Application of recurrence quantification analysis for early detection of lean blowout in a swirl-stabilized dump combustor

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ABSTRACT

Lean blowout (LBO) is a serious issue in modern gas turbine engines that operate in a lean (premixed) mode to follow the stringent emission norms. When an engine operates with a lean fuel–air mixture, the flame becomes unstable and is at times carried out of the combustion chamber by the unburnt flow. Thus, the sudden loss of the flame, known as lean blowout, leads to fatal accidents in aircrafts and loss of production in power plants. Therefore, an in-depth analysis of lean blowout is necessary as the phenomenon involves complex interactions between flow dynamics and chemical kinetics. For understanding the complex dynamics of this phenomenon, recurrence analysis can be a very useful method. In the current study, we observe a transition to LBO as the global fuel–air ratio is reduced from stoichiometric condition and perform recurrence quantification analysis (RQA) with the CH\textsuperscript{*} chemiluminescence data obtained experimentally. The extent of fuel–air mixing is varied with an objective of developing some robust early predictors of LBO that would work over a wide range of premixing. We find some RQA measures, such as determinism, laminarity, and trapping time, which show distinctive signature toward LBO and thereby can be used as early predictors of LBO for both premixed and partially premixed flames. Our analysis shows that the computational time for laminarity and trapping time is relatively less. However, computational time for those measures depends upon the dynamics of the combustor, size of the data taken, and choice of recurrence threshold.

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Modern gas turbine engines operate with lean premixed combustion to reduce the formation of NO\textsubscript{x}. Though lean combustion results in low NO\textsubscript{x} emission, the major drawback of this technology is the occurrence of flame blowout. The problem is very severe for both aircraft engines and power plants. This calls for the development of strategies for early detection of an impending blowout and adoption of appropriate measures to mitigate it. A few techniques recently reported are suitable for blowout detection for both premixed and partially premixed flames.1,2 However, the investigation needs to be further extended such that one can test the suitability of other parameters and find out early predictors of lean blowout (LBO) for these flames, which are robust enough to perform satisfactorily for a wide range of conditions. The present investigation of predicting the impending blowout uses the framework of recurrence analysis. Recurrence analysis is a powerful method in nonlinear dynamics, which has the capability to explore intrinsic dynamics of a system and has been implemented in diverse fields of research such as medicine,3,4 finance,5 chemistry,6 astrophysics,7 and engineering.8–10 In this study, CH\textsuperscript{*} chemiluminescence is considered as a measured variable on which the recurrence model is applied. The study includes premixed (higher degree of fuel–air mixing) as well as partially premixed (lower degree of fuel–air mixing) flames. We observe that the recurrence plots (RPs) alone are not very efficient to capture the transition to LBO in the case of partially premixed flames. However, the recurrence quantification analysis (RQA) measures such as laminarity, determinism, and trapping time show distinguishable trends as LBO is approached and, therefore, are good candidates to be used as early LBO predictors. Furthermore, the computational time for evaluating these RQA parameters is estimated and the analysis shows the prospect of laminarity and trapping time to be used in LBO prediction with...
less computational effort. To the best of our knowledge, the current research is the first effort on dynamic analysis using RQA measures of both premixed and partially premixed combustion where combustion noise precedes the lean blowout without any occurrence of thermoacoustic instability.

I. INTRODUCTION

The occurrence of flame blowout is a serious problem for gas turbine combustors used for both power generation and aerospace propulsion. As the presence of excess air reduces the temperature at the flame zone, lean fuel–air mixtures are used in combustors to reduce emissions of pollutants like oxides of nitrogen (NOx). This, however, increases the probability of blowout in lean premixed combustors. In simple terms, blowout refers to a condition when the local flow velocity of the reacting mixture exceeds the local flame speed of the reacting species. Under such a condition, the flame becomes unstable and is swept away by the flow from the unburned reactants leading to the extinction of the flame. The flame speed gradually decreases from its value at a stoichiometric condition as the air–fuel ratio is varied to a lean (i.e., fuel–lean) mixture. Hence, the flame becomes more susceptible to blowout at lean conditions. Such blowout events caused by an excessive leaness of the reacting mixture are commonly referred to as lean blowout (LBO).

In the case of land-based gas turbines, the threat of LBO originates from operating the combustor in a lean premixed mode to meet the stringent emission norms. Such an occurrence of LBO leads to a lengthy shutdown and restart procedure affecting power generation and causing a significant loss of revenue. On the other hand, in aircrafts, lean combustion happens during sudden deceleration of the engine. In such a situation, the fuel supply is reduced quickly by throttling; however, the air flow rate governed by the inertia of the compressor takes much longer time to respond to the changed condition. This gives rise to a situation where the fuel–air mixture becomes very lean and the flame becomes prone to blowout. For aero engines, LBO leads to an unexpected shutdown of engines during flight, causing fatal accidents.

Considering these detrimental effects of LBO, one must focus on the determination of LBO limit and should not follow any predefined curve because the parametric point at which such an instability occurs is uncertain. It is difficult to determine the LBO limit a priori during the design stage as the occurrence of lean blowout is closely related to the actual operating condition of the engine. This requires the LBO limit to be detected during the operation of the engine in real time. In the absence of any reliable detection system, the present practice in industry is to operate the combustor with a wide factor of safety. In the case of land-based gas turbines, this implies operating the engines at fuel–air ratios, which are considerably higher than the LBO limit, leading to increased NOx emission. In the case of aero engines, the absence of adequate LBO detection and control facility implies the loss of ability to maneuver the engine as the sudden throttling of fuel supply has to be avoided. This can be a serious problem particularly for military aircrafts. Thus, it is imperative to develop techniques for online detection of lean blowout in gas turbine combustors. The detection has to be done sufficiently ahead of the inception of the blowout event so that enough time is available for implementing any control action to prevent LBO.

The occurrence of lean blowout is a consequence of complex interactions between different physical factors such as hydrodynamics, thermochemistry, and chemical kinetics. Such a complex phenomenon can be simulated only by using models that are extremely intensive computationally. More attractive approach would be to develop data-driven techniques for detection and control of LBO. For instance, Mukhopadhyay et al. used Symbolic Time Series Analysis (STSA) by converting chemiluminescence data to a symbolic time series. The metric for LBO detection was an anomaly measure with reference to the symbolic time series data at a condition away from the LBO. Sarkar et al. extended the work of Mukhopadhyay et al. by constructing D-Markov machine. In another study, Dey et al. developed a cross wavelet transform aided rule-based approach for the early detection of lean blowout in a laboratory-scale combustor. Both the studies categorized the data for different operating conditions into three classes: healthy (i.e., away from LBO), moderate (i.e., approaching LBO), and close to LBO. Furthermore, Zhou used the spectral signatures of certain dominant tones and implemented them as a strategy for LBO detection and isolation in an industrial gas turbine combustor.

Over the last two decades, several researchers have focused on the early detection of lean blowout using experimental time series data obtained using different sensors. Nair and Lieuwen used acoustic pressure data for the early detection of lean blowout in three different designs of model gas turbine combustors: a swirl-stabilized burner, a bluff body stabilized burner, and a piloted burner. Toward that, they used wavelet-based time-frequency analysis, statistical analysis, and threshold-crossing based analysis of time series data. Muruganandam et al. used optical sensors [photomultiplier tubes (PMTs)] to record the instantaneous OH chemiluminescence data from flames. Using the chemiluminescence data, they did Fourier spectral analysis and statistical analysis in time domain and used them to locate extinction and reignition events prior to blowout for detection of LBO. Prakash implemented a moving window technique for determining LBO precursors using both optical and acoustic sensor data. In a subsequent study, Yi and Gutmark developed metrics for real time prediction of lean blowout in partially premixed liquid-fueled gas turbine combustors. They developed two metrics for LBO detection, namely, normalized chemiluminescence root mean square (NRMS) and normalized cumulative duration of LBO precursor events using time series of OH* chemiluminescence. Both the metrics were related to the statistical distribution of the flame characteristics, which was observed to shift from normal to Rayleigh distribution as LBO was approached. Recently, De et al. investigated the efficacy of different existing statistical tools for the early detection of LBO using CH* chemiluminescence time series. They also showed that the mean frequency, estimated using Hilbert transform (HT) based analytic signal approach, of the heat release rate (HRR) fluctuations can be a good early detector of LBO for premixed as well as partially premixed flames.

Apart from these, Chaudhari et al. adopted a very different approach and developed a metric for the early detection of LBO using the RGB intensities of flame color. De et al further extended the work of Chaudhari et al. by including a comparative study using a spectrometer and a color camera. With these, they showed...
that the implementation of color camera for LBO identification can be a more economical alternative as the results obtained using a spectrometer and a color camera show similar trends. They further studied the flame behavior using a wide range of color spectra with the help of a spectrometer. In short, the approaches described in Refs. 1, 20–23, and 28 are robust in the sense that they perform satisfactorily over a wide range of premixing.

Although a number of contributions (mentioned above) on lean blowout have been observed, the dynamics needs to be further explored especially in the zone which is close to the blowout. In such a situation, dynamical systems analysis may be the right choice as this approach has the ability to address the dynamics of a system properly. A number of recent studies have focused on the dynamic characteristics of combustors during the transition to thermoacoustic instability and approach to lean blowout. Some of them have utilized the approach of non-linear dynamics for identifying metrics to forewarn thermoacoustic instability. However, the efforts toward identifying quantitative measures for the early detection of LBO based on the dynamical systems approach have been relatively scarce. Gotoda and his co-workers have used dynamical systems theory and identified a few parameters such as translation error, trapping time, and permutation entropy as early LBO predictor. In their subsequent study, they have used average degree of nodes in a complex network as a metric for detection and control of LBO. However, the complex network was based on a modified visibility graph, which does not preserve the information related to geometry and structure of the attractor.

Despite several attempts using dynamical systems approach in recent years, recurrence plots (RPs) and recurrence quantification analysis (RQA) have not been widely used for the early detection of LBO. The ability of RP and RQA measures in distinguishing the dynamic characteristics has been found in the literature, where the system undergoes a drastic change, from intermittency to periodic state or vice versa. A number of researchers have used ε-recurrence plots and color recurrence plots to represent the dynamics of the combustor. Recently, Unni and Sujith applied RQA for detection of lean blowout in turbulent combustors but their study was limited to only one level of air–fuel premixing. Since recurrence is a fundamental property of deterministic dynamical systems, the choice of RQA measures as potential metrics for the early detection of LBO is logical.

The objective of the present work is to explore the suitability of different RQA measures as metrics for early detection of lean blowout in combustors with different levels of fuel–air premixing. Consideration of different levels of premixing would assess the suitability of RQA for developing LBO detection measures for different gas turbine applications, especially in the aircraft engines where fuel is injected very close to the flame zone and burns in a partially premixed mode. Moreover, unlike the majority of works by Sujith and co-workers and Gotoda and co-workers in the present work, full-blown thermoacoustic instability, characterized by high amplitude limit cycles, does not precede the approach to lean blowout.

We conduct experiments in a swirl-stabilized dump combustor and perform recurrence quantification analysis (RQA) on CH* chemiluminescence data. Here, we vary both equivalence ratio and the level of premixing. We find a few RQA measures such as laminarity, determinism, and trapping time to be robust LBO predictors for both premixed and partially premixed flames. Furthermore, a relatively less computational effort to evaluate laminarity and trapping time makes them more useful for prediction of an imminent lean blowout.

The rest of the paper is organized as follows. Section II describes the experimental setup and procedure adopted for the current analysis. Section III discusses about the technical approach that is used for the early prediction of the proximity to LBO. Section IV presents the results and discussions of the investigation including computational time analysis of different metrics. Finally, the major findings are summarized in Sec. V.

II. EXPERIMENTAL SETUP AND PROCEDURE

A. Combustor rig

Figure 1 represents the experimental setup with a swirl-stabilized dump combustor, which has a premixing chamber and a combustion chamber. The premixing chamber accommodates five fuel entry points at different locations (F1–F5) and one air entrance at a fixed location (Fig. 1). Variation in position of the fuel ports alters the length available for premixing of fuel and air. The extent of mixedness for a similar geometry was estimated numerically in our previous work and is found to decrease significantly as we changed the fuel port from F2 to F5. The fuel ports are placed 50 mm apart in the premixing chamber. Furthermore, at each air and fuel location, four inlet holes are provided circumferentially (90° apart from each other) to reduce the asymmetry in flow pattern. A swirler with swirler blade angle of 60° to the axial direction is provided in the flow path, 15 mm upstream of the dump plane. The swirl number ($S = 2/3(1 - (d_h/d)^3)/(1 - (d_i/d)^3)) \tan \theta$, where $d_h$ and $d$ are the hub and the tip diameter of the swirler, respectively, and $\theta$ is the swirler blade angle) is estimated to be 1.26. The swirler is used to obtain better flame stabilization as with a swirler, the flow takes a helical path and the residence time of the reactant mixture increases. Furthermore, at the dump plane, a sudden expansion in the flow area slows down the flow velocity and aids to create recirculation zones. In recirculation zones, the heat transfer from the hot gases to the unburnt mixture is ensured. A 200 mm long quartz tube (resistant to high temperature) with an outer diameter of 65 mm forms the combustor wall that provides optical access to the turbulent flame.

B. Experimentation and data acquisition

Air is supplied at ambient temperature from a compressor (ELGI equipment, capacity: 220 liters, maximum force bearing capacity: 12 kg/cm²) to the air entrance, the bottom most port in the premixing chamber (see Fig. 1). The air flow rate is metered through a mass flow controller (Alicat scientific, MCR series, range: 0–1500 SLPM). Fuel (liquefied petroleum gas, 60% butane, and 40% propane by volume) is supplied from a pressurized cylinder fitted with a needle valve to one of the fuel locations, which corresponds to a particular level of premixedness. The fuel flow rate is metered using separate mass flow controller (Alicat scientific, MCR series,
FIG. 1. Isometric view of the experimental setup (A) that is used for the LBO detection study, where the premixing chamber (a) and the combustion chamber (b) are indicated. A swirler (c) is used for flame stabilization. (B) The chemiluminescence measurement is done using a photomultiplier tube outfitted with CH\textsuperscript{*} filter. The data are acquired using NI-cRIO-9073 chassis and logged in a PC using LabVIEW.

range: 0–250 SLPM). Both the mass flow controllers are operated through Flow Vision software.

A photomultiplier tube (PMT, Hamamatsu, 931A) fitted with a CH\textsuperscript{*} interference filter (centered at 430 nm and FWHM of 10 nm) is used to acquire chemiluminescence data (see Fig. 1). Electric current produced by the PMT is converted into voltage by a built-in amplifier, which is operated with a 15 V DC source. The voltage output from the PMT is acquired using an NI-cRIO-9073 chassis and stored in a computer using the National Instruments software, LabVIEW.

The heat release rate (HRR) is one of the important properties of the combustion process as it characterizes the extent of energy conversion from chemical potential to thermal energy. The HRR is vital for understanding and predicting combustion instability\textsuperscript{42,43} in gas turbine engines. A direct measurement of the heat release rate is difficult and is mostly achieved through suitable chemiluminescence measurements.\textsuperscript{44–48} For premixed flames, CH\textsuperscript{*} chemiluminescence has been reported as a well-accepted marker of the heat release rate,\textsuperscript{44,46–48} whereas for partially premixed flames, there is a lack of unanimity regarding the CH\textsuperscript{*} chemiluminescence signal being the most representative of the heat release rate. However, Kim et al.\textsuperscript{52} proposed CH\textsuperscript{*} chemiluminescence as one of the acceptable candidates for the heat release rate marker for partially premixed flames also. In the present work, CH\textsuperscript{*} chemiluminescence signal as recorded by the PMT\textsuperscript{51} is used for analysis.

III. RECURRENCE BASED NONLINEAR MODEL

Recurrence is a fundamental property of a deterministic dynamical system and can be utilized to characterize the system’s behavior. Therefore, one way of addressing the changes of the flame dynamics and LBO detection would be the use of recurrence based model. The steps for the recurrence analysis followed in the present study are described in Secs. III A–III C.

A. Phase space reconstruction

The first step in recurrence analysis is the construction of a phase space. There are a couple of major problems associated with the construction of a phase space from experimental data: (i) a large number of variables are required to define a state and (ii) a limited number of sensors are available in experiments. Takens’ embedding theorem\textsuperscript{53} by which a high dimensional phase space can be reconstructed from the experimental time series of a scalar variable helps to overcome such problems. In the present study, we use the time series of CH\textsuperscript{*} chemiluminescence acquired through photomultiplier tube, \(\{I_i\}_{i=1}^{N_T}\) (where \(N_T\) is the total number of time steps acquired). The chemiluminescence data \(I\) are acquired for 20 s and at sampling frequency \((f)\) of 2 kHz; however, for some selective cases, we downsample the data to 1 kHz.
The phase space of the system can be reconstructed defining time delayed coordinates as

\[ y(t_i) = \{ I(t_i), I(t_i + \tau_d), I(t_i + 2\tau_d), \ldots, I(t_i + (D-1)\tau_d) \}, \]

where \( i = 1, 2, 3, \ldots, N \). The total number of phase space vectors in the reconstructed phase space, \( N = N_T - \tau_d(D - 1) \). \( \tau_d \) is the optimum time delay in seconds and \( D \) is minimum embedding dimension, which will be elaborated later. \( y(t_i) \) and \( I(t_i) \) represent the phase space vector and the chemiluminescence intensity at \( t_i \)th time step, respectively. \( t_0 \) is the initial time, \( i \) is the number of time steps (a positive integer), and \( \Delta t \) is the sampling time (\( \Delta t = 1/f \)). Therefore, \( i\Delta t \) represents the time span in seconds. We find that, for \( N_T > 10,000 \), the computation takes very long time. Therefore, the entire time series is split into four windows of length 5 s comprising of either 5000 or 10,000 data points depending upon \( f \) being 1 kHz or 2 kHz, respectively.

For constructing delay vectors [Eq. (1)], two crucial parameters are the minimum embedding dimension, \( D \) and the optimum time delay, \( \tau_d \). In this study, \( \tau_d \) is calculated using the methods of autocorrelation in which the optimal time delay corresponds to the time when the autocorrelation function [Eq. (2)] first crosses the zero value, \( C_1(\tau_0) \).

\[
C_1(\tau) = \frac{1}{N} \sum_{i=1}^{N} [I_{i+\tau} - \bar{I}] [I_i - \bar{I}],
\]

where

\[
\bar{I} = \frac{1}{N} \sum_{i=1}^{N} I_i.
\]

Here, \( I_i \) and \( I_{i+\tau} \) denote the instantaneous chemiluminescence data at \( t_i \) and \( t_{i+\tau} \) (where \( \tau \) is the time delay). On the other hand, the minimum embedding dimension is estimated using the method of false nearest neighbors (FNNs) in which the optimum embedding dimension corresponds to the case shown, it is observed that the percentage of FNN value first reaches zero, \( D_{\text{FNN}} \). The method of finding optimum threshold is given in Ref. 14.

In Fig. 2, the variation of false neighbors is shown by changing embedding dimension (\( D \)) for different equivalence ratios when port F3 is used as fuel entrance. This is a representative plot for evaluating minimum embedding dimension and similar approach is taken for other fuel locations at each operating condition. The minimum embedding dimensions are indicated for each case with arrows.

**B. Recurrence plot**

Once the phase space is reconstructed, we draw the recurrence plots, which show the time instances when a trajectory roughly visits the same area in the reconstructed phase space. The plots are drawn based on the recurrence matrix \( R_c(e) \) given as

\[
R_c(e) = H(e - ||\vec{y}_i - \vec{y}_j||),
\]

where \( i, j = 1, 2, \ldots, N \). \( \vec{y}_i \) and \( \vec{y}_j \) are two delay vectors in reconstructed phase space at time instances, \( i \) and \( j \), respectively, and \( H \) is the Heaviside function. In the reconstructed phase space, proximity of two phase space vectors, \( \vec{y}_i \) and \( \vec{y}_j \), can be described in terms of recurrence matrix, \( R_c(e) \). Here, the threshold, \( e \) ensures that if a state \( \vec{y}_i \) lies within a hypersphere of radius \( e \) centered at another state \( \vec{y}_j \), the two states \( \vec{y}_i \) and \( \vec{y}_j \) are considered to be recurrent. Therefore, the threshold value is a crucial parameter in recurrence analysis.

Earlier studies have reported different methods for determining the threshold value. Schinkel et al. used the area under curve (AUC) method for the selection of optimum threshold for quasiperiodic signals. Unni and Sujith considered the threshold value as 10% of the standard deviation estimated from the time series of a measured variable. A large number of studies suggested the threshold values to depend on the mean and maximum size of the attractor. The selection of the optimum threshold further depends on the dynamical behavior of the signal obtained from experiments. Therefore, for choosing the optimum threshold, we follow the adaptive threshold method proposed by Eroglu et al. The method of finding optimum threshold is given in Appendix A and the threshold value is found to be 18% of the attractor size.

Turning back to Eq. (3), \( R_c \) is a square symmetric matrix having values only 0 and 1. In recurrence plot, which is essentially a graphical representation of \( R_c \), white dots represent the zeroes and black dots represent ones. As \( R_{ij} = 1 \) always, the principal diagonal of the recurrence plot is composed of black dots throughout and is referred to as the line of identity (LOI).

**C. Recurrence quantification analysis**

Apart from the visual impressions provided by the recurrence plot (RP), the method of finding different measures, which quantify the small scaled structure in recurrence plots, is known as recurrence quantification analysis (RQA). In the following, we briefly...
describe the recurrence measures, which are used in this study for the detection of an impending lean blowout in a laboratory-scale gas turbine combustor.

**Recurrence rate (RR)** denotes the fraction of black dots in recurrence plots. This in other words indicates the percentage of the detection of an impending lean blowout in a laboratory-scale gas describe the recurrence measures, which are used in this study for the detection of an impending lean blowout in a laboratory-scale gas turbine combustor.

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Recurrence rate (RR) denotes the fraction of black dots in recurrence plots. This in other words indicates the percentage of the detection of an impending lean blowout in a laboratory-scale gas turbine combustor.\(^{(1)}\)

\[
RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{ij}
\]

where \(N\) represents total number of phase space vectors in the reconstructed phase space. **Determinism (DET)** represents the fraction of recurrence points that form diagonal structures and is estimated as

\[
DET = \frac{\sum_{l=l_{\text{min}}}^{N} |P(l)|}{\sum_{l=1}^{N} R_{ij}}
\]

where \(P(l)\) denotes the number of diagonal lines of length \(l\) and \(l_{\text{min}}\) denotes the minimum length of the diagonal line (here considered to consist of 4 black dots). On the other hand, **laminarity (LAM)** is a measure that determines the fraction of recurrence points forming the vertical structures, which indicate a close proximity of states (points in phase space) at successive time instants. **Laminarity** is calculated as

\[
LAM = \frac{\sum_{v=v_{\text{min}}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)}
\]

where \(P(v)\) indicates the number of vertical lines of length, \(v\), and \(v_{\text{min}}\) denotes the minimum length of the vertical line (here considered to consist of four black dots). The laminarity is an important recurrence parameter for the systems exhibiting intermittency,\(^{(1)}\) where two different dynamical states alternatively appear as the time progresses.

Diagonal structures in a recurrence plot are indicators of periodic oscillations. Therefore, the **average diagonal line length (L)** is a measure of divergence of an attractor in phase space. In other words, the shorter the average diagonal line length, the faster is the deviation of the system from periodic behavior. This is evaluated as

\[
L = \frac{\sum_{l=l_{\text{min}}}^{N} |P(l)|}{\sum_{l=1}^{N} P(l)}
\]

**Entropy (ENT)** or Shannon information entropy which is a measure of the complexity of the system is calculated as

\[
ENT = - \sum_{l=l_{\text{min}}}^{N} p(l) \ln(p(l)),
\]

where \(p(l)\) denotes the probability of occurrence of diagonal structures having length \(l\) in a recurrence plot. Another measure, **trapping time (TT)**, which denotes an average time for which a system remains at a particular state, i.e., the length of the vertical structure, is evaluated as

\[
TT = \frac{\sum_{v=v_{\text{min}}}^{N} vP(v)}{\sum_{v=1}^{N} P(v)}
\]

The procedure for calculating the lengths of diagonal or vertical line is given in **Appendix B**.

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**FIG. 3.** Time series of CH\(^+\) chemiluminescence (I) are shown using the fuel ports F1 (a1–h1) and F2 (a2–h2). For F1, conditions are (a1) \(\Phi/\Phi_{LBO} = 1.409\), (b1) \(\Phi/\Phi_{LBO} = 1.288\), (c1) \(\Phi/\Phi_{LBO} = 1.235\), (d1) \(\Phi/\Phi_{LBO} = 1.15\), (e1) \(\Phi/\Phi_{LBO} = 1.06\), (f1) \(\Phi/\Phi_{LBO} = 1.043\), (g1) \(\Phi/\Phi_{LBO} = 1.028\), and (h1) \(\Phi/\Phi_{LBO} = 1.008\). For F2, conditions are (a2) \(\Phi/\Phi_{LBO} = 1.40\), (b2) \(\Phi/\Phi_{LBO} = 1.288\), (c2) \(\Phi/\Phi_{LBO} = 1.235\), (d2) \(\Phi/\Phi_{LBO} = 1.15\), (e2) \(\Phi/\Phi_{LBO} = 1.08\), (f2) \(\Phi/\Phi_{LBO} = 1.05\), (g2) \(\Phi/\Phi_{LBO} = 1.025\), and (h2) \(\Phi/\Phi_{LBO} = 1.0\).
IV. RESULTS AND DISCUSSIONS

The experiments are conducted with a constant air flow rate, $Q_a = 80$ SLPM, and the fuel flow rate ($Q_f$) is varied in the range of 2.8–1.2 SLPM. In other words, the global equivalence ratio ($\Phi$) is decreased in steps of 0.04 from 1.0 (corresponding to the stoichiometric condition) to the value at which lean blowout occurs. As the equivalence ratio is varied, the change in dynamics is observed in the chemiluminescence fluctuations. In this section, we discuss the combustion dynamics using recurrence plots and recurrence quantification measures. We further vary the degree of premixing, which enables us to characterize the dynamics of partially premixed flames. We first show the dynamical features of the chemiluminescence fluctuations, followed by the quantification of such characteristics through recurrence analysis. It may be noted that in Figs. 4, 5, 7, and 9–11, values of embedding dimension, time delay and recurrence threshold vary with $\Phi/\Phi_{LBO}$ for a given level of premixing. However, as shown in Figs. S1 and S2 of the supplementary material, the choice of these parameters does not significantly alter the results. Hence, the variations of recurrence measures in Figs. 4, 5, 7, and 9–11 are primarily caused by the changes in system dynamics.

A. Transition to blowout with premixed flames

As the mixedness of air and fuel corresponding to the fuel ports F1 and F2 (the mixing length, $l_f = 330$ mm and 280 mm, respectively) have been numerically quantified to be nearly equal and also maximum compared to those of the other fuel ports, the flame produced using these fuel ports is referred to as the premixed flame. At this degree of premixing, the time series of CH$^*$ chemiluminescence ($I$) are captured by varying fuel–air ratio from stoichiometric to the lean condition where blowout occurs. Figures 3(a1–h1) and 3(a2–h2) exhibit the oscillations of CH$^*$ chemiluminescence, which reflect the behavior of the flame at different equivalence ratios as it approaches blowout. At a high equivalence ratio ($\Phi/\Phi_{LBO} = 1.4–1.288$ for both F1 and F2, Figs. 3(a1–b1) and 3(a2–b2)), the fluctuation is seemingly aperiodic when the flame remains attached to the dump plane of the combustion chamber indicating the stable operation of the combustor. This aperiodic and broadband fluctuations in CH$^*$ chemiluminescence is a characteristic feature of combustion noise.\cite{16} We consider this range of operating condition as a normal state. At $\Phi/\Phi_{LBO} = 1.235$ [Figs. 3(c1) and 3(c2)], the flame is lifted and stabilized at certain standoff distance from the dump plane.\cite{17} As a consequence, the pattern of the chemiluminescence fluctuations changes significantly. Thus, the condition indicates a transition state, where the flame becomes significantly weak.\cite{18}\cite{19} The flame becomes even leaner as the equivalence ratio reduces from $\Phi/\Phi_{LBO} = 1.235$ toward LBO. Prior to LBO, for $\Phi/\Phi_{LBO} = 1.06–1.0$ [Figs. 3(e1–h1) and 3(f2–h2)], the nature of fluctuations in the time series changes significantly due to the occurrence of frequent extinction and reignition. The regime of $\Phi/\Phi_{LBO} = 1.235–1.0$ can be further divided and categorized as the post-transition state (around $\Phi/\Phi_{LBO} = 1.235–1.06$) and critical state ($\Phi/\Phi_{LBO} = 1.06–1.00$) for both ports F1 and F2. It is very difficult for the flame to sustain when it enters into the critical state as the flame becomes largely unstable in this state. Therefore, the prediction of LBO needs to be done well before this critical state.\cite{20}\cite{21}

The high amplitude bursts in the signal occur at fairly regular intervals in the critical state are commonly known as chemiluminescence excursion.\cite{22}\cite{23} At this condition (prior to lean blowout), the flame detaches from the dump plane and moves toward the downstream direction in the combustion chamber. For a short while, the flame almost disappears from the field of view due to the weak reaction rate. This temporary disappearance of the flame from the combustion chamber is referred to as “local extinction.” The local extinction or flame loss in the combustion chamber is indicated by the low amplitude phase of the oscillations [Figs. 3(e1–h1) and 3(f2–h2)]. On the other hand, the high amplitude phases correspond to the re-ignition of the unburned fuel, which has entered and accumulated in the combustion chamber, when the flame packets from the downstream direction are convected back to the upstream
FIG. 5. Variation of recurrence rate (a), determinism (b), entropy (c), laminarity (d), trapping time (e), and average diagonal line length (f) as the equivalence ratio is varied from near stoichiometric condition to lean blowout when fuel ports F1 and F2 are used as fuel entrance. The above measures are calculated for four consecutive windows, each of which has 10,000 data (N). Subscripts 1 and 2 are used in the plot to distinguish the states for F1 and F2, respectively.

The cycle of flame loss and reappearedance of the flame in combustion chamber is observed to increase toward lean blowout. In Figs. 3(e1–h1) and 3(f2–h2), such events of extinction-reignition are manifested as the intermittent occurrence of high and low amplitude fluctuation in the heat release rate. Although the changes in flame dynamics are observed in time series of the CH emission, measured with a photomultiplier tube qualitatively, the transition needs to be quantified through some suitable measures. In this connection, we considered some statistical parameters like mean, variance, and so on, in our previous investigation. These parameters, in general, were observed suitable for early prediction of lean blowout for premixed flames but not for partially premixed flames. Therefore, the objective of the present study is to explore more robust nonlinear tools, which will not only capture the flame dynamics but also predict the impending LBO.

Having described the time series of chemiluminescence, we next turn to explore the dynamics qualitatively and quantitatively through recurrence analysis. The recurrence plots are shown in Fig. 4 for a few selective time series of chemiluminescence (I) shown in Fig. 3. We consider 10,000 data points (N) to construct the recurrence matrix (R). It may be noted that the time-range of the plots in Fig. 4 is not the same for all cases as the number of delayed vectors varies due to different values of D and τ (values are mentioned in the caption of Fig. 4) estimated. Figures 4(a)–4(b) show the grainy structure where recurrence points are very few and randomly scattered. Such a random appearance of black and white dots indicates the dominance of noise in the system dynamics. As the system approaches LBO, black dense patches are clearly observed in the plot [Figs. 4(c)–4(e)]. Furthermore, the black patches with perforated upper and right edges are observed prior to LBO [Fig. 4(e)], which confirms the behavior of type-II intermittency. This type of intermittency is observed due to the low frequency events occurring near blowout. However, the recurrence plots alone may not be suitable for early prediction of blowout as the intermittency becomes more pronounced close to LBO. Therefore, we explore different RQA measures as they are able to extract relevant information for different ranges of data size.

The requirement of a quantitative characterization of the dynamical transition further motivates us to resort to recurrence quantification analysis. In a recent study, recurrence measures have been evaluated to describe the flame dynamics for both high and low fuel flow rates. The investigation showed the variation of flame dynamics using three RQA parameters, namely, recurrence rate, entropy, and trapping time for a fixed level of air–fuel premixing. Furthermore, to perform the analysis, a constant threshold (ε), which is 40% of the amplitude of maximum fluctuation obtained in the operating range of the air–fuel ratio, was considered by Unni and Sujith. Therefore, the study needs to be further extended for different levels of premixing of air–fuel mixture using threshold better tailored to the system dynamics. The variation of different RQA measures for varied level of premixing is shown in Sec. IV B. In this section, behavior of the RQA measures is shown only for the premixed flame.

The RQA measures such as recurrence rate (RR), determinism (DET), entropy (ENT), laminarity (LAM), trapping time (TT), and average diagonal line length (L) are computed at different...
equivalence ratios (Φ) and shown in Fig. 5, RR turns out to be very low at high equivalence ratios or at normal state [Fig. 5(a)] due to the aperiodic oscillations of CH⁺ chemiluminescence. This is consistent with the recurrence plot shown in Fig. 4(a), which displays a very few recurrence points. However, as LBO is approached, the recurrence rate starts increasing and is quite high near LBO (critical state). The increase of the recurrence rate at the critical state (close to blowout) can be attributed to the high amplitude bursts occurring at fairly regular intervals. Thus, RR quantifies the transition in dynamics. However, as the transition in the value of RR occurs only in the critical state, it may not be very suitable as a LBO predictor.

The parameters based on the diagonal structure such as determinism and entropy [see Figs. 5(b) and 5(c)] take higher values at the critical state. Determinism shows a significant increase as the flame reaches the transition state for F2. On the other hand, the trend of DET for F1 is almost monotonic. Although the CH⁺ chemiluminescence fluctuations for F1 and F2 look almost similar, we can notice a difference in the trend of the RQA measure. However, in both the cases, the impending LBO can be predicted at or near the transition state as there is a substantial change in the RQA measure. On the contrary, there is a gradual increment of entropy after the transition state (Φ/Φ_{LBO} = 1.235). Therefore, determinism can be a better early predictor of LBO than entropy. Similar to determinism, laminarity can be used as an early predictor of blowout as a significant jump is observed at the transition state [Fig. 5(d)]. Another parameter based on the vertical structure of RP, trapping time increases gradually from the normal state as the flame approaches toward the critical state. As the flame goes into the critical state, the parameter becomes high, which also signifies that the state of the system is trapped in a small volume of phase space for a relatively longer time [Fig. 5(e)]. This essentially reflects the presence of vertical lines in recurrence plots close to LBO [Figs. 4(c)–4(e)] for which laminarity is also observed to be high [Fig. 5(d)]. The average diagonal line length [see Fig. 5(f)] takes a higher value at both close to and away from LBO. Hence, it may not be suitable as an early predictor of LBO for premixed flames.

In short, among different RQA measures, determinism, laminarity, and trapping time exhibit excellent sensitivity to changes in the equivalence ratio. On the other hand, though the recurrence rate shows a steep increase close to LBO, the increase is observed only in the critical state. Similarly, we find significant increase in entropy (2.8–3.3) only in the range of Φ/Φ_{LBO} from 1.15 to 1.05, which is too close to LBO for early prediction. Thus both of these measures are
not very suitable for early detection of LBO. Furthermore, the increment of the average diagonal line length toward LBO is observed for a very short span of equivalence ratios, which may not be helpful for the early detection of an impending LBO. In Sec. IV B, we turn our attention toward exploring the suitability of recurrence parameters in detecting LBO for partially premixed flames.

B. Transition to blowout with partially premixed flames

When the fuel is introduced to the mixing chamber through the remaining ports, F3–F5, the degree of fuel–air premixing varies, which in turn affects the combustion dynamics. In other words, dynamics prior to blowout can be different as one changes the mixing length ($l_f$). The reduced mixing length results in an imperfect mixing of fuel and air, here referred to as partially premixed mixtures. In this section, our primary focus is to quantify the shift in combustion dynamics of partially premixed flames using the framework of recurrence quantification analysis. The results of partially premixed flames are presented here using fuel ports F3 ($l_f = 230$ mm) and F4 ($l_f = 180$ mm) as the premixedness for these ports varies significantly as obtained in the numerical analysis.

The variation of chemiluminescence oscillations at different equivalence ratios using port F3 [time series plots are provided in Figs. 6(a)–6(f)] is quite similar to that observed using ports F1 and F2. The similarity between the dynamics using port F1 (Fig. 4) and port F3 is further observed in recurrence plots (Fig. 7). It shows few recurrence points at higher equivalence ratios [Figs. 7(a)–7(c)]. As the flame approaches lower equivalence ratio, recurrence points gradually increases, which signifies the growth in regularity of the flame dynamics [Figs. 7(c)–7(e)]. Furthermore, close to blowout, we can notice the black patch in the recurrence plot having perforated upper and right edges, which confirms the nature of type-II intermittency [Fig. 7(f)]. We do not find any significant change in dynamics as we shift from port F1 to F3, and therefore we next focus on the flame dynamics further reducing the level of air–fuel mixing with port F4.

The port F4 corresponds to a situation with a lower level of premixing than that with port F3 due to the shorter $l_f$. Figure 8 depicts the time series of CH$^*$ chemiluminescence for port F4. The combustion oscillations exhibit a similar pattern when the system is away from LBO [Figs. 8(a)–8(c)] but the dynamics prior to lean blowout is quite different. The events of local flame loss and the re-ignition observed with ports F1–F3 are not noticed prominently for port F4 and, therefore, high amplitude peaks in the time series are absent [Figs. 8(d)–8(f)]. Consequently, we do not find any intermittent behavior at LBO regime (see recurrence plots in Fig. 9). Therefore,
we explore the applicability of RQA measures in detecting such a dynamical transition.

Figure 10 shows the variation of different RQA measures for the ports F3 and F4. Similar to F1 and F2, the values of determinism and laminarity are significantly high at the critical state for F3. Also, there is a gradual increment in the post-transition zone [Figs. 10(a) and 10(b)]. However, early jump of these parameters at the transition state (Fig. 10(c)) makes them suitable for LBO prediction for F3. These measures for F4 show marginally lower values in the critical regime compared to those for F3. However, the sudden increase of such measures near \( \Phi_{LBO} = 1.375 \), where the flames become elongated is referred to as the transition state. This observation for F4 is quite similar to those for F3 [Figs. 10(a) and 10(b)]. It is important to note here that the ranges of different states obtained for F3 and F4 are different. The fluctuations in the measurement for F4 occur at the critical state due to the ultra-lean flame, where the intensity of CH\(^*\) chemiluminescence is reduced to a very low value leading to a low signal to noise ratio. Control action for mitigating LBO has to be initiated in the transition region where the RQA measures show a sharp change in value. However, the exact value of the measures at which the control will be initiated will depend on the control hardware.

In the case of F3 and F4, recurrence rate does not show any monotonic trend toward LBO [Fig. 10(c)]. Furthermore, the variation of average diagonal line length (L) to the changes in equivalence ratio is not monotonic [Fig. 10(d)] and entropy (ENT) increases at a slower rate toward LBO for both fuel locations [Fig. 10(e)]. Therefore, the variations of RR, L, and ENT are not very suitable to be used for blowout prediction in the case of partially premixed flames. However, trapping time increases monotonically toward lean blowout [Fig. 10(f)]. Hence, it can be marked as an early sensor of lean blowout for partially premixed flames and threshold for initiating control can be taken in the region of \( TT = 6.9-8.24 \) (corresponding to range of \( \Phi/\Phi_{LBO} \) from 1.25 to 1.16) for both F3 and F4, which is well before the flame going into the critical state. Thus, with the proper setting of a threshold value, it is possible to take necessary control actions at an early stage using suitable measures such as DET, LAM, and TT.

To summarize, some of the RQA measures such as DET, LAM, and TT are found suitable as the early predictor of blowout for both premixed and partially premixed flames (for different degrees of mixing). Furthermore, the significance of the results is that the trend of these parameters toward LBO is similar although the characteristics of the flame varies with the reduction in premixing length (\( l_f \)).

C. Comparison of different metrics

In the early prediction of LBO, the computing time of a measure is very important. The predictability of an impending blowout using any quantitative metric rests on the minimum computational effort to calculate the parameter. Therefore, we evaluate the computational times for different RQA measures and an existing metric, translational error\(^{36}\) for comparison. In a recent study,\(^{16}\)
translational error ($E_{\text{trans}}$) was reported to be a suitable metric to predict lean blowout online. Therefore, we perform a comparative study in terms of computational time to assess the suitability of the RQA measures for online control of LBO. $E_{\text{trans}}$, which measures the diversity in the directions of neighboring trajectories, is expressed as

$$E_{\text{trans}} = \frac{1}{K+1} \sum_{k=0}^{K} \frac{| |v(t_k) - \bar{v}| |^2}{| |v| |^2},$$

where $\bar{v} = \frac{1}{K+1} \sum_{k=0}^{K} v(t_k)$. Here, $v(t_k) = I(t_k + \tau_d) - I(t_k)$. $K$ is the number of neighboring points of $I(t_k)$. We consider the value of $K = 5$ as prescribed in an earlier literature. In the estimation of the translational error, we calculate the median of $E_{\text{trans}}$ values evaluated for 100 randomly chosen phase space vectors, $I(t_k)$. All the parameters are considered same as those prescribed in the literature.

We choose RQA measures, laminarity, determinism, and trapping time for comparison in terms of predictability [Figs. 11(a)–11(c)] and computational time [Figs. 11(d)–11(f)]. The value of translational error gradually reduces as the system approaches LBO when optimum $D$ is considered [Fig. 11(a)]. Following Unni and Sujith, we further calculate RQA measures without embedding the data in high dimensional phase space (i.e., taking $D = 1$) to reduce computational time. The results of RQA measures with $D = 1$ and with optimum $D$ show similar qualitative trends [Figs. 11(a)–11(c)]. However, $E_{\text{trans}}$ does not show monotonic trend for $D = 1$ [Fig. 11(a)]. Furthermore, we observe that the reducing sampling frequency from 2 kHz to 1 kHz does not alter the variation of the metrics much [Figs. 11(a)–11(c)]. Therefore, the dynamical transition can be well captured through RQA metrics when the data are sampled at 1 kHz and embedded in one dimension.

Figures 11(d)–11(f) depict the time ($t_c$) required to compute RQA metrics for different data sizes (5000 and 10 000) and embedding dimensions ($D = 1$ and optimum $D$) along with the time to compute $E_{\text{trans}}$ at different values of $\Phi$. Note that all computational efforts are tested on a personal computer with i5-4460 CPU and 32 GB RAM. We observe that the time required to compute laminarity is higher when $\Phi$ is far away from LBO and the computational time reduces as the system approaches LBO when $f$ is considered to be 2 kHz and an optimum $D$ is chosen through FNN...
[Fig. 11(d)]. However, the computational time reduces substantially (here in the range of 2–4s) with $f = 1$ kHz and an optimum $D$ [Fig. 11(d)]. The computational time required for calculating $LAM$ for one dimensional system ($D = 1$) and $f = 1$ kHz further reduces to $t_{c} = 1.686–1.874$ s and becomes slightly lower than that of $E_{b0}$ for which $t_{c} = 1.737–2.206$ s [Fig. 11(d)].

We observe a similar time requirement of trapping time ($TT$) at different operating conditions [Fig. 11(f)] while the computational time required for $DET$ is slightly higher [Fig. 11(e)] at $f = 1$ kHz. It may be noted that the computing times of laminarity and trapping time with both $D = 1$ and optimum $D$ are similar to that of translational error, especially for $\Phi / \Phi_{BO} \leq 1.25$. Thus, laminarity (LAM) and trapping time ($TT$) can be recommended to be used as online LBO control variables where the data size and embedding dimension should be chosen in such a way that the dynamical transition of the system is well captured and at the same time, the metric takes less computational effort. In addition, determinism can be used as a control variable depending upon the speed at which the control variable is being changed.

V. CONCLUSIONS

We experimentally investigate the dynamics near LBO and attempt to find out the early predictors of it using recurrence quantification analysis (RQA). The nonlinear analysis is performed on $CH^{\ast}$ chemiluminescence, which is assumed to be proportional to the chemical reaction rate. Thus, an effort is given to characterize the flame behavior in terms of chemiluminescence fluctuations. Although the transition in dynamics prior to blowout is quite visible in the chemiluminescence time series for different levels of premixing, the observation of the transition is qualitative in nature and some suitable measures are needed to quantify such transition. The present study finds recurrence analysis to be effective for capturing the transition in combustion dynamics prior to LBO. In this regard, one needs to evaluate the governing parameters such as the embedding dimension ($D$), optimum time delay ($\tau_{d}$), threshold value ($\epsilon$) and size of the attractor ($Z$) properly to reveal the combustion dynamics, with consideration of both accuracy and computational effort.

The significant observations from the present study can be summarized as follows:

- Recurrence plots (RP) capture the variation of combustion characteristics from stable operation to lean blowout for premixed flames through which only the detection of LBO may be possible. However, significant variation is not found using RP as the degree of the premixedness reduces (see the results using fuel port F4).
- Several RQA measures such as laminarity, determinism and trapping time appear to be robust as the variations of these measures are most sensitive to the changes in the equivalence ratio and can predict an impending LBO at all levels of premixing. Moreover, we observe that the measures exhibit similar trend with lower data size and without embedding the data in high dimension. Therefore, the computational times estimated for the calculation of laminarity and trapping time are low. However, the computational time for these measures would vary with the length of the data set, apart from the system dynamics.

- Recurrence rate may be used only as a detector of LBO for premixed flames and may not be useful as an early predictor of LBO for both premixed and partially premixed flames. Furthermore, average diagonal line length and entropy are not found to be useful as early predictors at any level of premixing.

Furthermore, to the best of the authors’ knowledge, this is the first application of recurrence quantification for characterizing and developing robust signatures for early detection of LBO in premixed as well as partially premixed flames.

SUPPLEMENTARY MATERIAL

See the supplementary material for the results of RP (Fig. S1) and RQA (Fig. S2) are shown for the variation of embedding dimension ($D$), threshold ($\epsilon$), and time delay ($\tau_{d}$).

ACKNOWLEDGMENTS

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APPENDIX A: SELECTION OF THE OPTIMUM THRESHOLD ($\epsilon$)

In network theory, the phase space vectors are considered as nodes of a network. In other words, if the nodes are sufficiently close to each other, then a link exists between them. The connection between the network nodes are described in terms of adjacency matrix, $A_{ij} = R_{ij}(\epsilon) - \delta_{ij}$ (where $\delta_{ij} = 1$ if $i = j$; otherwise, $\delta_{ij} = 0$). In the present work, we form the Laplacian matrix, $L_{ij}$, as $b_{i}b_{j} - A_{ij}$, where $b_{i}$, degree of $i$th node, is calculated as $\sum A_{ij}$. The eigenvalues, $E$, are obtained from $L_{ij}$ using eig command in Matlab. Now, the eigenvalues are sorted in ascending order to find out the second minimum value ($\lambda_{2}$). In Figs. 12(a) and 12(b), we vary $\epsilon$ parameter with $\lambda_{2}$ until a connected network is ensured, where $\lambda_{2} > 0$. We can consider the range of optimum threshold as 0.26–0.32 (for $\Phi / \Phi_{BO} = 1.06$) and 0.30–0.38 (for $\Phi / \Phi_{BO} = 1.235$), respectively, in two cases to gain the confidence of having fully connected network for estimating the recurrence matrix. Furthermore, we calculate the attractor sizes ($Z$) corresponding to different equivalence ratios for the port F1 and find that the threshold values becomes 18% of the attractor size (Table 1) and this remains same for the data with other ports (F2–F5).

APPENDIX B: CALCULATION OF LENGTHS OF DIAGONAL OR VERTICAL LINE STRUCTURES

a. Length of vertical line structure $v$

For the measurement of laminarity (LAM) and trapping time ($TT$), the length of a vertical line ($v$) is an important parameter. A recurrence point $(i, j)$ on column $i$ belongs to a vertical line if $(R_{ij} + R_{i,j+1}) > 0; j \in [1, \ldots, N]$ with $R_{i0} = R_{in+1} = 0$. The


length of the vertical line is obtained as $\epsilon$ if the recurrence points corresponding to states $R_{ij+1}, R_{ij+2}, \ldots, R_{ij+L}$ satisfy the above relation but those corresponding to $R_{i+j+1}$ and $R_{ij}$ do not.1

b. Length of diagonal line structure I

For the measurement of determinism (DET), entropy (ENT) and average diagonal line length ($L$), calculation of length of a diagonal line ($l$) is very crucial. A recurrence point $(i, j)$ belongs to a diagonal line if $(R_{ij} - R_{i+1,j+1}) + (R_{ij} - R_{i-1,j-1}) > 0: j \in [1, \ldots, N]$ with $R_{0,0} = R_{N+1} = 0$. The length of the diagonal line is obtained as $l$ if the recurrence points corresponding to states $R_{i+j+1}, R_{i+j+2}, \ldots, R_{i+j+L}$ satisfy the above relation but those corresponding to $R_{i+j+1+j+1}$ and $R_{ij}$ do not.1

Table I. The size of the attractor ($Z$) and optimum threshold ($\epsilon$) estimated are given below for different $\Phi/\Phi_{LBO}$ using port F1. Attractor size is defined as the maximum value of phase space distance. The variability in attractor size and optimum threshold is evaluated considering four sliding windows in time, each having 10,000 data points.

<table>
<thead>
<tr>
<th>Operating conditions ($\Phi/\Phi_{LBO}$)</th>
<th>Attractor size ($Z$)</th>
<th>Optimum threshold taken where $\lambda_2 &gt; 0$</th>
<th>Ratio of optimum threshold and attractor size ($\epsilon/Z$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.008</td>
<td>1.718 ± 0.173</td>
<td>0.308 ± 0.0306</td>
<td>~0.18</td>
</tr>
<tr>
<td>1.026</td>
<td>1.951 ± 0.366</td>
<td>0.351 ± 0.0655</td>
<td>~0.18</td>
</tr>
<tr>
<td>1.043</td>
<td>1.79 ± 0.220</td>
<td>0.323 ± 0.0445</td>
<td>~0.18</td>
</tr>
<tr>
<td>1.060</td>
<td>1.738 ± 0.485</td>
<td>0.319 ± 0.0874</td>
<td>~0.18</td>
</tr>
<tr>
<td>1.11</td>
<td>1.404 ± 0.104</td>
<td>0.258 ± 0.0205</td>
<td>~0.18</td>
</tr>
<tr>
<td>1.235</td>
<td>2.133 ± 0.158</td>
<td>0.389 ± 0.0306</td>
<td>~0.18</td>
</tr>
<tr>
<td>1.288</td>
<td>2.543 ± 0.140</td>
<td>0.457 ± 0.0252</td>
<td>~0.18</td>
</tr>
<tr>
<td>1.409</td>
<td>2.814 ± 0.103</td>
<td>0.523 ± 0.0170</td>
<td>~0.18</td>
</tr>
</tbody>
</table>

Figure 12. Observation of the second smallest eigenvalue ($\lambda_2$) of the Laplacian matrix with variation of threshold value ($\epsilon$) at two operating conditions, $\Phi/\Phi_{LBO} = 1.06$ (a) and $\Phi/\Phi_{LBO} = 1.235$ (b). The thresholds at which $\lambda_2 = 0$ indicates the existence of unconnected components in the network. The dashed line parallel to abscissa is a reference for zero value of $\lambda_2$. $\lambda_2$ values are calculated using CH$^+$ chemiluminescence data for port F1. 10,000 data points ($N_T$) are considered for this calculation.

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