solar maximum (2001), we have analyzed both solar wind and $Dst$ data. However, the interpretation of our recurrence network transitivity as a generalized fractal dimension (Donner et al., 2011) may permit further comparison between the respective results when operating with the same data and analysis protocol. In this regard, we emphasize that Wei et al. (2004) had studied the exponent of the power spectral density, which actually provides a measure of long-range dependence, which is often (but not necessarily always) closely associated with a fractal scaling. More generally, it is important to note that fractal scaling and dynamical complexity are commonly closely interrelated aspects of nonlinear dynamical systems. While we have focused solely on the latter aspect as quantified by recurrence characteristics, combining these different aspects may provide additional information that could be helpful for integration into future space weather applications. A systematic assessment of the fractal or multifractal properties of is, however, beyond the topical focus of the present work.

The consideration of multiple measures from RQA and RNA — all based on the same underlying structure of the corresponding recurrence plots but generally characterizing different aspects of dynamical complexity — allowed us to distinguish periods of magnetic storms and quiescence based on the dynamical complexity of $Dst$ index and solar wind parameter fluctuations. The thus obtained results confirm previous findings that the dynamical characteristics of magnetospheric activity are fundamentally different between storm and nonstorm periods, while we have for the first time reported evidence that a corresponding distinction is not possible for the potential driving variables associated with characteristic observables of the solar wind variability. In this regard, we have found considerable small-scale differences between the recurrence plots of the $Dst$ index and the three solar wind parameters, IMF $B_z$, $VB_{South}$, and $P_{dyn}$. Moreover, the large-scale structure of the recurrence plots is distinctively different for $Dst$ and its potential drivers. In particular, the recurrence characteristics for the IMF $B_z$ exhibit rather distinct behaviors during the transition from periods of relative quiescence to magnetic storms, which can be explained by the nonlinear response of the terrestrial magnetosphere to the geoeffective southward component of the IMF as CMEs or magnetic clouds approach the Earth’s neighborhood.

In the past, March et al. (2005a) employed recurrence plots to visualize nonlinear correlations between the $AE$ index and $VB_{South}$ times series. While they also observed many differences between the obtained recurrence plots at small scales that were attributed to fast fluctuations, the overall large-scale structure displayed by both variables was qualitatively similar. It seemed that the shared structure on the recurrence plots covering periods of both high and low activities is the result of electric fields in the solar wind related to effects on
Earth observed on a timescale of the order of hours. Even though our analysis has not yielded any discernible change in the degree of determinism $\text{DET}$ during magnetic storms for $V_{\text{Bsouth}}$ time series, a multiple-measure approach promises to provide interesting information from the study of the Earth’s magnetic field variations, even below the general storm/quiescence variability. This perspective shall be further explored in future work, thereby extending this study to other geomagnetic activity indices employed for tracing magnetospheric phenomena like substorms.

Finally, we note that various methodologies have been applied in past studies to model the dynamics of the $\text{Dst}$ index (Boynton et al., 2011; Chandorkar et al., 2017; Valdivia et al., 1996; Vassiliadis et al., 1999). These attempts have been mostly originating from machine learning approaches and usually employed solar wind parameters and/or solar wind-magnetosphere coupling functions. These methods have been demonstrated to exhibit high performance with respect to their forecasting ability of magnetic storms. In turn, the recurrence measures applied in the present study clearly indicate different levels of stochasticity, complexity, and fractal dimensionality between the quiet-time and storm-time magnetosphere. In this context, their added value is that recurrences may be utilized to offer an alternative viewpoint on the space weather conditions and the state of the solar wind-magnetosphere system, thereby providing complementary information beyond the former well-established forecasting techniques. In our opinion, this is potentially relevant for future improved models, since our results underline that (i) $\text{Dst}$ variations during periods of magnetospheric activity and quiescence fundamentally differ from each other and (ii) solar wind input signals with different complexity signatures may lead to similar reactions of the magnetosphere. Taking these two observations together, it is not unlike that a single empirical (data-driven) model cannot optimally describe and forecast variability in both dynamical regimes, as well as corresponding transitions between both regimes. In turn, including a priori knowledge on the existence of such two distinct regimes in creating some state-dependent empirical dynamical model could help greatly improving the skills of contemporary models. We therefore outline further investigations along the aforementioned lines as a prospective field for future research.

6. Conclusions and Outlook

We have applied a set of three complementary recurrence-based measures to study the temporal changes in dynamical complexity exhibited by the $\text{Dst}$ index together with three characteristic variables of the solar wind during 1 year of observations near a solar maximum. Our results demonstrate that in the case of $\text{Dst}$, all three measures are able to trace variations associated with the time-dependent dynamical complexity of magnetospheric variability during a succession of storm and nonstorm periods. Nonetheless, the measures exhibit different degrees of sensitivity with respect to such changing conditions resulting from characteristic phenomena like CMEs and magnetic clouds imprinted in the solar wind’s dynamical properties. Specifically, the degree of determinism $\text{DET}$ takes relatively large values for the $\text{Dst}$ index, the IMF $B_z$, and the $V_{\text{Bsouth}}$ during storm time periods, but typically much lower values during periods of quiescence, indicating a more stochastic behavior. Low values of the trapping time $TT$ indicate fast changes of the system’s state, whereas high values observed in the $\text{Dst}$ index, the IMF $B_z$ and $P_{\text{dyn}}$ point toward slower changes, which are characteristic for a decrease in the complexity of the solar wind driver of magnetospheric disturbances. The recurrence network transitivity $T$ can be used to define an easily calculable generalization of a fractal dimension and reaches higher values (indicating lower dimensionality) for both the $\text{Dst}$ index and $B_z$.

Our results, together with other recent findings characterizing the multifractality of the interplanetary plasma, suggest that measures associated with recurrence plots provide valuable insights into the temporal structure of solar wind measurements with geomagnetic effects observed through the $\text{Dst}$ index. The recurrence-based complexity measures employed here serve as tools to detect characteristic dynamical structures embedded in the temporal variations of solar wind properties that may initiate intense magnetospheric events.

We emphasize that the quantitative differences in the considered measures applied to different variables are clearly affected by the spectral properties of the respective time series. Specifically, for signals with a strong high-frequency content and hardly any variability at lower frequencies, large-scale structures in the recurrence plot are unlikely to emerge, and this would be reflected by relatively low values of our complexity measures. In turn, signals with a low level of high-frequency variability are prone to highlight long-term changes, which would result in large-scale recurrence patterns and, consequently, a higher probability of elevated values of the three considered recurrence characteristics. Future work should address this aspect more explicitly, for
example, by considering sophisticated filtering of the selected observables to retain only the variations below a certain minimum frequency.

In this work, we have mostly focused on 1 year of continuous measurements of the variables of interest. In this context, we have chosen the year 2001 for various reasons. On the one hand, it includes two of the most intense magnetic storms of solar cycle 23 as well as several commonly observed (weaker) storm events along with time intervals of (lower) background geomagnetic activity. On the other hand, it presents a well-studied case using various time series analysis techniques and complexity measures (e.g., wavelet transforms, Hurst exponent, and entropies). The goal of our study has been to provide an initial attempt to demonstrate the applicability of recurrence measures to revealing various dynamical aspects (e.g., stochastic behavior and level of complexity) hidden in the variability of the coupled solar wind-magnetosphere system. Follow-up work will systematically expand the present study to one or more complete solar cycles in order to draw statistically more robust conclusions on the nonlinear variability of magnetospheric activity and the associated solar wind driver.

Finally, it is arguable that solar wind forcing and possible magnetospheric response are characterized by different temporal variability scales, which have not yet been accounted for in our present analysis (in particular, by keeping the embedding delay as a fundamental parameter of our analysis intentionally fixed at the same value during storm time and quiescence periods). To shed further light on the relevance of different scales, it might be advisable to desentangle variability at different scales (Alberti et al., 2017) and consider the dynamical complexity associated with fluctuations at individual timescales (as seen by recurrence properties or other nonlinear characteristics) independently. Even more, studying the possible transfer of information on the fluctuation properties across different scales might provide another fruitful subject of future studies. In this context, we note that the recurrence characteristics employed in the present work have not been selected to unveil any cause-effect relationships. For the latter purpose, there exist further sophisticated approaches based upon recurrence plots (e.g., based on conditional recurrence probabilities [Romano et al., 2007; Zou et al., 2011] or inter-system recurrence networks [Feldhoff et al., 2012] or related phase space-based techniques like convergent cross-mapping [Sugihara et al., 2012]. However, applying such approaches has been clearly beyond the scope of the present work. An alternative approach to this problem has recently been made in terms of causal inference methods based on information theory and graphical models (Runge et al., 2018).

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