6

SINGLE-CHANNEL TECHNIQUES
FOR INFORMATION EXTRACTION
FROM THE SURFACE EMG SIGNAL

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6.1 INTRODUCTION

Signal processing techniques are mathematical procedures that can be usefully applied to extract information from biomedical signals. This chapter describes some of the most commonly used techniques for processing single-channel surface EMG signals. Some basic knowledge of signal theory (the concepts of complex numbers, convolution, Fourier transforms, autocorrelation, and stochastic processes) is assumed [94]. The single-channel techniques described in this chapter are used to study the interference pattern that results from the simultaneous activation of many motor units (MUs). These techniques do not
resolve, or decompose, the signal into the individual MUs, rather they provide a global
description of the electric potential observed at the recording site. Such information is
usually not directly related to physiological phenomena or events. For example, represen-
tation of a signal as a sum of sine waves (harmonics) does not imply that the physiology
formed the signal by generating and then adding sine waves; modeling of a random
signal as the output of a filter with random noise as input does not imply that the signal
was generated in such a way. Nevertheless, these mathematical representations are pow-
ful tools used to detect and quantitatively describe the recorded signal resulting from
physiological events.

Different approaches—some traditional, others emerging—are discussed in this
chapter. In the time domain, the dominant change in the single-channel EMG is a modu-
lation of the signal amplitude due to muscular effort and/or fatigue. As muscle effort
increases, the signal strength (or amplitude) grows. Estimates of the EMG amplitude are
used as the control input to myoelectric prostheses and as indicators of muscular activity
or fatigue. In the frequency domain, the dominant change in the single-channel EMG
during sustained contractions is a compression of the signal spectrum toward lower
frequencies. Measures of this compression are associated with metabolic fatigue in the
underlying muscle. Spectral changes can also be evaluated in the time domain with tech-
niques based on zero crossings of the signal or on spike analysis. Moreover emerging
methods based on nonlinear signal analysis are being applied. These techniques, known
as recurrence quantification analysis (RQA), are based on detecting deterministic struc-
tures in the signals that repeat throughout a contraction.

The current state of the art of these methods is described in the subsequent sections.
Two sections will precede these descriptions. The first provides a review of spectral esti-
mation, and the second describes “traditional” stochastic models for the observed EMG
signal. These models are used to develop and interpret most of the signal processing
techniques described in the chapter.

6.2 SPECTRAL ESTIMATION OF DETERMINISTIC SIGNALS AND
STOCHASTIC PROCESSES

6.2.1 Fourier-Based Spectral Estimators

The energy spectral density of a finite energy deterministic discrete time signal \( x(k) \) is, by
definition, the magnitude squared of the discrete time Fourier transform of the signal
\( |X(e^{j\omega})|^2 \). It represents, as a consequence of Parseval’s relation [94], the distribution
of signal energy as a function of frequency of the signal’s harmonics. The power spectral
density (PSD), \( S_{mm}(e^{j\omega}) \), of a wide sense stationary (WSS) discrete time stochastic process\(^1\)
with zero mean is by definition the discrete time Fourier transform of its autocorrelation
sequence:

\[ S_{mm}(e^{j\omega}) = \mathcal{F}\{r_{mm}(l)\} = \mathcal{F}\{E[m(k+l)m(k)]\}, \]

where \( \mathcal{F}\{\cdot\} \) is the expectation operator and \( k \) and \( l \) are discrete time indexes. The EMG signal recorded during
isometric constant force contractions can be considered a WWS process, at least for time intervals short enough
to exclude fatigue (see below in the text). In the following we will consider only zero mean WSS stochastic
processes.

\(^1\) When both the mean and autocorrelation of a discrete time stochastic process are invariant to time shifts (and
the second moment is finite), the process is said to be wide-sense stationary (WSS). In this case the mean is a
single number \( \mu \) and the autocorrelation can be represented by a sequence of numbers \( r_{mm}(l) = E[m(k+l)m(k)] \),
where \( E[\cdot] \) is the expectation operator and \( k \) and \( l \) are discrete time indexes. The EMG signal recorded during
isometric constant force contractions can be considered a WWS process, at least for time intervals short enough
to exclude fatigue (see below in the text). In the following we will consider only zero mean WSS stochastic
processes.
where $e^{j\omega k}$ represents the $k$th sinusoidal harmonic and $r_{km}(k)$ is the autocorrelation function defined as: $r_{km}(l) = E[m(k + l)m(k)]$.

In Eq. (1) the computation of the autocorrelation sequence implies an expectation whose calculation would require the availability of all the realizations of the process. It is clear that in practical applications, collection of these data is not possible. However, in the case of ergodic processes the autocorrelation sequence can be estimated from a single realization by substituting the expectation operation with a temporal average [94]. Given a limited number of samples, the autocorrelation sequence can therefore be estimated as

$$
\hat{r}_{km}(k) = \frac{1}{L} \sum_{l=0}^{L-1-k} m(k + l)m(l), \quad 0 \leq k < L
$$

where $m(k)$ is a single process realization and $L$ the number of acquired signal samples. It can be shown that estimator (2) is a biased estimator of the autocorrelation sequence. Replacing $r_{km}(k)$ with $\hat{r}_{km}(k)$ in Eq. (1) provides an estimate of the power spectrum of the process. The estimated autocorrelation sequence is generally windowed in order to reduce estimation bias, leading to a class of correlogram-based estimators.

It can be shown that the power spectral estimate based on the discrete Fourier transform of the correlation sequence estimated by (2) is equivalent to the following estimation:

$$
\hat{S}_{km}\left(e^{j\omega}\right) = \frac{1}{L} |M(e^{j\omega})|^2
$$

where $|M(e^{j\omega})|^2$ is the energy spectral density of the finite energy signal obtained by windowing one realization of the stochastic process. The estimator defined in (3) is called periodogram. The periodogram is an asymptotically unbiased estimator of the power spectrum (i.e., as $L \to \infty$, the expected value of the periodogram is equal to the true spectrum) but not consistent in the mean square sense, since its variance tends to the square of the spectrum value as $L \to \infty$. To reduce estimation variance, different approaches have been proposed, such as the average of the estimates obtained by different consecutive or partially overlapped signal epochs [109]. Moreover different window shapes have been introduced to enhance frequency resolution (in practical cases, indeed, the expected value of the periodogram estimator is the true spectrum convolved by the spectrum of the observation window).

### 6.2.2 Parametric Based Spectral Estimators

An alternative approach to spectral estimation is based on the methods referred to as parametric or model based. The theoretical basis for this class of spectral estimation techniques is the representation of the stochastic process under study as the output of a linear time-invariant (LTI) filter with white noise as its input. If the LTI filter, called the generator model, is identified, the spectrum of the process is also known. The parametric approach is based on estimation of the generator model from the available data. While the Fourier approach implicitly considers the signal to be periodic outside the observation
window, the parametric methods propose an estimate that is based on the global process whose characteristics are estimated from the available data. Thus, in theory, there are no limitations to the frequency resolution; nevertheless, some assumptions on the generator model are required. In practical applications the generator model is considered physically realizable; thus the system has to be causal, implying that the LTI filter has a rational transfer function and a finite number of poles:

\[ H(e^{j\omega}) = \frac{\sum_{k=0}^{q} b_k e^{j\omega}}{\sum_{i=0}^{p} a_i e^{j\omega}} \]  

(4)

It can be shown that the power spectrum of a process generated by filtering white noise with a LTI filter is the multiplication of the power spectrum of the input (a constant in the case of white noise) with the squared magnitude of the filter transfer function. Equation (4) provides the general transfer function that defines a so-called ARMA (autoregressive moving average) model. If \( a_i = 0 \) \((i = 1, \ldots, p)\) and \( a_0 = 1 \), a MA (moving average) model results, and if \( b_i = 0 \) \((i = 1, \ldots, q)\) and \( b_0 = 1 \), an AR (autoregressive) model results.

The problem of spectral estimation is thus converted into the problem of estimating a finite number of parameters (from which the term “parametric methods”), which are sufficient to completely describe the entire process from the spectral content point of view. For details about the methods for estimating the model parameters, the interested reader can refer to [51,63].

There are many limitations to the parametric approach. The first issues encountered in dealing with parametric methods are selecting the type (AR, MA, or ARMA) and the order of the model (number of parameters). In the ideal case the model type should always be the most general one (ARMA) and the order must be chosen larger or equal to the real order: the extra parameters will theoretically be estimated to be zero. In practical applications, AR parameters are much easier to compute than MA parameters; moreover it is always possible to represent an ARMA or MA model by an infinite AR model [63] that, in practice, will be truncated at a specific order. Thus AR models are widely used for spectral estimation. In the following we will always refer to AR models. In practice, the choice of an arbitrarily large number of parameters is not appropriate, since the unnecessary parameters are never estimated as zero. As indicated by Parzen [83], the variance of the AR spectrum estimate for large sample sizes is directly related to the number of parameters and inversely related to the number of samples (available data). Thus, in order to make the variance small, the model order \( p \) should be small. However, a small \( p \) may lead to a poor AR approximation of the true spectrum, resulting in increased bias of the estimated spectrum and lower resolution of closely spaced spectral peaks. The trade-off between estimation variance and model order is the counterpart of the trade-off between variance and frequency resolution of the Fourier-based methods. Many criteria have been proposed to estimate the appropriate model order from the available data. These criteria include Akaike’s final prediction error (FPE) [1], Akaike’s information criterion (AIC) [1], Parzen’s autoregressive transfer function criterion (CAT) [83], and Rissanen’s minimum description length criterion (MDL) [91,92]. However, often, as in the case of the surface EMG signal, the selection of the model order is based on signal simulations and is adapted to the specific application.
6.2.3 Estimation of the Time-Varying PSD of Nonstationary Stochastic Processes

If the statistical properties of a process change with time, the process is said to be nonstationary, and spectral analysis with the estimators introduced above may not be appropriate. In particular, the preceding spectral analysis techniques give frequency information without any time localization. For example, in the case of surface EMG, the signal changes its characteristics as a consequence of muscle fatigue (see Chapter 9) or as a consequence of changes in the MU pool. If we consider an EMG signal detected during a prolonged, high-effort muscle contraction of one minute and compute one power spectrum from the entire contraction, we will not obtain any information about the changes that occurred throughout the contraction. We will get some “averaged” information about the frequency content of the signal during the entire contraction. The simplest approach to obtain both time and frequency information is to divide the signal into many segments (epochs) and estimate a power spectrum for each. If the changes we want to monitor are slower than one second, for example, it would be sufficient to divide a one minute duration signal into 60 contiguous epochs (each of one second duration). In each epoch the signal can be considered as a realization of a WSS stochastic process; thus an estimate of its spectrum is feasible. This partitioning is the basic idea of the short-time Fourier transform (STFT), from which the spectrogram is defined; it is currently the most widely used method for studying non-stationary signals. In the same manner a time-varying autoregressive (TVAR) approach results if the spectra of the epochs are estimated with an AR model applied to contiguous signal epochs. The STFT and TVAR approaches can be refined by including epoch overlapping and/or windowing of the data. More advanced time-frequency approaches may be more appropriate in cases of strong nonstationarity of the signals under study [17], and these will be discussed in Chapter 10.

6.3 BASIC SURFACE EMG SIGNAL MODELS

The surface EMG signal detected during voluntary contractions is the summation of the contributions of the recruited MUs that are observed at the recording site. A simple analytical model of the generated signal, $m(k)$ (k being the discrete time index) is the following:

$$m(k) = \sum_{i=1}^{R} \sum_{l=1}^{\infty} x_{il}(k - \Phi_{il}) + \nu(k)$$  \hspace{1cm} (5)

where $R$ is the number of active MUs, $x_{il}(k)$ the $l$th MU action potential (MUAP) belonging to MU $i$, $\Phi_{il}$ the occurrence time of $x_{il}(t)$, and $\nu(k)$ an additive noise/interference term [24]. The additive noise/interference represents electrode-electrolyte noise, the noise of the electronic amplifiers, line interference, biological noise and the interference activity of MUs far from the detection point. Equation (5) is an example of a stochastic process represented by an analytical expression containing parameters which are random variables (the occurrence times of the MU firings). For the case of uncorrelated discharges, it can be shown that the resultant spectrum is the summation of the spectra of the MUAP trains. The spectrum of a MUAP train is the product of the spectrum of the MUAP (deterministic finite energy signal) with that of the point process describing the firing pattern (random
process). For the case of a Gaussian distributed interpulse interval, the spectrum of the point process is given by [56]

\[ S_f(\omega) = \frac{1}{\mu} \frac{1-e^{-\sigma^2\omega^2}}{1+e^{-\sigma^2\omega^2} - 2e^{-\sigma^2\omega^2/2} \cos(\mu\omega)} \]  

(6)

where \( \mu \) and \( \sigma \) are the mean and standard deviation of the interpulse interval, respectively. As \( \omega \to \infty \) the spectrum of the point process tends to a constant value. Substituting into (6) values of the mean and standard deviation of the interpulse interval for normal physiological conditions (e.g., a mean firing rate of 8–35 Hz and coefficient of variation of the mean interpulse interval of approximately 15%), one finds that the spectrum of the point process is nonconstant in a rather small frequency region, mainly below 30 Hz. Thus, because high-pass analog filters are used for surface EMG conditioning (see Chapter 5), the influence of the firing patterns of the MUs on the surface EMG power spectrum can be neglected in many applications. The high-frequency range (above 30 Hz) of the EMG power spectrum represents the morphology of the recorded MUAPs, influenced by the relative positions of the MUs with respect to the recording system, the electrode configuration and electrode shape and size, and the conduction velocity (CV) at which the action potentials propagate [30,57,101] (see Chapters 4 and 8). Note that in the case of correlated firing patterns, the global EMG spectrum would also contain cross-terms [111]. Dependent firing patterns are due, for example, to short-term synchronization (MUAPs of different MUs firing at approximately the same time more frequently than would be expected by chance alone) or to common drive (the common modulation of firing rates).

For certain applications, the surface EMG signal can be modeled by functional models in a coarser fashion than that provided by Eq. (5). Functional models of the EMG seek to capture the observed stochastic behavior of the EMG signal without including the complexity that would be involved in modeling the activity of each individual MU (see also Chapter 8). A complete model of this type for a single channel of EMG is shown in Figure 6.1a. This model produces a measured surface EMG \( (m_k) \) with statistical properties similar to real EMG, during both fatiguing and non-fatiguing contractions. In the model, a zero-mean, WSS, correlation-ergodic (CE), white process of unit variance \( w_k \) passes through the stable, inversely stable, linear, time-variant shaping filter \( H_{time}(e^{j\omega}) \). The white random Gaussian process and the shaping filter account for the first-order probability density of the EMG and the spectral shape of the EMG, respectively. The signal is then multiplied by the EMG amplitude \( s_k \), which modulates the EMG standard deviation based on the level of muscular activation. The filter \( H_{time}(e^{j\omega}) \) preserves signal variance so that all modulation in the standard deviation of the noise-free EMG signal \( (n_k \text{ in Figure 6.1a}) \) is attributed to changes in EMG amplitude. Finally, a zero-mean, WSS, CE noise process \( \nu_k \) is added to the signal to form the measured surface EMG \( m_k \). This noise process represents measurement noise (e.g., due to the electrode-amplifier circuitry and due to noise at the electrode–skin interface) and cannot be completely eliminated. The processes \( w_k \) and \( \nu_k \) are assumed to be uncorrelated with each other. In Figure 6.1b examples of realizations of the stochastic process defined in Figure 6.1a are reported together with the expected spectra. The shaping filter has been selected as suggested by Shwedyk et al. [97]. The expression of this suggested spectrum has two parameters \( f_s \) and \( f_i \) that allow to change the shape of the spectrum. Nonstationarity may be generated by changing these parameters during time. The first moment (mean frequency; see also below) of this spectrum can be computed analytically [26]:

\[ S_s(\omega) = \frac{1}{\mu} \frac{1-e^{-\sigma^2\omega^2}}{1+e^{-\sigma^2\omega^2} - 2e^{-\sigma^2\omega^2/2} \cos(\mu\omega)} \]
where \( a \) is the ratio \( f_h / f_l \). This model is fundamentally phenomenological and, of course, cannot be used for understanding how physiological events are reflected in the surface EMG signal features. Nevertheless, it assumes importance for the analysis of the statistical properties of estimators of signal features, such as amplitude and frequency content.

### 6.4 SURFACE EMG AMPLITUDE ESTIMATION

EMG amplitude estimation can be described mathematically as the task of best estimating the standard deviation of a colored random process in additive noise (refer to the model of Fig. 6.1). This estimation problem has been studied for several years. Inman et al. [49] are credited with the first continuous EMG amplitude estimator. They implemented a full-wave rectifier followed by a resistor-capacitor low-pass filter. Subsequent early investigators.

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**Figure 6.1.** Simulation of EMG signals by filtering white Gaussian noise. (a) The output signal is obtained by filtering white Gaussian noise with a shaping filter (from [14]). (b) Examples of expected spectra obtained from Shwedyk's expression [97] for different values of the two parameters \( f_h \) and \( f_l \). Two simulated signals obtained by filtering white Gaussian noise with the inverse Fourier transform of the square root of \( H_{time}(f) \) are also shown. (From [26] with permission)

\[
f_{mean} = \frac{2f_h}{\pi} \left[ \frac{2\alpha^2}{\alpha^2 - 1} \ln \alpha - 1 \right]
\]

where \( \alpha \) is the ratio \( f_h / f_l \). This model is fundamentally phenomenological and, of course, cannot be used for understanding how physiological events are reflected in the surface EMG signal features. Nevertheless, it assumes importance for the analysis of the statistical properties of estimators of signal features, such as amplitude and frequency content.
Gators studied the type of nonlinear detector that should be applied to the waveform. This work was primarily empirical, and led to the routine use of analog rectification and low-pass filtering to estimate amplitude. Most modern systems are digital, and use mean absolute value (MAV), also called average rectified value (ARV), and root-mean-square (RMS) indicators. Other amplitude estimators, not based on simple rectification or squaring, were later investigated.

From these later works, a standard cascade of sequential processing stages emerged to form a single channel processor for EMG amplitude estimation. The stages are (1) noise and interference attenuation, (2) whitening, (3) demodulation, (4) smoothing, and (5) relin-earization (Fig. 6.2). Noise and interference attenuation seek to limit the adverse effects of motion artifacts, electronic noise, power line interference, for example, as described in Chapter 5. The correlation between neighboring EMG samples is a consequence of the limited signal bandwidth, which reflects the actual biological generation of EMG and the low-pass filtering effects of the tissues (see Chapter 8). Decorrelation, that is whitening, makes the samples statistically uncorrelated, increases the “statistical bandwidth” (defined in [8]) and reduces the variance of amplitude estimation. Demodulation rectifies the whitened EMG and then raises the result to a power (either 1 for MAV processing or 2

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**Figure 6.2.** Cascade of processing stages used to form an EMG amplitude estimate. The acquired EMG signals are assumed to be from bipolar electrodes. The EMG amplitude estimate is $\hat{s}_k$. In the “detect” and “relinearize” stages, $d = 1$ for MAV processing and $d = 2$ for RMS processing. (Adapted from [16] with permission)
for RMS processing). Smoothing filters the signal, whereas relinearization inverts the power law applied during the demodulation stage, returning the signal to units of EMG amplitude. Powers higher than 1 or 2 could be used. The quality of EMG amplitude estimates and techniques for implementing the processing stages will be discussed in the following sections.

6.4.1 Measures of Amplitude Estimator Performance

Before describing the processing stages for amplitude estimation, it is important to describe objective assessment measures of amplitude estimator performance. When the contraction is isometric, of constant force, and nonfatiguing, it is generally assumed that the EMG amplitude should be constant. To quantify the “quality” of the amplitude estimate, it is common to define a dimensionless signal-to-noise ratio (SNR) as the amplitude estimate mean, computed over a number of signal segments, divided by the standard deviation of the estimates. This measure does not vary with the gain of the EMG channel and makes no assumption as to any relationship between EMG and muscle force. Because SNR is a measure of the random fluctuations of the EMG amplitude estimate, better estimators yield higher SNR’s. Some authors have used the square of this measure as a performance index (see Chapter 18).

When force or posture is changing, SNR is no longer meaningful and alternative measures of performance must be used. One approach has been to display a real-time amplitude estimate to the subject as a form of biofeedback. The experimenter generates a target display for the subject to track. The target is moved over the range of desired EMG amplitudes, usually via computer control. The tracking error (e.g., RMS error between the target amplitude and the estimate) serves as a performance measure, with better EMG amplitude estimators presumably providing lower error. This technique also makes no assumption of an EMG-force relationship.

Finally, a common application of surface EMG is to estimate joint torques. Again, better amplitude estimation is assumed to provide better EMG-torque estimation. Note that the amplitude of EMG is affected by many confounding factors other than joint torque, such as the subcutaneous layer thickness, the inclination of the fibers with respect to the detection system, and the interelectrode distance selected [30] (Chapter 4). As a consequence EMG-based joint torque estimates must account for these confounding factors, for example by normalization with respect to a subject and muscle specific reference value, and results must be appropriately interpreted.

6.4.2 EMG Amplitude Processing—Overview

For the functional model for sampled EMG presented in Figure 6.1, the goal is to estimate \( s(k) \) based on samples of \( m(k) \). If the myoelectric samples were comprised of independent, identically distributed (IID), noise-free random samples, then estimation of \( s(k) \) would be very simple. If the noise-free IID samples were Gaussian distributed, then classic estimation results (e.g., see [45]) show that the maximum likelihood (ML) estimate is the RMS. If the noise-free IID samples were Laplacian distributed (a more centrally peaked distribution than Gaussian), then the ML estimate is the MAV [15]. Unfortunately, EMG samples are neither IID nor noise free. Formal optimal solutions to the complete model do not exist. However, a simplifying approach can be taken to explain existing solutions (Fig. 6.2).
Stage 1: Noise and Interference Attenuation. The goal of the first stage is to eliminate additive noise, artifacts, and power line interference that are acquired along with the “true” EMG. Methods to do so are described in Chapter 5 and will not be repeated here. Note that these methods can incorporate proper skin preparation and electrode setup, analog filtering in the amplifier apparatus, adaptive digital filtering during postprocessing, and so on.

Stage 2: Whitening. Because successive samples of the EMG signal are correlated, direct information extraction from the signal is confounded (in a probabilistic sense). That is, the signal correlation temporally “weights” the information. Whitening resolves this problem by transforming the signal so that successive samples have equal “weight.” A frequency domain measure of the degree of correlation in the data is the statistical bandwidth [8]. Hogan and Mann [45] showed that as the statistical bandwidth of a signal is increased (and correlation is decreased, e.g., via whitening), the SNR of the EMG amplitude estimate (for the constant force case) increases as the square root of the statistical bandwidth. Additional details of the relationship between SNR and signal bandwidth are derived in Chapter 18. For contractions above 10% MVC, whitening has led to a 63% improvement in the SNR [14] (Fig. 6.2).

A whitening filter outputs a theoretically constant, or “whitened” power spectrum in response to an input. This filter is formed by first estimating the PSD of the EMG signal. Then the inverse of the square root of the PSD is the magnitude of the whitening filter. As long as the EMG PSD estimate is nonzero at all frequencies below the Nyquist frequency, this inverse will exist (Fig. 6.3). The phase of the whitening filter is arbitrary, but it is treated as causal for the causal EMG processor. For isometric, constant-force, non-fatiguing contractions, it is common to model the EMG as a WSS, amplitude modulated, AR process (software for doing so is readily available [87]). With this model, the PSD of EMG, denoted $S_{mm}(e^{j\omega})$, can be written as

$$S_{mm}(e^{j\omega}) = \frac{b_0}{1 - \sum_{i=1}^{p} a_i e^{-j\omega i}}$$

where the $a_i$ are the AR coefficients, $p$ is the model order, and $\omega$ is the angular frequency in rad/s. These coefficients are estimated from a calibration contraction that is typically a few seconds in duration. Once these coefficients are determined, whitening can be performed on subsequent recordings with a discrete-time MA filter, which operates as follows (Fig. 6.3):

$$y(k) = \frac{1}{\sqrt{b_0}} x(k) + \frac{-a_1}{\sqrt{b_0}} x(k - 1) + \ldots + \frac{-a_p}{\sqrt{b_0}} x(k - p)$$

where $x(k)$ are the generic data input to the whitening filter and $y(k)$ are the whitened output data. Model orders of 4–6 have been found sufficient to model the PSD for whitening purposes [14,43,104].

D’Alessio et al. [11,18,19] used a generalization of this whitening approach based on the assumption that the PSD of the EMG can vary in a general manner (i.e., not just restricted to an amplitude modulated PSD), and thus the MA whitening filter must do so as well. In this case the EMG signal is considered to be nonstationary, and thus the PSD model and the whitening filter must be continuously updated.
The whitening techniques can fail at low contraction levels because of the presence of additive background noise. Clancy and Farry [13] implemented an alternative adaptive MA whitening technique (again based on an amplitude modulated AR PSD model), by incorporating the fact that EMG is invariably acquired in the presence of an additive, broadband background noise. Thus the whitening filter should be adapted, but the adaptation is not as general as that proposed by D’Alessio et al. [19]. The adaptation scheme for the time-varying MA whitening coefficients is determined in a calibration phase based on the estimated PSD of the additive noise (measured from a rest contraction) and a reference contraction (e.g., at 50% MVC).

**Stages 3 and 5: Demodulation and Relinearization.** After the whitening stage, the signal is assumed to be noise free and uncorrelated. The standard deviation of the “true” EMG is now well approximated by the standard deviation of the resulting signal. In order to estimate this standard deviation of the EMG samples, some form of nonlinearity must be applied to the signal. In general, the nonlinearity consists of raising the absolute value of each sample to a power (demodulation). The two most common powers...
(1 and 2) will be discussed. After raising the signal to a power, the signal is smoothed (as discussed in the next section) and then relinearized. Relinearization consists of raising the signal to the inverse of the demodulation power. Hence demodulation and relinearization are considered here together. To approach this problem theoretically, it will also be assumed that the signal is WSS over the short interval being considered (this point will be addressed in more detail in the smoothing section). For testing purposes, WSS EMG samples from isometric constant-force, nonfatiguing contractions will be used.

Usually EMG samples are modeled as conforming to a zero-mean Gaussian probability density function (PDF) [35,45,46]. With a ML estimation, a second-power (or RMS) demodulator is used to give the highest theoretic SNR performance of $\text{SNR}_{\text{RMS}} \approx \sqrt{2N}$, where $N$ is the number of statistical degrees of freedom in the EMG [8,45]. For perfectly whitened data, $N$ equals the number of data samples in the smoothing window. More recently Clancy and Hogan [15] modeled an EMG sample with the Laplacian PDF (which is more peaked near zero) and showed that a first-power (or MAV) demodulator can give the best performance in this case, yielding a theoretical SNR of $\text{SNR}_{\text{MAV}} = \sqrt{N}$, which is smaller, by a factor of $\sqrt{2}$, than that of the Gaussian distribution. Their experimental comparison of MAV and RMS indicated that MAV processing had a higher SNR than RMS processing, but only by 2.0% to 6.5% [15], suggesting that, in practical conditions, RMS or MAV processing are nearly indistinguishable.

**Stage 4: Smoothing.** In the smoothing stage, several demodulated samples are time-averaged to form one amplitude estimate. A sliding window selects the demodulated samples for each successive amplitude estimate, thereby forming an averaging filter. Because EMG amplitude is, in general, changing during contraction, an appropriate smoothing window length over which the signal is “quasi-stationary” must be selected. It is found that random fluctuations in the EMG amplitude estimate are diminished with a long smoothing window; however, bias (deterministic) errors in tracking the signal of interest are diminished with a short smoothing window. This trade-off can be expressed by considering the mean square error (MSE) of the amplitude estimate: $\text{MSE} = \sigma^2 + b^2$, where $\sigma^2$ is the variance and $b$ is the bias of the estimate. An appropriate balance needs to be established. Clancy [12] derived a method for optimal selection of a fixed window length. Both the variance and bias were written as a function of the window length, and then optimization was used to determine the best length. Different results were derived for causal and noncausal (midpoint moving average) processing. For noncausal processing, the optimal window length was found to be

$$N_{\text{Noncausal}} = \left[ \frac{72}{8} \right]^{\frac{1}{8}} \left[ \frac{s_{\text{ave}}^2}{(\bar{s}^2)_{\text{ave}}} \right]^{\frac{1}{8}}$$

where $N$ is the window length (samples), $f$ is the sampling frequency (Hz), $s_{\text{ave}}^2$ is the average value of the square of EMG amplitude, and $(\bar{s}^2)_{\text{ave}}$ is the average value of the square of the second derivative of EMG amplitude. The quantities $s_{\text{ave}}^2$ and $(\bar{s}^2)_{\text{ave}}$ assume different values for different tasks and must be estimated by the user. The constant $g$ is related to the number of statistical degrees of freedom in the data, determined by the statistical bandwidth of the EMG, the number of EMG channels and the detector type (see [16,19] for details). For causal processing, the optimal window length was found to be
where $\left( \hat{s} \right)_{\text{Ave}}$ is the average value of the square of the first derivative of EMG amplitude.

Several studies have attempted to improve the amplitude estimate by dynamically adapting the window length to the local characteristics of the EMG (see [12] for a review). In direct comparison to the best fixed-length smoother, these adaptive smoothers have found little or no advantage for generic applications, with a few exceptions. Mathematically other smoothing methods are possible (e.g., Evans et al. [25] used a Kalman filter) and other techniques may prove better than the simple approaches presently discussed. Thus future work in nonstationary EMG processing is needed to develop and compare such methods to existing methods.

### 6.4.3 Applications of EMG Amplitude Estimation

For many years myoelectrically controlled upper-limb prosthetics have been a driving force in the development of EMG amplitude estimation algorithms. EMG from remnant muscles has been used to control the operation of a prosthetic elbow, wrist, and/or hand. Most frequently the electrical activity of two muscle sites is monitored. If biceps and triceps muscles remain, these sites are usually selected. A common scheme is to estimate the EMG amplitude from the two sites, and determine the difference amplitude. If the biceps amplitude is larger, flexion occurs (i.e., elbow flexion, hand closure). If the triceps amplitude is larger, extension occurs (i.e., elbow extension, hand opening). For these applications small variance estimations are required (for a more detailed description of myoelectrically controlled prosthetics, see Chapter 18).

Clinically EMG amplitude is used to study muscle coordination and activation intervals. For example, EMG amplitude is used in gait analysis to determine when various muscles are active throughout the gait cycle (see Chapter 15). Finally, surface EMG amplitude is computed as an indicator of muscle fatigue, often together with spectral analysis (see Chapter 9), since it is an indicator of CV decrease, MU pool changes, and other mechanisms occurring with fatigue. In occupational field studies, joint amplitude and spectral surface EMG analysis have also been proposed to get better insight into the development of muscle fatigue in non-isometric contractions (see below).

### 6.5 EXTRACTION OF INFORMATION IN FREQUENCY DOMAIN FROM SURFACE EMG SIGNALS

Changes in the power spectrum of the surface EMG signal during muscle contraction were observed for the first time by Piper [86] who detected a decrease in the dominant oscillation of the recorded surface EMG signal during maximal voluntary contractions, as a consequence of muscle fatigue. Subsequent investigators quantified the changes in the frequency content of the EMG signal using various spectral descriptors, such as the centroid frequency [96], the median frequency [100], and the high/low frequency ratio [41]. Moreover parameters extracted from the signal in the time domain, such as the rate of zero crossings [42,48] or the spike properties [36], were proposed as alternative indicators of changes in the surface EMG spectral content.
The theoretical basis for the interpretation of the evolution of the power spectrum of the surface EMG signal during fatigue and for understanding the factors influencing it follow from the works by Lindstrom, DeLuca, and Lago [20, 56, 57]. These works developed the theory needed to interpret frequency changes in the EMG signal with respect to the underlying physiological events. The conclusions by Lindstrom and Magnusson [57], especially those related to the effect of the spatial filter on the PSD of the detected signal, have been extensively validated experimentally. Lindstrom and Magnusson [57] also provided the basis for the interpretation of the changes of the characteristic spectral frequencies as a consequence of changes in MU CV. They proposed the following expression for the PSD of the surface signal generated by an intracellular action potential traveling along the muscle-fiber:

\[ P(f) = \frac{1}{\nu^2} G\left(\frac{fd}{v}\right) \]  

(7)

where \( \nu \) is the propagation velocity of the action potential and \( G(fd/v) \) includes the spectrum of the action potential and various geometrical factors. This expression implies that the spectrum is scaled by \( \nu \). Thus its shape does not change but only the frequency axis is scaled when \( \nu \) changes.

These concepts were further developed by DeLuca [20, 101], who clarified the fundamental role of spectral analysis in the study of muscular fatigue. The work by Lago [56] outlined the effect of the firing pattern of the active MUs on the surface EMG PSD and spectral variables (refer to Eq. (6), to be used together with Eq. (7) to derive the PSD of a MUAP train).

Since these pioneering works, a large number of studies reported the use of spectral analysis of the surface EMG signal for the investigation of muscle fatigue or MU recruitment strategies. From the experimental evidence provided by Merletti, DeLuca, and Arendt-Nielsen et al. [4, 5, 21, 77, 73], it is clear that the use of spectral shift of the EMG signal as a measure of muscle fatigue offers a more objective assessment technique compared to the more subjective clinical techniques based on mechanical fatigue. This observation led, in the early 1980s, to the development of analog and digital instruments for monitoring spectral parameters of the signal [39, 67, 84, 102].

This section describes the basic body of knowledge concerning EMG spectral analysis (by Fourier and parametric techniques) and how physiological parameters are reflected by surface EMG power spectra. The section is mostly focused on the assessment of muscle fatigue during isometric constant force contractions, since this assessment is by far the most prevalent application of surface EMG spectral analysis.

### 6.5.1 Estimation of PSD of the Surface EMG Signal Detected during Voluntary Contractions

Spectral analysis of EMG signals detected during voluntary constant force isometric contractions is usually performed using the STFT with nonoverlapping epochs of 0.25 to 1 s as described in Figure 6.4a. A few studies in the literature have reported EMG results from parametric analysis [22, 26, 64, 74, 82]. The estimation model chosen in these cases was the AR model with an order between 4 and 11.
Figure 6.4. Estimation of EMG signal spectrum. (a) Spectral estimation during voluntary contractions. The signal is divided into a sequence of epochs during which the signal is assumed stationary. (b) Spectral estimation of a signal detected during electrically elicited contractions by dividing the quasi-periodic signal into epochs containing many M-waves (a line spectrum estimate is obtained). (c) Spectral estimation of an electrically elicited signal by averaging a number of M-waves within each epoch. (d) Spectra obtained by averaging the estimations obtained from each M-wave. (From [72] with permission)
6.5.2 Energy Spectral Density of the Surface EMG Signal Detected during Electrically Elicited Contractions

Electrically evoked signals may be considered deterministic and quasi-periodic signals, with the period determined by the stimulation frequency imposed by the stimulator. Each M-wave is a finite energy, finite duration signal whose frequency content can be described by the energy spectral density. The energy spectral density reflects the properties of the detected MUAPs, in particular, their CV.

Different strategies can be applied to estimate the spectrum of an electrically elicited EMG signal during a prolonged contraction [72] (Fig. 6.4b, c, and d):

1. The signal can be divided into epochs, and the frequency attributes are computed over each epoch. The signal of each epoch includes many M-waves and is periodic; the spectral lines are separated by an interval equal to the stimulation frequency.
2. The spectral features can be computed for each M-wave, which is thus seen as a finite length, nonperiodic signal. In this case the spacing between spectral lines is determined by the epoch length and can be reduced by zero padding, which provides interpolation of an otherwise coarse spectral estimate. The resulting spectra can then be averaged to improve the quality of the estimate.
3. The signal can be divided into epochs, and the M-waves in each epoch are averaged. The spectral features are then computed for each averaged M-wave. Similar considerations as in the previous case can be drawn, but the signal to noise ratio is increased by the averaging process and the computational cost decreased.

The method usually applied is the third one.

Spectral analysis of electrically evoked EMG signals is mainly used for detecting changes of scale of the M-waves. An alternative approach for estimating scale factors in deterministic signals is to process the signals directly in the time domain. Merletti et al. [69] proposed a maximum likelihood approach to solve this problem, while more recently, Muhammad et al. [79] developed a pseudojoint estimator of the time scale factor and time delay between signals for applications of M-wave analysis during fatigue. These approaches were shown to be, in general, more robust than spectral analysis for estimating the scaling of the M-wave due to fatigue for the case of significant truncation of the wave (when the stimulation frequency is above 30–35 Hz, the stimulus interval may be shorter than the total M-wave length and thus the M-wave is truncated). Alternative approaches have been proposed by Lo Conte et al. [58], who decomposed the M-wave into a particular series of functions, and by Olmo et al. [81], who proposed the matched continuous wavelet transform to estimate the M-wave scale factor. The distribution function technique proposed by Rix and Malengé [93] can also be directly applied to pairs of M-waves for estimating the scale factor between them.

6.5.3 Descriptors of Spectral Compression

During both voluntary and electrically elicited fatiguing contractions the PSD of the EMG signal progressively moves toward lower frequencies. This phenomenon can be described to a large extent as a compression of the spectrum, meaning the shape of the spectrum does not change but only the scale factor of the frequency axis changes (refer to Eq. (7)).
and to Chapter 9). In the case of a pure scaling, a single parameter would give all the information about the phenomenon of spectral compression; thus any reference spectral frequency can be used as an estimator of spectral compression. One of the possible spectral descriptors is the mean or centroid frequency (MNF) which is defined as

$$f_{\text{mean}} = \frac{\int_{0}^{f_s/2} f S(f) df}{\int_{0}^{f_s/2} S(f) df}$$

(8)

where $S(f)$ is the PSD of the signal and $f_s$ is the sampling frequency.

MNF is the moment of order one of the power spectrum. In general, the central moments of order $k$ are defined as [95]

$$M_{ck} = \frac{\int_{0}^{f_s/2} (f - f_{\text{mean}})^k S(f) df}{\int_{0}^{f_s/2} S(f) df}$$

(9)

Merletti et al. [71] proposed a time domain technique to estimate the central moments of any order that avoids direct estimation of the PSD and is particularly suitable for real-time implementation. Other characteristic frequencies $f_p$ are the $p$th fractile frequencies, indirectly defined as

$$\int_{0}^{f_p} S(f) df = p \int_{0}^{f_s} S(f) df, \quad 0 < p < 1$$

(10)

Equation (10), with $p = 0.5$, defines the median frequency (MDF), whereas $p = 0.25$ and $p = 0.75$ define the other interquartile frequencies. Although any fractile frequency can be used to estimate spectral compression, it has been recently suggested that a better estimate of spectral compression due to CV decrease may be obtained by computing the average change of a number of properly selected percentile frequencies. Each percentile frequency indeed may provide a different indication of fatigue due to the other factors affecting the PSD of the signal apart from CV. The analysis of a number of percentile frequencies enables a distinction between the spectral changes due to CV and those due to other factors that affect spectral shape. It has been shown [59] that some percentile frequencies are better correlated with CV changes.

In addition to spectral moments and fractile frequencies, all of the parameters from an AR model spectral estimate can be used to characterize spectral changes in the EMG signal [53]. Finally the frequency at which the first dip introduced by the detection system (see Chapter 5) can be used as a descriptor of spectral changes. In this case the spectral descriptor reflects only the CV of the active MUs [57], but its variance of estimation is much higher with respect to spectral moments [65].

MNF and MDF are the most commonly used spectral descriptors. Only a few studies have used higher order moments to describe the EMG spectrum [68]. In the case of voluntary contractions, spectral descriptors are random variables with particular statistical properties that depend on the nature of the descriptor, epoch signal length, amount of epoch overlapping, type of window, and the spectrum estimator adopted. The influence of
these parameters can be evaluated by the use of simulation models such as that shown in Figure 6.1 [7,26,66].

**Properties of MNF and MDF.** MNF and MDF provide some basic information about the spectrum of the signal and its changes versus time. They coincide if the spectrum is symmetric with respect to its center line, while their difference reflects spectral skewness. A tail in the high-frequency region implies MNF higher than MDF. A constant ratio \( f_{\text{mean}}/f_{\text{med}} \) versus time implies spectral scaling without shape change, while a change in this ratio implies a change of spectral skewness or shape. It can be shown that the standard deviation of the estimate of MDF is theoretically higher than that of MNF [102], as confirmed in experimental studies [101]. However, it can also be shown that MDF estimates are less affected by additive noise (particularly if the noise is in the high-frequency band of the EMG spectrum) [101] and more affected by fatigue (since the spectrum becomes more skewed with fatigue). Because of these pros and cons and because of the additional information that is carried jointly by the two variables, researchers often use both in their reports. However, in cases where the signal-to-noise ratio may be very low, at least during particular intervals of time (e.g., at the beginning of a ramp contraction), MDF is often preferred [9,10].

**Fourier versus Parametric Approach.** The Fourier and parametric approaches have been compared using simulated EMG signals (model of Fig. 6.1) for different window lengths and degree of nonstationarity [26]. It was found that in both stationary and nonstationary conditions the two approaches lead to similar results, in terms of variance and bias of estimation, for MNF and MDF over a large range of epoch lengths. For very short epochs (below 0.25 s) the parametric approach performs better than the Fourier approach, but the difference can be negligible in practical applications. Figure 6.5 shows the comparison between Fourier and AR estimation of the PSD of experimental surface EMG signals for different epoch lengths. MNF and MDF estimates are also reported.

**Window Shape.** The type of window shape determines the bias in the power spectrum estimation, but it is difficult to predict analytically how the bias in spectral lines is reflected in the bias of spectral descriptors. Again, simulation studies provide an evaluation of this effect [66]. In the case of the generation model in Figure 6.1, it was shown that the choice of the window is not critical for MNF nor MDF estimation. The rectangular window has been used in the majority of the experimental studies.

**Epoch Length and Epoch Overlapping in Stationary and Nonstationary Conditions.** In stationary conditions (model of Fig. 6.1 with fixed parameters), the larger the duration of the signal epoch, the lower the variance and bias of estimation of the spectral descriptors. In the case of nonstationary conditions, two sources of bias are present, the bias related to the spectral estimator applied to a finite observation window and that due to the nonstationarity. Bias due to both effects increases with epoch length. On the contrary, variance of MNF and MDF estimates decreases with increased window duration. In the case of isometric, constant force, fatiguing contractions, the signal can be considered stationary for epoch durations of the order of 1 to 2 seconds. The spectral descriptors are computed from several sequential (possibly overlapping) epochs. Usually the parameters of the linear regression that best fits the time-changing values of descriptors are used as a fatigue index (see Chapter 9). A large number of short epochs (i.e., of
Figure 6.5. Comparison between periodogram (dashed lines) and AR (solid lines) spectrum estimates of an experimental surface EMG signal (muscle biceps brachii) for different lengths of the observation window. MNF and MDF are also reported. (From [26] with permission)
experimental points) in the regression interval will reduce the variance of the polynomial parameter estimates but will increase the spread of the experimental points. An additional factor of interest is the degree of epoch overlapping, which allows an increase of the number of experimental points without increasing their scatter but increasing their statistical dependence.

The standard deviation of the slope and intercept of the regression line fitting MNF and MDF in simulated fatiguing contractions (with low to medium nonstationarity) has been found to be minimal for epoch durations between 250 and 500 ms, which therefore seems to be the most suitable for regression line parameter estimation [26]. Epochs shorter than 250 ms lead to high variance and bias of estimation. Overlapping does not provide significant benefits as it increases the computational load [26].

6.5.4 Other Approaches for Detecting Changes in Surface EMG Frequency Content during Voluntary Contractions

Another possible indicator of changes in the frequency content of the signal is the rate of zero crossings, meaning the number of sign changes of the signal in the unit time. When the signal is noise free and comprised of only one sinusoidal function, this rate reflects the frequency of oscillation of the function. According to Rice [89], if the signal has a Gaussian stationary amplitude distribution, the expected number \(Z\) of zero crossings per second can be expressed by the following relationship:

\[
Z = 2 \sqrt{\frac{\int_0^{f_s/2} f^2 S(f) df}{\int_0^{f_s/2} S(f) df}}^{1/2}
\]  

(11)

As indicated above, the distribution of amplitudes of the surface EMG signal is in between Gaussian and Laplacian; thus the Gaussian hypothesis is almost verified in practical cases. From (11), it is easy to show that, as MNF and MDF, \(Z\) is scaled by CV and can thus be used for evaluating spectral compression. Although the standard deviation of estimation of the zero crossing rate has been shown to be higher than that of MNF (but similar to that of the percentile frequencies) [44], the technique does not require spectral estimation, and it is particularly easy to implement in hardware. Based on this method, Hägg [42] and Inbar et al. [48] developed simple real-time fatigue monitors in the early 1980s.

Another approach to evaluate spectral changes in the surface EMG is based on the automatic detection of spikes in the signal. A spike is defined as a segment of signal shaped by an upward and downward deflection [36]. Both deflections of a spike cross the zero isoelectric baseline and should be at least 100 \(\mu V\) in amplitude. The analysis of spike activity has a long history in both clinical neurophysiology [62] and kinesiology [105] but has received less attention for surface EMG analysis in favor of more sophisticated techniques. Mean spike frequency, that is, the average number of spikes per unit time, has been shown to be highly correlated to MNF [37], and thus, as the characteristic spectral frequencies, it can be used to monitor spectral changes in the signal. This technique has been indicated as potentially useful in EMG analysis, since it does not directly require stationarity of the signal. However, the mean spike frequency, as well as the other spike parameters (see [36] for their definition), imply the use of a signal segment for their com-
6.5.5 Applications of Spectral Analysis of the Surface EMG Signal

Spectral analysis of the surface EMG signal has been extensively applied for the study of muscle fatigue in both voluntary and electrically elicited contractions [73] (see Chapter 9). The preferred application of these techniques has been the analysis of isometric constant force, short duration, and medium–high level contractions. It is now accepted that EMG spectral variables reflect fatigue with the possibility of detecting some differences in muscle fiber composition [55, 76, 70]. Applications of spectral analysis of the EMG signal for fatigue assessment during nonconstant force and dynamic contractions have been proposed in more recent years together with advances in spectral estimation techniques based on time-frequency representations [54]. Nevertheless, many artifacts, mainly related to geometrical and anatomical factors of the EMG generation system, may be associated with these approaches [31]. The relevance of these artifacts is still not fully clear; thus caution should be taken in extending the considerations drawn for isometric conditions to dynamic exercises.

The analysis of the surface EMG PSD has also been applied to the investigation of MU recruitment strategies, in an attempt to extract information about central nervous system’s (CNS) motor control strategies from a global analysis of the surface EMG signal (i.e., without decomposing the individual MU activities). It was speculated that MNF and MDF should reflect the recruitment of new, progressively larger and faster MUs and increase until the end of the recruitment process. They should then reach a constant value (or decrease) when only rate coding is used to track the desired target force level [9, 98]. However, Farina et al. [29] indicated that in general, the establishment of a relationship between force and characteristic spectral frequencies is confounded by anatomical factors (see Chapter 4). Figure 6.6 shows CV and MNF of simulated EMG signals detected during ramp contractions.

6.6 JOINT ANALYSIS OF EMG SPECTRUM AND AMPLITUDE (JASA)

In nonisometric contractions that imply changes of muscle activity with recovery periods—a situation typical, for example, in occupational field studies—it is very difficult to interpret amplitude and spectral changes of the surface EMG signal independently. Recently the joint analysis of EMG spectrum and amplitude has been proposed to overcome some of the problems that arise in these situations. As both amplitude and spectral variables depend on muscle force and fatigue, amplitude increases with force and fatigue (see Chapter 9). Spectral variables decrease with fatigue, while their dependency on muscle force is not completely clear and contradictory results are reported in the literature [38, 85, 110]. Farina et al. [29] recently showed that it is not possible to establish a general relationship between muscle force and surface EMG spectral variables. However, these authors also indicated that, in agreement with past results from the literature, the most probable behavior is an increase with force (see also Fig. 6.6). The joint analysis of spectrum and amplitude (JASA) method [61] is thus based on the assumption of an increase of amplitude with fatigue, and force, a decrease of MDF with fatigue, and an
increase of MDF with force. By comparison of MDF and amplitude it is thus possible to identify four regions of muscle activity during a dynamic task: (1) force increase (both amplitude and MDF increase), (2) fatigue (amplitude increases and MDF decreases), (3) force decrease (both amplitude and MDF decrease), and (4) recovery (amplitude decreases and MDF increases). Although based on many simplifying assumptions, the method is easy to apply and does not require complex algorithms [60].

6.7 RECURRENCE QUANTIFICATION ANALYSIS OF SURFACE EMG SIGNALS

Recently nonlinear methods, such as recurrence quantification analysis (RQA), were introduced for the study of single-channel surface EMG signals, mainly in fatigue assessment. In general, the range of applications of nonlinear techniques to problems in biomedicine
is rapidly expanding and spans from studies of the heart beat [40,50,88,90,114] to brain rhythms [6,112], from the neuromuscular system [34,52,78,80] to blood pressure regulation [3,106], from the breathing system [2] to cardiorespiratory coordination [47]. Non-linear analysis was introduced in the study of surface EMG first by Webber et al. [107] and Nieminen and Takala [80]. RQA, described by Eckmann et al. [23], is based on a graphical method originally designed to locate recurring patterns (hidden rhythms) and nonstationarities (drifts) in experimental data sets. By mapping the signal in a bi-dimensional space (as described below), it is possible to identify time recurrences that are not readily apparent in the original recordings, either by qualitative visual inspection or by evaluating some specific variables, derived from the bi-dimensional maps, which quantify the deterministic structure and complexity of the plot itself. This method was recently used in some experimental surface EMG studies [32,33] that showed its potential in detecting changes of muscle properties due to fatigue.

6.7.1 Mathematical Bases of RQA

If we consider the neuromuscular system to be investigated as a dissipative dynamical system governed by a set of $D$ first-order differential equations, the states of the system can be represented by points in a $D$-dimensional space where the coordinates are the values of the state variables. In most biological experiments, where it is not possible to measure all components of the vector giving the state of the system, it is feasible to reconstruct the system dynamics from a one-dimensional output of the system by mapping it in a $D$-dimensional space using delay coordinates. This procedure follows Taken’s embedding theorem [103].

The main steps of the algorithm for projecting the original time series of the surface EMG samples into the phase space by means of the time-delay embedding procedure are given in Figure 6.7. The procedure is applied to a signal segment of $K$ samples:

$$s(k) = [s(0) \ s(1) \ldots s(K-1)]$$

Recent applications of RQA to surface EMG [32,34,108,107] have selected a $K$ such that the interval of observation of the signal is equal to that used for spectral or amplitude analysis in order to compare the results obtained by different techniques.

The surface EMG samples in the epoch selected are time shifted by an integer number $\lambda$ of samples usually estimated as the first zero of the autocorrelation function [80]. This choice uncorrelates the elements of the $D$-dimensional vectors $v(n)$ that are extracted from the original myoelectric time series as

$$v(0) = [s(0) \ s(\lambda) \ldots s((D-1)\lambda)]$$

$$v(1) = [s(1) \ s(\lambda+1) \ldots s((D-1)\lambda+1)]$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$v(N-1) = [s(N-1) \ s(\lambda+N-1) \ldots s(K-1)]$$

The last component of the last vector corresponds to the last sample of the original time-series and the number of vectors is

$$N = K - (D-1)\lambda$$
Choice of the Embedding Dimension $D$. Nieminen et al. [80] showed that based on a correct evaluation of the correlation dimension, it is possible only to define a minimum necessary embedding dimension $D = d_{\text{min}}$, which is related to the muscular task under consideration. More recently Filligoi and Felici [34] have analyzed the problem of a correct and unique evaluation of $D$ remaining valid throughout the course of a complete experiment. The criterion that has been used is based on the evaluation of saturation effects of the nonlinear variables under consideration (see below for their definition), when varying $D$ over a range of values with a technique analogous to that applied to studies of classic chaotic systems, such as the Lorentz–Mackey-Glass differential delay equation or the Henon map (e.g., see [113]). The results showed that the best value to be given to the embedding dimension, as a trade-off between sufficient resolution in the phase space and the need to limit computational time, is $D = 15$ for both constant and nonconstant force contractions.
**Distances Matrix DM.** The states of the dynamic system under consideration are represented by the vectors \( \mathbf{v}(n) \) defined in (13). The closeness of the vectors is then evaluated by the definition of a distance between vectors. The distance commonly used is Euclidean:

\[
d(i, j) = \left[ \langle (\mathbf{v}(i) - \mathbf{v}(j))^2 \rangle \right]^{1/2}
\]  

(15)

In order to make the results of the analysis independent of the energy of the observed signal, the usual effective values adopted in RQA are either expressed as a percentage of the maximal distance (considered as 100) or normalized with respect to the average distance between vectors:

\[
d_{av} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} d(i, j)}{N(N - 1)/2}
\]  

(16)

where the denominator represents the number of distances \( d(i, j) \).

The collection of these normalized distances provides a symmetric matrix called the distance matrix, \( \mathbf{DM} \):

\[
\mathbf{DM} = \begin{bmatrix}
    d(1, 1) & d(1, 2) & \ldots & d(1, N) \\
    d(2, 1) & d(2, 2) & \ldots & d(2, N) \\
    \vdots & \vdots & \ddots & \vdots \\
    d(N, 1) & d(N, 2) & \ldots & d(N, N)
\end{bmatrix}
\]  

(17)

**Recurrence Map RM.** A recurrence plot is finally obtained as a map of pixels, which assume the values 0 or 1 on the basis of a threshold (RADIUS) set on the distance matrix, \( \mathbf{DM} \). The following comparison

\[
d(i, j) \leq \text{threshold} \quad \Rightarrow \quad \text{YES} \rightarrow \text{pixel } b(i, j) = \text{ON}
\]

\[
\text{ON} \rightarrow \text{pixel } b(i, j) = \text{OFF}
\]  

(18)

provides the recurrence map (\( \mathbf{RM} \)) as the collection of all \( b(i, j), \forall i, j \). By the mathematical development above, recurrence maps (1) are symmetric, since \( d(i, j) = d(j, i) \) and (2) are characterized by a main diagonal of pixels ON, since obviously \( b(i, i) = 1 \).

The threshold operation is conceptually equivalent to considering two states of the dynamical system as close to each other when the embedded vectors \( \mathbf{v}(i) \) and \( \mathbf{v}(j) \) are enclosed in a \( D \)-dimensional hypersphere with radius equal to the selected threshold. The choice of high threshold values leads to considering too often the system states as near neighbors (pixel ON). The choice of low values for the threshold leads to opposite results. Fine-tuning of the threshold value will be described below, since it is strictly related to the variables extracted from the procedure of quantifying the \( \mathbf{RM} \). Figure 6.8 shows examples of recurrence maps of synthetic signals.
Quantification of the Recurrence Map. Often recurrence maps contain subtle patterns that are difficult to detect by visual inspection. Hence quantitative descriptors that emphasize different features of the map have been proposed [108]. Among them the percentage of recurrence structures (%REC) and the percentage of determinism (%DET) have been the most used and describe the following quantities:

1. %REC is the percentage of pixels on with respect to the total number of pixels in RM and measures the number of embedded vectors close to each other. This parameter indicates how far the time series is from a purely random dynamic system (typically represented by %REC in the range 5–15%) and is highly influenced by the threshold value (see below).

2. %DET is the percentage of points that form upward diagonal lines (with length greater than a prefixed cutoff value LINE) with respect to the number of pixels on in RM. Points organized into diagonal patterns represent strings of vectors re-occurring at different times and are indicative of systems progressing through similar states in the phase-space.

Selection of RADIUS and LINE. RADIUS should assume the minimum value compatible with the baseline level of noise. A way to accomplish this task is to record surface EMG signals while the muscle is relaxed and then select RADIUS such that %REC is very low (e.g., between 5% and 15%). LINE value is not critical. Nevertheless, the choice of an excessively low value (2 or 3) could produce an erroneous detection of deterministic structures which are randomly present in the system. Again, when the results

Figure 6.8. RQA applied to a sinusoid (a), a sinusoid plus additive noise, (b) with two SNRs, and white noise (c). The values of the five variables extracted in these cases are also indicated.
obtained on white noise series were compared, a value of 20 of the parameter LINE appeared to be appropriate [32,34].

### 6.7.2 Main Features of RQA

The aim of the recurrence analysis is to enhance the presence of repetitive patterns within the surface EMG time series. Subtle time correlations are more easily revealed in a bi-dimensional perspective than from direct observation of the original mono-dimensional time series. From this perspective the main features of the recurrence representation $RM$ are the following:

- Single isolated points are due to chance recurrences and are characteristic of stochastic behavior.
- Upward diagonal lines result from strings of vector patterns repeating themselves in time. Therefore the presence of diagonal lines indicates that deterministic patterns are present in the phenomenon under observation.
- Downward diagonal lines occur whenever the vector sequences at different locations are mirror images of each other.
- A horizontal or a vertical line results when a specific vector is closely matched with other vectors separated in time.
- Bands of white space are due to the presence of transients (i.e., events that lie far outside the normal distribution of values).
- Non-uniform texture or paling away from the central diagonal line is an indicator of nonstationarities in the time series.

Examples of recurrence maps and their RQA variables applied to computer generated signals (a sinusoid, a sinusoid plus additive noise with signal to noise ratio equal to 10 and 25 dB, and a pure random noise) are given in Figure 6.8. The squared diagonal structure presented by the $RM$ in the left side is obviously characteristic of the repetitive structure of the sinusoid. The high value of %DET is due to the fact that most points in the $RM$ belong to diagonal lines, as it would be expected for a purely rhythmic signal. When some noise is superimposed on the sinusoid, the regularity of the structure is partially broken and recurrent points are more spread. This phenomenon is detected by %DET with a consistent reduction of its value. In the case of pure noise, points are randomly distributed in the map and %DET is drastically reduced.

### 6.7.3 Application of RQA to Analysis of Surface EMG Signals

As explained above, RQA primarily extracts information on recurrence structures that are repeated along the signal. For surface EMG, MUAPs are repeating during time regularly, forming an interference pattern when the number of active MUs is high. Thus we have a situation of deterministic structures repeating in a background activity. If the contraction level increases, it is expected that the number of deterministic structures also increases, while with fatigue RQA may detect short-term synchronization among MUAP trains (which should increase the determinism of the signal) or changes in muscle fiber CV. However, due to the nonlinear nature of the RQA approach, it is difficult to predict ana-
lytically the sensitivity of recurrence variables in response to the different changes in muscle properties.

An example application of the RQA technique to an experimental surface EMG signal is provided in Figure 6.9. Signals recorded at very low contraction levels (near 0% of the maximal voluntary contraction, MVC) are often corrupted by several sources of environ-
mental noise (Fig. 6.9, box a1). When the muscle effort increases (Fig. 6.9, box a2, where the muscle load is 20% MVC), several physiological and experimental variables interact in a complicated manner [32,75,99]. When the number of active MUs is large enough for the action potentials to overlap (tract of surface EMG data corresponding to 80% MVC, between second 35 and second 53 in Fig. 6.9), the surface EMG is well described as a Gaussian or Laplacian distributed stochastic process with zero mean and variance related to the muscular effort. Deterministic structures are clearly recognized by RQA in this case (%DET increases with the muscular effort). When this strong muscular effort is exerted for a sufficiently long interval, fatigue phenomena appear (in Fig. 6.9, box a3) and are detected by RQA with a further increase of %DET.

Experimental fatigue studies on sedentary subjects and athletes indicated that RQA is a suitable method for assessing muscle fatigue with some advantages with respect to classic spectral and amplitude analysis [32]. However, the main issue is to verify which properties of the neuromuscular system RQA mostly reflects, and this can be done only by modeling the generation of the surface EMG signal taking into account the effects of anatomical, physiological, physical, and detection system parameters (i.e., with a structure based model rather than a phenomenological model). As mentioned in the introduction to this chapter, most of the techniques and the variables described here are not directly related to physiological phenomena (i.e., they do not provide a direct measurement of any physiological variable). This limitation is particularly true for RQA. The most commonly used variable extracted from the recurrence maps, %DET, does not, of course, directly indicate any physiological events. It is (as are MNF, MDF, zero crossing rate, and almost all the variables previously introduced) a mathematical indicator that may be sensitive to physiological events of potential interest.

The most difficult task is to define the relationships between these mathematical variables and the properties of the system under study. For RQA, this task has yet to be done, except for a recent modeling study by Farina et al. [28]. These authors have generated surface EMG signals with a model varying the mean MU CV and the degree of MU short-term synchronization. Then they applied classic spectral analysis and RQA to the synthetic signals. It was shown that %DET is sensitive to both CV and degree of synchronization changes during fatigue, as it happens with traditional spectral techniques.

In addition, %DET and results of traditional spectral techniques were highly correlated, thus being similarly sensitive to the muscle parameters. However, %DET was shown, both experimentally [28,107] and in simulation [28], to be more sensitive to fatigue-induced changes than spectral analysis. Thus RQA may be a potential alternative technique for muscle assessment with respect to spectral analysis to characterize different muscles and/or subjects in fatiguing conditions.

Figure 6.10 shows the power spectral densities and the recurrence maps of two simulated surface EMG signals (the model used is described in [27]), generated from MUs with different mean CV and degree of short-term synchronization. The recurrence maps clearly show the differences between the two signals, and %DET quantifies these differences. In particular, a higher number of rule-obeying structures in the signal with decreasing CV and increasing the degree of short-term synchronization are clearly shown by a higher regularity in the recurrence map. Other details on the applications of this technique will be described in Chapter 14.

The application of RQA to surface EMG still needs, however, additional research efforts to determine its sensitivity to other factors of variability between subjects and muscles, such as the thickness of the subcutaneous layers, the orientation of the detection system, the interelectrode distance, the electrode location, the recruitment of MUs, and
additive noise. While the influence of all these parameters on spectral variables has been analyzed in past works, there is a complete lack of this information for RQA.

6.8 CONCLUSIONS

Single-channel processing methods for surface EMG based on the interference pattern of the signal (without aiming at separating the contributions of individual MUs) have been
discussed. This research focuses on the establishment of the relationships between the
global variables obtained from the signal and the underlying physiological processes.
Amplitude and spectral analysis have been used for many years for assessing muscle activ-
ity and fatigue. Although they have some limitations (primarily related to the impossibil-
ity of extracting information of single MU properties), these techniques are useful in a
number of basic and clinical research studies. Other global processing techniques, less
commonly applied, have been also proved to be potentially useful for muscle assessment,
and they are currently being investigated in experimental and modeling studies. Recently
large research efforts have been devoted to the development of advanced signal process-
ing and detection methods (see Chapters 7 and 10) for more detailed analysis of the surface
EMG signal. These methods are mostly focused on the extraction of more localized infor-
mation on muscle activity (for example related to a small number of MUs). However, fre-
cquency and amplitude analysis are still by far the most widely used methods both in basic
and applied studies. Their great advantage with respect to other techniques is indeed the
simple applicability to a number of experimental conditions and recording systems.

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