Relative phase analyses of 10.7 cm solar radio flux with sunspot numbers

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1. Introduction

The Sun is a complex and dynamic body, releasing energy on all wavelength intervals of the electromagnetic spectrum. The changes in the output energy can be traced from many solar activities, including the sunspot numbers (SNs), 10.7 cm solar radio flux (F10.7), total solar irradiance (TSI), and so on (Kilcik et al., 2011a). The most frequently used to describe long-term solar variability has been the SNs, which have been proven invaluable in studies of terrestrial climate and space weather (Eddy, 1976; Hoyt and Schatten, 1998; Hathaway et al., 2002). Although SNs are highly correlated with other solar-activity indicators, they are not the best indicator of solar activity because they do not contain information on how magnetically energetic sunspot regions are (Kilcik et al., 2011b).

SNs are the weighted sum of the numbers of sunspots and sunspot groups on the solar disk at one time. This index is not the numbers of sunspot but a relative number including groups on a given day. Anyhow, they are convenient as an index of almost all other activities on the Sun including active regions, plages, flares, prominences, and to some extent changes in coronal form and in the solar wind (Greenkorn, 2009). F10.7 is the integrated emission from the whole solar disk at the radio wavelength of 10.7 cm (a frequency of 2800 MHz), it results from thermal ionization at the photosphere-chromosphere interface and from magnetic resonance above sunspots and plages (Tapping and DeTracey, 1990; Tapping and Charrois, 1994). It is a general indicator of the full-disk flux in Ca II and Mg II, solar ultraviolet (UV), extreme ultraviolet (EUV) and X-ray emissions, and even solar irradiance. Previous studies show that the origin of F10.7 is mainly due to thermal free-free process that is a widely-distributed emission. This index is more complex than SNs because it contains both active regions and plage-related contributions. If the magnetic-flux emergence is taken into account, the situation will become more complicated.
Although SNs are more frequently used than F10.7 in practice, the latter is considered to have more explicit physical significance than the former.

Various studies show a well-pronounced relationship between F10.7 and sunspot-related indicators, and F10.7 tends to lag behind sunspot activity by several months or even a few years in a hierarchical manner. Dodson et al. (1974) first noted that F10.7 lags behind SNs in solar cycle 20. Vitinsky et al. (1986) pointed out that the correlation between the two does not show a linear behavior during solar cycles 18, 19 and 20, this conclusion was confirmed by Bruevich and Yakunina (2011). Wilson et al. (1987) found that the maximum of F10.7 occurs about 1.5 years after the maximum of SNs in solar cycles 20 and 21, whereas there is no lag between them during solar cycles 18 and 19. They argued that F10.7 is sensitive to the magnetic complexity of active regions and the average magnetic complexity near solar maximum varies from cycle to cycle. Bachmann and White (1994) found that the former lags behind the latter and the phase difference between the two is about 20 days. de Toma et al. (2004) and Kilcik et al. (2011a) showed that F10.7 reaches its maximum two years later than SNs in solar cycle 23. However, Most of these studies on the phase relation between the two activity indicators are based on linear tools. It is well known that synchronization of the Sun is one key aspect for understanding the origin and evolution of active regions on the Sun and their various manifestations in the solar corona (Zolotova and Ponyavin, 2007), and traditional linear approaches are inappropriate for recognized complex nonlinear coupling between different solar activities. Fortunately, there are many advanced nonlinear tools that can be used to examine the relative phase relationship between two time series (Marwan et al., 2002; Marwan and Kurths, 2002, 2005), and they have been used in numerous scientific researches, such as geophysics, astronomy, and other research fields in the last several decades. (Zolotova and Ponyavin, 2006, 2007; Donner and Thiel, 2007; Chang, 2008; Li et al., 2008, 2009; Gao et al., 2011; Xie et al., 2012; Deng et al., 2012, 2013; Qu and Xie, 2013 and references therein).

An understanding of complexity of the phase relation between F10.7 and SNs may provide insight into the complexly dynamic behavior of the solar magnetic fields in the lower and the upper atmosphere of the Sun. The aim of this paper is to perform a detailed analysis to study their phase relationship by means of modern nonlinear techniques. The rest of this paper is organized as follows. Section 2 derives the data and methods employed in this paper. The results are revealed in Section 3. Finally, Section 4 gives the main conclusions and discussions.

2. Data and methods of analysis

2.1. Data

The data series analyzed in this paper are listed as follows:

1. The monthly counts of F10.7 are obtained from National Geophysical Data Center (NGDC) and can be downloaded from the web site (ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SOLAR_RADIO/FLUX). The data set covers the period from 1947 February to 2012 June. Measurements of F10.7 have been taken daily by the Canadian Solar Radio Monitoring Programme since 1946 and observations were made in the Ottawa area from 1946 to 1990. Another flux monitor was installed at Penticton, British Columbia in 1990 and run in parallel with the Ottawa monitor for six months before moving the Ottawa monitor itself to Penticton as a back-up.

2. The monthly numbers of SNs can be also publicly downloaded from the NGDC’s Web site (ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS). The data series is from 1874 May to 2012 June and is updated every month. We use the data from 1947 February to 2012 June, the common period to the F10.7.

Fig. 1 shows the monthly counts of these two time series and their corresponding 13-point smoothed values. From the figure one can see that the smoothed data of the two generally coincide during the ascending and descending phases of a solar cycle, but they slightly differ from during the minimum and maximum phases of a solar cycle. For example, F10.7 reaches its maximum and decrease later than SNs in solar cycles 20, 21 and 23. Thus, the two time series seem to synchronize at the ascending and descending phases of a cycle, while they are slightly asynchronous at the minimum and maximum.

2.2. Methods

In this study we use three methods to study the phase relationship between F10.7 and SNs, the first one is the cross-recurrence plot (CRP), line of synchronization (LOS) is the next, and the third one is the cross-wavelet transform (XWT).

2.2.1. Cross-recurrence plot

Recurrence Plot (RP), which was first proposed by Eckmann et al. (1987) and developed later by Zhilut et al. (1998), Marwan and Kurths (2002, 2005) and Groth (2005), is a tool for visualizing the recurrence of states of dynamical systems in phase space. Its extension, the so-called CRP, is a two-dimensional representation of high dimensional phase-space trajectories. The mathematical definition of the CRP is

\[
CR_{ij}^{Rm} = \Theta(\epsilon_i - ||x_i - y_j||),
\]

(1)

where \(x_i, y_j \in Rm\) \((i = 1, \ldots, N_x; j = 1, \ldots, N_y)\) with \(N_x\) and \(N_y\) the number of considered states \(x_i\) and \(y_j\), respectively, \(\epsilon_i\) is the threshold distance, \(|| \cdot \||\) is the norm (e.g., the Euclidian norm), and \(\Theta\) is the Heaviside function. Visualization of the CRP is a graphical pattern of the matrix \(N_x \times N_y\) and all elements \((CR_{ij})\) of which are either zero or one.

The CRP of the two corresponding time series will not contain a main diagonal, but if the two are similar, a fluctuated line in the CRP liking a distorted main diagonal can occur. This line is called

![Fig. 1. Monthly counts of F10.7 (top panel) and SNs (bottom panel) from 1947 February to 2012 June. The black and red curves represent the original and their 13-point smoothed values. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)](image-url)
the LOS. In the more common non-stationary case, an off-set of the LOS away from the main diagonal is an indication of a phase shift or a time delay between the two considered time series (for details, see Marwan and Kurths, 2002, 2005).

2.2.2. Cross-wavelet transform

Wavelet transform is good for detecting the localized and quasi-periodic fluctuations in time–frequency space (Torrence and Compo, 1998). One of its extensions, the XWT, can be used to examine the relative phase relationship between the two time series (Grinsted et al., 2004).

The XWT of the two time series \( X \) and \( Y \) is defined as:

\[
W_{XY} = W_X^* W_Y^*
\]  

(2)

where \( W_X^* (W_Y^*) \) is the complex wavelet transform of the time series \( X \) and \( Y \) and \( * \) denotes the complex conjugation. The complex argument \( \arg (W_{XY}) \) can be interpreted as a local relative phase between \( X \) and \( Y \) in time–frequency space. We employ the Morlet wavelet with the dimensionless frequency \( \omega_0 = 6 \) in our analysis, since it provides a suitable balance between the time and frequency localization (Grinsted et al., 2004).

3. Results

3.1. CRP and LOS analyses of F10.7 and SNs

We apply the CRP tool to compare their relative phase difference on a point-by-point basis, which is shown in Fig. 2. The bold red\(^1\) line in the figure indicates the LOS. From this figure on can see that the LOS lies practically close to the main diagonal, marking a level of synchronization between F10.7 and SNs. However, there still have some deviations from the main diagonal during some time points (e.g., around the years 1958, 1971, 1982 and 2009), implying that the existence of phase differences between these two time series is indeed a distinct feature in solar activity.

Fig. 3 illustrates the LOS extracted from the CRP. Here, the change of sign reflects the change of leading role between these two time series. In each time point, LOS > 0 means F10.7 leading in phase and LOS < 0 means SNs leading in phase. The minimum and maximum times of solar cycles 18 to 23, taken from papers of Makarov et al. (2003) and Tlatov (2009), are indicated by vertical solid and dashed lines respectively. From this figure one can see that, (1) F10.7 lags SNs in phase during most parts of the considered time interval because the values of the LOS are mostly less than 0; (2) the two time series are more asynchronous around the minimum and maximum times of a solar cycle than at the ascending and descending phases of the cycle; (3) The mean value of all LOS values is about \(-1.5\) months, implying that F10.7 lags behind SNs on the average of the considered time interval. However, the LOS values fluctuate so violently that we are not sure about which activity index leads the other at different time scales. For example, their phase difference ranges from 15 to 25 months during the minimum and maximum times of a solar cycle, whereas it is less than \(\pm 5\) months at other time points. Therefore, the wavelet transform tools are needed to study their different phase relationship at different periodic scales.

3.2. Wavelet transform analysis of F10.7 and SNs

To know their phase relationship varying with time and periodic scales respectively, the codes provided by Grinsted et al. (2004) are employed to show the XWT of them, which is displayed in Fig. 4. Their relative phase relation in time–frequency space is shown by arrows in this figure. Arrows point to the right when the two processes are in phase and to the left when they are in anti-phase. If an arrow points up, then the first process lags behind the second one. From this figure one can see that almost all of the arrows are horizontal and point to the right in the low-frequency components, suggesting that F10.7 and SNs are in phase around the periodic scales of about 7–15 years. However, there still have small phase deviations within these areas, the arrows subtend a small angle with the horizontal upward direction, which implies that F10.7 slightly lags behind SNs. In the high-frequency components, namely the periodic scales less than 7 years, the arrows are fairly randomly distributed showing strong phase mixing. That is to say, the two time series display a strong phase asynchrony in the high-frequency modes. Therefore, the regions of the low-frequency components show a closely positive correlation (in-phase relation) between the two while the high-frequency components are responsible for the obvious phase asynchronization.

Donner and Thiel (2007) stated that there are no regular oscillatory patterns in both the smaller and larger periodic scales, and the availability of a physically meaningful phase definition depends crucially on the appropriate choice of reference frequencies or periodicities. For the study on the relative phase relationship between F10.7 and SNs, their relative phase differences and the corresponding standard deviations varying with periodic scales are shown in Fig. 5 to illustrate their dependence on the considered frequencies. For the periodic scales less than 7 years, the phase differences fluctuate violently and their corresponding standard deviations are very large, implying that there are no regular oscillatory patterns on periodic scales less than 7 years, as reported earlier by

\[^1\] For interpretation of color in Fig. 2, the reader is referred to the web version of this article.
For the periodic scales of 7–15 years, most of the phase differences are less than 5% (about 7 months) of the mean length of the Schwabe cycle. Furthermore, the phase differences are negative for the periodic scales of 7–10.5 years and positive for the periodic scales of 10.5–15 years, indicating that the phase relationship between them is frequency-dependent.

We also calculated their relative phase differences for all time points of the considered time interval on both the periodic scales of 0–7 years and 7–15 years, as displayed in Fig. 6. It is no doubt that the phase relationship between the two are asynchronous in the periodic scales of 0–7 years. For the periodic scales of 7–15 years, the phase differences show continuous transitions from those conditions where F10.7 leads in phase during the years of 1947–1971 and 2008–2012 to those where F10.7 lags behind SNs during the years of 1971–2008. Namely, the phase relationship between the two is also time-dependent. Furthermore, the relative phase differences between the two vary from 4 to 3 months in the low-frequency regions. According to the above analyses, we should be careful in choosing the periodic scales and time ranges when we study the phase relation between different solar activities.

4. Conclusions and discussions

Using the data of F10.7 and SNs in the time interval from 1947 February to 2012 June, we investigate the phase relationship between them by means of modern nonlinear tools. The results suggest that F10.7 lags behind SNs during most parts of the considered time interval, and their relative phase differences vary from −4 to 3 months with a mean value of −1.5 months. These two activity indicators are more asynchronous around the minimum and maximum of a solar cycle than at the ascending and descending phases of the cycle. Moreover, the phase relationship between them is not only time-dependent but also frequency-dependent. These indicate that the phase asynchrony between these two solar activities is not a simple linear relationship, although they are highly correlated with each other. Our results are fully in agreement with the previous findings reported by Holland and Vaughn (1984). This confirms that advanced nonlinear approaches are more powerful than traditional linear tools when they are applied to analyze nonlinear behavior of solar activity.

According to the definition of SNs, they are directly related to the number of sunspot group that are present on the solar disk on a given day. Generally speaking, large sunspot groups (LSGs), which contain the majority of long-living complex active regions, tend to reach their maxima in the phase of about 0.5 (the second cycle maximum), while small sunspot groups (SSGs) generally peak much earlier at a phase of about 0.32 (the first cycle maximum). Moreover, when compared to SSGs, LSGs show better correlation with F10.7, facular area and other activity parameter (Kilcik et al., 2011a). That is to say, SSGs have a dominant effect on SNs in the early phase of a solar cycle, while their contribution to solar irradiance is not significant because of their small size. By contrast, LSGs may play in contribution to the integral power of output energy or magnetic flux emergence, and result in high value of F10.7 (Wenzler et al., 2005). Therefore, F10.7 and SNs are more asynchronous around the epoch of cycle maximum.

On the other hand, F10.7 is more complex than SNs because it contains both active regions and bright regions (such as faculae and plages) contributions. The result is that their variations with time of contribution to F10.7 is more complicated than SNs, especially the magnetic flux emergence is taken into account. The strong correlation of F10.7 with plage area and faculae area has been reported earlier (Oster, 1990; Kilcik et al., 2011a). In any active region, solar faculae and plage are usually peak later than SNs and they are still present after all sunspots disappear on the solar disk (Greenkorn, 2012). The active region’s contribution to sunspot-related indicators peaks before that to faculae- and plage-related indicators, so the temporal variation of F10.7 and SNs behave differently in a solar cycle. We thus arrive at the conclusion that the time delay between sunspot-related indices and peak much earlier at a phase of about 0.32 (the first cycle maximum). Moreover, when compared to SSGs, LSGs show better correlation with F10.7, facular area and other activity parameter (Kilcik et al., 2011a). That is to say, SSGs have a dominant effect on SNs in the early phase of a solar cycle, while their contribution to solar irradiance is not significant because of their small size. By contrast, LSGs may play in contribution to the integral power of output energy or magnetic flux emergence, and result in high value of F10.7 (Wenzler et al., 2005). Therefore, F10.7 and SNs are more asynchronous around the epoch of cycle maximum.

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Appendix A. Supplementary data

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References