Recurrence Plots for Dynamic Analysis of Type-I ELMs at JET With a Carbon Wall
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Abstract—In this paper, the dynamic characteristics of type-I edge-localized modes (ELM) time series from the JET tokamak, the world’s largest magnetic confinement plasma physics experiment, have been investigated through recurrence plots (RPs). The analysis has been focused on RPs of pedestal temperature, line averaged electron density, and outer divertor $D_{\alpha}$ time series during experiments with a carbon wall. The analysis of RPs shows the patterns similar to those characteristics of signals exhibiting type-2 intermittency, in particular, a characteristic kite-like shape; this gives useful hints to model the temperature signal as well as the $D_{\alpha}$ radiation time series, with simple nonlinear maps capturing the nearly periodic behavior of type-I ELMs.

Index Terms—Edge-localized modes (ELM), ELM modeling, recurrence plots (RPs).

I. INTRODUCTION

In 1982, it was first observed at the axially symmetric divertor experiment tokamak that externally heated tokamak plasmas can rapidly reach an operating regime of improved confinement. The operating regime transition is normally accompanied by the appearance of recurrent magnetohydrodynamic instabilities, known as edge-localized modes (ELMs). ELMs manifest themselves as short repetitive bursts of energy and particles at the plasma edge. The loss of energy and particles, which flow along the magnetic field lines toward the divertor plates, deteriorates the confinement, which may cause damage to the first wall. In addition, the short but intense particle and power loads on the divertor cause erosion of the plates, which might become a serious concern in the future machines, such as the International Thermonuclear Experimental Reactor (ITER). Despite these drawbacks, ELMs are beneficial in expelling exhaust impurities and helium ash that otherwise would accumulate in the plasma and eventually terminate the fusion burn. They also provide a means for density control. Hence, the understanding and control of the level and nature of the ELM activity are crucial for the achievement of the fusion power.

Although a number of ELM types have been classified, the physics of ELMs is still to a large extent unresolved. The information about the ELM dynamics is fundamental to develop a proper dynamical model, which could provide guidance in mitigating ELMs in view of the steady-state operation of ITER. Several attempts are reported in the literature to identify the nature, stochastic, or deterministic of ELMs, and the results are sometimes conflicting. In [1], the results of a nonlinear dynamic analysis of type-I ELM time series excluded the presence of chaos, leading to the hypothesis of pseudoperiodic behavior.

In this paper, the dynamic characteristics of type-I ELM time series from the JET tokamak, the world’s largest magnetic confinement plasma physics experiment, have been investigated through recurrence plots (RPs). This tool, together with the recurrence quantification analysis, is a modern method of nonlinear data analysis. The tool of RPs, introduced by Eckmann et al. [2], allows the visualization of a square matrix, in which the matrix elements correspond to those times at which a state of a dynamical system recurs, revealing if the phase space trajectory of the dynamical system visits roughly the same area. The advantage of RPs is that they can also be applied to rather short and even nonstationary data.

A huge amount of data for each JET experiment is available up to several tens of gigabytes. Time series of relevant plasma parameters are stored in the JET data warehouse. For ELM studies, the analysis has been focused on the pedestal temperature, the electron density at the edge of the plasma, and outer divertor deuterium visible light emission ($D_{\alpha}$) time series.

Fig. 1 shows the pedestal temperature, the line averaged electron density at the edge, and the $D_{\alpha}$ time series for shot 74444.

All temperature, density, and $D_{\alpha}$ time series exhibit early regular behavior (laminar flow) intermittently interrupted by outbreaks (bursts) at irregular intervals. From a physical point...
of view, the ELM cycle consists of two phases: an ELM crash due to the MHD instability inducing energy and particle losses from the pedestal, and a quiescent phase recovering the pedestal pressure between ELMs (inter-ELM phase). The delay period between ELMs is apparently random; for this reason, the nature of the irregularities is still under debate, and the literature reports several attempts to establish if the ELM nature is deterministic or stochastic [3]–[9]. As it can be noted, each ELM cycle is characterized in all the time series by asymmetry, i.e., temperature and density decrease ($D_n$ increases) more rapidly than they increase (it decreases). This type of behavior is typical for intermittent systems [10]. Such signals are copious in natural and physiological systems, e.g., annual sunspot numbers [11], [12], laser output [13]–[17], and human electrocardiogram [18], [19].

This paper is organized as follows. Section II describes the RPs. Section III presents the database. Section IV reports the results of the analysis. In Section V, the conclusion is drawn.

II. Recurrence Plots

Let us consider a dynamical system

$$\frac{dx(t)}{dt} = f(x)$$

(1)

and a scalar observable $y(t)$. The first step to build an RP is to reconstruct the dynamics by embedding the 1-D time series $y(t)$ in a $d$-dimensional space using the method of delay coordinates [20]. The general topological result of Mañé [21] and Takens [22] states that the complete dynamics of a system can be reconstructed from a set $y_n = [y_{n-(m-1)\tau}, y_{n-(m-2)\tau}, \ldots, y_{n-\tau}, y_n]$ of time-delayed versions of a suitable scalar measurement $y_n$ derived from the system at multiples of a fixed sampling time, where the embedding dimension $m$ and the time delay $\tau$ are the two critical parameters. Mañé [21] and Takens [22] proved that if $m \geq 2D + 1$, where $D$ is the box counting dimension of the attractor, there exists a one-to-one correspondence between the state space reconstructed and the original one. This is a sufficient but not necessary condition. In practice, an attractor may also be reconstructed successfully with an embedding dimension satisfying $m \geq D$. The embedding dimension $m$ can be estimated through the method of false nearest neighbors (FNNs) [23]. A simple criterion to compute the time delay $\tau$ is to set it equal to the first minimum of the average mutual information $I_\tau$ [24], when present.

After reconstructing the embedding space, the RP can be mathematically expressed by a 2-D squared matrix, the so-called recurrence matrix

$$r_{i,j} = \Theta(\varepsilon - ||y_i - y_j||),$$

$$(2)$$

where $N$ is the number of considered samples $y_i$, $\varepsilon$ is a threshold distance, $|| \cdot ||$ is a norm, and $\Theta(\cdot)$ is the Heaviside function. Each element of the matrix represents the recurrence at a different time $j$ of a state at time $i$, equal to 1 if the distance between the two states is lower than $\varepsilon$ and equal to 0 otherwise. The RP is obtained by plotting the matrix in a square map, where both axes are time axes and assigning black dots to ones and white dots to zeros.

Since $r_{i,j} = 1, \forall i$, the RP has always a black main diagonal line, named the line of identity. Furthermore, the RP is symmetric by definition with respect to the main diagonal ($r_{i,j} = r_{j,i}$).

Iwanski and Bradley [25] proposed the unthresholded RP (UTRP), a variation of the RP which plots directly the distance matrix

$$\Delta_{i,j} = ||y_i - y_j||, \quad y_i, y_j \in \mathbb{R}^m, \quad i, j = 0, \ldots, N - 1.$$ (3)

By using an appropriate color bar for the values of the threshold distance $\varepsilon$, it is possible to highlight different recurrence structures at different thresholds.

Iwanski and Bradley [25] found that the appearance and statistics of RPs for certain low-dimensional systems are not significantly altered by a small change in the embedding dimension, suggesting that these statistics may be important for a new invariant characteristics of a system.

III. ELM Database

In [26], a statistical analysis of 60 JET $D_n$ signals’ characteristic of JET type-I ELMs was performed. The reference experimental campaigns, all in deuterium, were C21–C27b (June 9, 2008–October 23, 2010). The wall was in graphite. The analysis was restricted to time intervals with fixed engineering conditions: only the experiments and time intervals characterized by variations of $\pm 4\%$ for toroidal magnetic field $B_t$, $\pm 2\%$ for plasma current $I_p$, $\pm 10\%$ for neutral beam injection (NBI) input power $P_{\text{NBI}}$, and lower triangularity $\delta_{\text{low}}$, were taken into account. All tolerances were related to the signal resolutions. The 60 shots correspond to 24 experimental conditions with different values of $I_p$, $B_t$, $P_{\text{NBI}}$, and $\delta_{\text{low}}$ for a total of 3448 type-I ELM time intervals.

After the localization of the inter-ELM time intervals, the memorylessness test suggested the presence of memory in the ELM time intervals for the considered type-I ELMs, in agreement with [27] and [28]. From a statistical point of view, the probability distribution of inter-ELM periods did not show the same properties with varying the experimental conditions. Pulses related to similar inputs were grouped into 24 groups, including 10 singles, 4 pairs, and 10 cliques. The Kruskal–Wallis test [29] was applied to inter-ELM intervals of pulses of the same group to verify whether they belonged to the same population. Four groups (6, 10, 16, and 19) were identified with 5% confidence level (see Table I). The following analysis is restricted to pulses in these four groups. Nevertheless, the obtained results can be extended to all type-I ELMs in the same plasma conditions (carbon wall (CW) and deuterium plasma) since they are obtained on the base of qualitative considerations on the signals’ shape.

The same database has been used in [1] to investigate the pseudoperiodicity of ELM time series.

In Section IV, the previously described database is analyzed with RPs; a subscript, which indicates the membership clique, is added to the name of each pulse for ease of notation.

IV. Results

A. Recurrence Plots

The embedding parameters have been evaluated for temperature, density, and $D_n$ time series before creating the RPs.
TABLE I
GROUPS WITH DIVISION BY ENGINEERING CONDITIONS (PLASMA CURRENT, TOROIDAL MAGNETIC FIELD, NBI INPUT POWER, AND LOWER TRIANGULARITY) AND BELONGING TO THE SAME EXPERIMENT

<table>
<thead>
<tr>
<th>Pulses</th>
<th># of intervals</th>
<th>Engineering conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>74375, 74376</td>
<td>54 2.5MA, 2.5T, 15.1-15.5MW, 0.33-0.35</td>
</tr>
<tr>
<td>10</td>
<td>74443, 74444</td>
<td>90 2.5MA, 2.7T, 14-15.8MW, 0.32</td>
</tr>
<tr>
<td>16</td>
<td>76428, 76430, 76431, 76437, 76438</td>
<td>197 2MA, 2T, 7.5MW, 0.35</td>
</tr>
<tr>
<td>19</td>
<td>76470, 76471, 76472, 76473, 76474, 76475, 76476, 76477, 76478, 76479</td>
<td>363 2MA, 2T, 14.5-16.8MW, 0.35-0.37</td>
</tr>
</tbody>
</table>

Fig. 2. 3-D projection for (a) pedestal temperature, (b) density at the edge, and (c) outer divertor $D_{\alpha}$, with $\tau = 1$.

If we consider that the rise/crash time of a burst is about 2–3 sampling times, it can be assumed that a small $\tau$ is necessary. Fig. 2 shows that, with $\tau = 1$, the attractor is well unfolded. Thus, the chosen value is $\tau = 1$.

To determine the appropriate embedding dimension, the FNNs method has been used. The estimated embedding dimension could be evaluated as the shortest $m$ for which the percentage of FNNs is lower than 30%. The obtained results on the list of shots showed $m = 3$ for the temperature and density time series and values in the interval 4–7 for the $D_{\alpha}$ time series. It is worth noting that this estimation might be greater than the right embedding dimension due to the presence of noise in the signals. Fortunately, RPs are not particularly sensitive to the choice of embedding parameters, and we have found only marginal, almost invisible, differences when choosing the embedding dimension and time delay within reasonable ranges.

In Fig. 3(a), the UTRP related to temperature time series for shot 7647119 is shown, where black and white points represent, respectively, the smallest and the highest distances between the points. White corridors are present when ELMs appear, i.e., the distance between the points of the burst and those of the laminar phase are high. Squares or rectangular blocks among white corridors correspond to laminar phases. By introducing the threshold $\varepsilon$, the shape of the blocks varies [Fig. 3(b)]. The RPs of the temperature time series have a checkerboard structure, which is typical of periodic and quasi-periodic systems [30]. In particular, for $\varepsilon$ chosen such that the recurrence point density is approximately 30%, squares and rectangles with an elongated lower left corner are present. Moreover, the presence of close secondary peaks gives rise to larger and closer white corridors in the UTRP and white corridors in the central region of the RP.

The UTRP and RP shown in Fig. 4 for shot 743756 highlight the presence of nonstationarity. Indeed, large bright regions on the higher left part and lower right part give evidence of increasing distances among points of the first and second parts of the series in the embedding space, harder to note from the visualization of the time series alone. This is due to the fact that the second part of the time series is characterized by slightly higher values during the laminar phase.

Figs. 5 and 6 show the UTRP and RP of density and $D_{\alpha}$ time series for shot 743756. As it can be noted easily, the same typical kite-like shape appears in the RP image, although the effect of nonstationarity is less visible, both for density and $D_{\alpha}$. Moreover, the RP for density shows a more complex structure inside the kite, mainly due to the high-frequency oscillations in the time series.

B. From Type-2 Intermittency to ELMs

As stated above, the shape of squares and rectangles with an elongated lower left corner resembles a uniformly black kite.
This is a characteristic common to all the cliques. The same shape is characteristic of RPs for type-2 intermittency systems [31], model of weakly chaotic behavior. It is important to clarify that there is no connection between the intermittency systems [31], model of weakly chaotic behavior. It is important to clarify that there is no connection between the intermittency system and those of temperature and \( D_u \) time series, i.e., a slow growth toward an equilibrium value \( T_{eq} \), interrupted by a fast crash after crossing a threshold \( T_{th} \) (see Fig. 9). Once this critical value has been exceeded, the system relaxes to a status of lower energy by rapidly decreasing the pressure gradient. Then, the cycle repeats and can continue indefinitely if the conditions remain stationary. This kind of instability is guided by pressure, and the ELM frequency increases with the heating power. This translates into an inverse proportionality between the heating power and the mean laminar phase length.

The first equation describes the relaxation and dissipation of the perturbations, and the second equation describes the drive process in the plasma. The model can be described by

\[
T_{n+1} = \begin{cases} h_T(T_n), & T_{min} \leq T_n \leq T_{th} \\ g_T(T_n), & T_{th} < T_n < T_{max} < T_{eq} \end{cases}
\]

\[
h_T(T) = T_{eq} - f^{-1}(T_{eq} - T)
\]

\[
g_T(T) = T_{min} + \gamma_T \frac{h_T(T) - h_T(T_{th})}{h_T(T_{max}) - h_T(T_{th})}
\]

where \( T_{min} \) is the minimum value obtainable by \( T_n \) during the drive phase, \( T_{max} = h_T(T_{th}) \) is the maximum value obtainable by \( T_n \) during the relaxation phase, and the parameter \( \gamma_T \in (0, T_{max} - T_{min}) \) is an arbitrary value which determines the maximum temperature attainable during the drive process.

The map in (5) exhibits different dynamical behaviors, depending on \( \gamma_T \) and \( T_{th} \), both chaotic and periodic [see Fig. 10(a) when \( T_{min} = 0 \) and \( T_{eq} = 1 \)]. When the map
is periodic, the period of the time series can vary between one and many ELM cycles. In Fig. 10(b), different dynamical behaviors are shown, where $N$ is the number of ELM cycles inside one period of the time series.

In particular, when $T_{\text{eq}} = 1$, $T_{\text{th}} = 0.9793$, and $T_{\text{min}} = 0$, the map is periodic for $\gamma_T < 0.3821$ and chaotic for $\gamma_T > 0.3821$. When the map is periodic, $N = 1$ for $\gamma_T < 0.316$ and $N > 1$ for $\gamma_T > 0.316$. A property of the map, common to all values of $T_{\text{th}}$, is that for low values of $\gamma_T$, and the system is periodic with the period equal to one ELM cycle; then, after increasing the value of $\gamma_T$, the map is subjected to a continuous series of bifurcations that alter the size of the period, until the system becomes chaotic. Fig. 11 shows the three time series, respectively, obtained when $\gamma_T = 0.2, 0.35$, and 0.7.

The system in (5) can exhibit also a pseudoperiodic behavior [1] when in the periodic region is corrupted by a dynamical noise. In this case, the resulting system is described by

$$T_{n+1} = \eta_{n+1} + \begin{cases} h_T(T_n) & T_{\text{min}} - \eta_{\text{max}} \leq T_n \leq T_{\text{th}} \\ g_T(T_n) & T_{\text{th}} < T_n \leq T_{\text{max}} + \eta_{\text{max}} \end{cases}$$

(6)

where the dynamical noise $\eta_n$ is uniform in the interval $[-\eta_{\text{max}}, \eta_{\text{max}}]$ and is responsible for the irregular behavior in frequency and amplitude of the signal.

A map, which graphically represents the behavior of the model in (6), is shown in Fig. 12. The gray stripes indicate the tolerances due to noise.

Fig. 13 shows the $T_n$ time series obtained for $\alpha = 0.001$, $T_{\text{th}} = 0.9793$, $T_{\text{eq}} = 1$, $T_{\text{min}} = 0$, $\eta_{\text{max}} = 0.003$, and $\gamma_T = 0.1$.

The RP and UTRP for the obtained time series are shown in Fig. 14. The threshold $\varepsilon$ of the RP has been chosen such that the recurrence point density is approximately 30%.

A uniformly black kite-like shape appears for the laminar phase oriented as in temperature time series.

With respect to type-2 intermittency, the inversion and the convexity modification along the laminar phase change the equilibrium point and its stability. In fact, in type-2 intermittency, there is a single unstable equilibrium point in the origin, which destabilizes the trajectory and a reinjection phase, which reinjects the trajectory in a zone close to the equilibrium point.

In the new map, there is a single stable equilibrium point in $T_{\text{eq}}$ toward which the trajectory tends and a crash phase, in which the trajectory is sent far from the equilibrium point.

Moreover, with an analog procedure, it is possible to reproduce a behavior similar to the one characteristic of the $D_n$ time series taking into account that the $D_n$ burst (decay) occurs when the temperature crashes (increases). Thus, a $D_n$ model can be obtained by setting $T = T_{\text{eq}} + D_{\text{eq}} - D$ in (6) obtaining a model of the form

$$D_{n+1} = \eta_{n+1} + \begin{cases} g_D(D_n) & D_{\text{min}} - \eta_{\text{max}} \leq D_n < D_{\text{th}} \\ h_D(D_n) & D_{\text{th}} \leq D_n \leq D_{\text{max}} + \eta_{\text{max}} \end{cases}$$

$$g_D(D) = D_{\max} - \gamma_D \frac{h_D(D_{\text{th}}) - h_D(D)}{h_D(D_{\text{th}}) - h_D(D_{\text{min}})}$$

$$h_D(D) = D_{\text{eq}} + f^{-1}(D - D_{\text{eq}})$$

(7)
where $D_{\text{min}} = h_D(D_{\text{th}})$ is the minimum value obtainable by $D_n$ during the relaxation phase when $\eta_{\text{max}} = 0$, $D_{\text{max}}$ is the maximum value obtainable by $D_n$ during the drive phase when $\eta_{\text{max}} = 0$, $D_{\text{eq}}$ is the equilibrium point toward which the trajectory tends during the relaxation phase, and $\gamma_D$ is an arbitrary value which determines the minimum value reachable during the drive process.

A map that graphically represents the behavior of the model in (7) is shown in Fig. 15. The gray stripes indicate the tolerances due to noise.

Fig. 16 shows the $D_n$ time series obtained for $\alpha = 0.001$, $D_{\text{th}} = 0.0207$, $D_{\max} = 1$, $D_{\text{eq}} = 0$, $\eta_{\text{max}} = 0.003$, and $\gamma_D = 0.1$.

The RP and UTRP for the obtained time series are shown in Fig. 17.

Also, in this case, a kite-like shape appears, similar to the one shown in $D_n$ and temperature time series.

V. CONCLUSION

RPs have been applied to investigate the characteristics of type-I ELM time series of JET tokamak with a CW. Even if the discussion is developed at a heuristic level, avoiding all the delicate mathematical questions, we think it may shed some light on currently still puzzling problem of modeling ELMs.

RPs of temperature, density, and $D_n$ time series show patterns similar to those characteristics of signals exhibiting type-2 intermittency, in particular, a characteristic kite-like shape. Starting from this similarity, a qualitative model of temperature and $D_n$ time series has been derived starting from a type-2 intermittency model. The models presented in this paper describe the qualitative behavior of temperature and $D_n$ time series. Oscillations can be either periodic/pseudoperiodic with period varying between one and many ELM cycles, or chaotic: in case of temperature, the modeled time series is characterized by a slow growth toward an equilibrium value which is interrupted by a fast crash; for $D_n$, the fast growth and the slower crash are reproduced.

This is the first attempt at modeling type-I ELMs, which needs to be validated and connected with physical parameters and existing physical models. First, a parameter identification will be made to fit the models to the individual signals in particular experimental conditions. The first objective of this identification phase is to find a connection between the threshold parameter in the model and the critical pressure gradient characteristic of the majority of physical models describing the ELM behavior.

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Authors’ photographs and biographies not available at the time of publication.