Temperature dependence of phase and spike synchronization of neural networks

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\section*{Abstract}
We simulate a small-world neural network composed of 2000 thermally sensitive identical Hodgkin-Huxley type neurons investigating the synchronization characteristics as a function of the coupling strength and the temperature of the neurons. The Kuramoto order parameter computed over individual neuron membrane potential signals, and recurrence analysis evaluated from the mean field of the network are used to identify the non-monotoniuous behavior of the synchronization level as a function of the coupling parameter. We show that moderated high temperatures induce a low variability of the inter-burst intervals of neurons leading to phase synchronization and further increases of temperature result in a low variability of inter-spike intervals leading the network to display spike synchronization.

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\section*{1. Introduction}

Due to advances of computational methods, real neural networks can now be satisfactorily studied by using mathematical neural models coupled through complex networks. In fact, beyond neural networks, chemical, biological, physical and even social problems can be analyzed using complex networks models and coupled oscillators [1–5]. Neural networks are composed of interconnected neural cells and can be found in small systems of hundreds of neurons as the C. Elegans [6] neural network as well as forming much more complex problems such as the human brain network [7]. To simulate mathematically the real neural network, neurons are considered as nodes possessing their own dynamics and linked to others through a connection matrix. Usually, random, scale-free and small-world topologies are used to simulate the connection schemes of neural networks [8–11] since such topologies find support in real situations [6,12,13]. For all these connection schemes, a transition from unsynchronized to synchronized states is observed.

An important characteristic of complex systems consists of the emergent collective behavior, where global phenomena are, in general, different and richer than the simple sum of individual behavior of system elements [3]. A common collective behavior of a neural network is the presence of synchronized states. In special, a large number of networks exhibit a quite complex transition to synchronized states, characterized by a non-monotonic dependence on the coupling parameter [14–17] leading the network to an intermittent and nonstationary transition to synchronization [9,14]. Neural networks composed of bursting neurons display synchronization in two time scales [18]: a fast one, related to spike synchronization, and a slow one, associated to phase (bursting) synchronization. These dynamical properties are very useful to the neural system understanding since there are neural diseases associated to an excess of (sometimes anomalous) synchronization, as observed in [19,20] and neurological disorders related to the intermittent (nonstationary) behavior [21] of the network dynamics. A particularly interesting point consists of the effect of the temperature on the neural system since it is known that temperature increases can lead the brain to undergo an epilepsy scenario [22,23]. Other studies suggest that high body temperature can be associated with an acute crisis in Parkinson disease [24]. On the other hand, a theoretical approach of dynamical properties as a function of the temperature of the neural system is given in [25], where the temperature increases result in a decrease of individual neuron behavior variability.

Here 2000 thermally sensitive identical bursting neurons in a small-world topology are simulated. Synchronization characteristics are studied as a function of the temperature and the coupling parameter. The model of Braun et. al. [26] is used to simulate the neuron behavior, a modified version of the original Hodgkin-Huxley model [27]. Following many real neural systems [28], the neural behavior considered is the bursting regime, which consists of a sequence of chaotic spikes related to the fast time scale of the
dynamics, followed by a resting period that is related to a slow time scale [29].

We use the Kuramoto order parameter [30], recurrence quantification analysis, particularly the determinism [31] and standard deviation of membrane potential signal [17] to evaluate unequivocally the synchronization characteristics. The order parameter is able to quantify the phase synchronization level through the individual signal of each neuron which is associated with a geometric phase of the bursting activity. On the other hand, the determinism measures the ratio of recurrent points belonging to diagonal structures of the recurrence plot and is evaluated from the potential mean field of the neural network as proposed in [14]. At last, the standard deviation of the membrane potential of the neurons is able to bring information about synchronization in slow (phase synchronization) and fast (spike synchronization) time scales and is used to corroborates the results. Through these quantifiers, it is possible to investigate the synchronization level in a general way, where is observed that the neural system can display non-monotonous synchronization phenomena as a function of coupling strength and temperature changes, including synchronization in two time scales, namely phase synchronization related to the bursting time scale, and spike synchronization associated to the time scale of the neuron spikes. We show that exist a lower coupling regime where phase and spike synchronization can be closely related to the individual dynamics of the neurons. For such regimes, the collective dynamics is strongly related to the small variability of the neurons and the synchronization frequency is closely related to the frequency of the neurons itself. For strong coupling, the neuron variability is not a limiter and phase synchronization can be obtained for a larger interval of frequencies.

This paper is organized as follows. In Section 2 the neural model and topology characteristics are discussed, in Section 3 the synchronization quantifiers are defined and in Section 4 the results and discussions are shown to support the conclusion in Section 5. At last, the acknowledgment and references are presented.

2. Neural model and connection architecture

The model of Braun et al. [26] is a variation for the original Hodgkin-Huxley model [27], where the temperature dependence and new ionic fluxes are considered. The original work of Braun et al. was developed in order to investigate the temperature dependence and its effects in the crayfish cold receptors [26]. However, many authors have developed adaptations to the model to mimic different biological scenarios related to the temperature influence in the neural system [17,32,33]. Recently, Burek et al. have investigated the temperature influence on neuronal firing rates and bursting transitions [34] by using the model of Braun et al. The results have shown the temperature influence on the neural activity and the ability of the model to reproduce some experimentally observed phenomena such as the increase of the depolarization time for low temperatures [35].

The model is able to reproduce bursting neuronal activity and the main equation is based on capacitor modeling

\[ C_m \frac{dV_i}{dt} = -J_{i,Na} - J_{i,K} - J_{i,Na} - J_{i,L} + J_{i,\text{coupling}}, \]

where the membrane capacitance is given by \( C_m = 1.0 \ \mu F/cm^2 \), \( J_{i,Na} \) is the ionic flux (nA/cm²) over the ith neuron membrane and related to Sodium (Na), Potassium (K) and leak (L) currents. Additionally, two slow fluxes due to Calcium, (sd) and (sa) are considered. The flux due to coupling to other neurons is represented by \( J_{i,\text{coupling}} \). The parameters of the model were chosen following [36].

The fluxes related to each ion are given by their specific maximum conductances,

\[ J_{i,Na} = 1.5 \rho \sigma_{i,Na}(V_i - 50), \]

\[ J_{i,K} = 2.0 \rho \sigma_{i,K}(V_i + 90), \]

\[ J_{i,Na} = 0.25 \rho \sigma_{i,Na}(V_i - 50), \]

\[ J_{i,sa} = 0.4 \rho \sigma_{i,sa}(V_i + 90), \]

\[ J_{i,L} = 0.1(V_i + 60). \]

The Nernst potential associated with each ion is supposed as defined in [36]. The first temperature factor is given by

\[ \rho = 1.3^{\frac{r_T}{\mathcal{T}}}, \]

where \( T \) is the temperature of the neural system, \( T \in [37, 40] \) measured in Celsius degrees. The activation channel variables \( \alpha \) have dynamical evolution given by

\[ \frac{d\alpha_{i,Na}}{dt} = \frac{\phi}{0.05}(\alpha_{i,Na} - \alpha_{i,Na}^0), \]

\[ \frac{d\alpha_{i,K}}{dt} = \frac{\phi}{2.0}(\alpha_{i,K} - \alpha_{i,K}^0), \]

\[ \frac{d\alpha_{i,Na}^0}{dt} = \frac{\phi}{10}(\alpha_{i,Na}^0 - \alpha_{i,Na}^0^0), \]

\[ \frac{d\alpha_{i,sa}^0}{dt} = -\frac{\phi}{20}(0.012J_{i,sa} + 0.17\alpha_{i,sa}^0), \]

where \( \phi = 3.0^{\frac{\mathcal{T}}{\mathcal{T}_0}} \) is the second temperature dependence. The membrane potential dependencies of the activation variable are given by

\[ \alpha_{i,Na}^0 = \frac{1}{1 + \exp[-0.25(V_i + 25)]}, \]

\[ \alpha_{i,K}^0 = \frac{1}{1 + \exp[-0.25(V_i + 25)]}, \]

\[ \alpha_{i,Na}^0 = \frac{1}{1 + \exp[-0.09(V_i + 40)]}, \]

The coupling term is modeled by excitatory chemical synapses [37] through a small-world (Newman–Watts model [38]) binary matrix, \( e_{ij} \), given by

\[ J_{i,\text{coupling}} = \frac{\varepsilon}{N} \sum_{j=1}^{N} e_{ij} r_j (V_{syn} - V_i), \]

where \( \varepsilon \) is the coupling strength (mS/cm²), \( \chi = 4.8 \) is the average number of connections, \( V_{syn} = 20 \) mV is the synaptic reversal potential, \( N = 2000 \) is the number of neurons and \( r_j \) is the kinetic term representing the fraction of receptors available to receive connections [37]

\[ \frac{d_r_j}{dt} = \left( \frac{1}{\tau_r} - \frac{1}{\tau_d} \right) \frac{1 - r_j}{1 + \exp[-50(V_i - V_0)] - \tau_d}, \]

where \( s_0 = 1 \) (mV)⁻¹, \( V_0 = -20 \) mV, \( \tau_r = 0.5 \) ms and \( \tau_d = 8 \) ms are constants.

Fig. 1 depicts the typical dynamics of a neuron, where the black line depicts bursting behavior and the magenta line is the temporal behavior of the variable defined as \( U = 1/\alpha_{i,sa} \), which assumes a maximum every time a fast sequence of spikes starts (the burst), making possible a geometric phase association to each slow time scale bursting activity. For all coupling and temperature values used here, the bursting behavior is obtained and the phase association is valid. Here, we simulate the slow bursting frequencies centered around 1 Hz and the spike frequencies are centered around 0.05 Hz. Such values reflect biologically acceptable values but the
model is general since Eqs. (1) – (17) can be re-scaled in time and other suitable sets of constants lead to the same dynamical behavior.

The total number of connection of the network is given by

\[ K = 4N + \frac{N(N - 5)p}{\text{local}} \frac{\sqrt{1 - p}}{\text{nonlocal}} \]  \hspace{1cm} (18)

where \( p \) is the probability to add a nonlocal connection. \( p \to 1 \) leads to a globally connected network. Here, \( p = 0.0004 \) is used resulting in 1590 nonlocal connection added by 8000 local ones resulting in a small-world network with 9590 connections.

A 12th Adams’ predictor-corrector method with an absolute tolerance of 10^{-8} [39] is used to integrate the differential equations. The initial conditions are randomly distributed in the intervals given by \( V_i \in [-65, 0] \) mV, \( \alpha_{i,j} \in [0.1, 1] \) when \( \nu \) represent Na, K, sd, sa, L and \( r_1 = 0.1 \), avoiding any synchronization trend and/or numerical divergence.

### 3. Synchronization quantifiers

The Kuramoto order parameter can constitute a quantifier to phase synchronization since the variable \( U \) assumes a maximum every time a burst starts, as shown in Fig. 1, such that we can associate a geometric phase \( \theta_j \) to each neuron, increasing it by \( 2\pi \) every maximum of \( U \). To obtain a continuous variation of phase we use simple linear interpolation [40,41]

\[ \theta_j(t) = 2\pi k_i + 2\pi \frac{t - t_{k_{i,j}}}{t_{k_{i+1,j}} - t_{k_{i,j}}} , \quad t_{k_{i,j}} < t < t_{k_{i+1,j}}. \]  \hspace{1cm} (19)

where \( t_{k_{i,j}} \) is the time when the \( k \)th burst occurs in the \( j \)th neuron.

To obtain the Kuramoto order parameter of the entire network it is necessary to evaluate the phase of all neurons through [30]

\[ R(t) = \left[ \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j(t)} \right]. \]  \hspace{1cm} (20)

If the system is (not) in a phase synchronized state, \( R \to 1 \) (\( R \to 0 \)).

Better visualization of the synchronization as a function of the coupling parameter or the temperature is obtained from the mean value of the Kuramoto order parameter, defined as

\[ \langle R \rangle = \frac{1}{A} \int_{t_0}^{t_f} R(t) \, dt. \]  \hspace{1cm} (21)

where \( t_f \) is the total simulation time, \( t_0 \) is the transient time and \( A = (t_f - t_0)/h \), where \( h = 0.01 \) ms.

Recurrence analysis is an additional tool to evaluate phase synchronization [14,16]. Since it does not use information of neuron phases, it can be computed over the mean field of the network computing a recurrence matrix defined as [42]

\[ R_{ab}(\delta) = \Theta(\delta - ||x_a - x_b||), \quad x_a, x_b \in \mathbb{R}, \quad a, b = 1, 2, \ldots, S. \]  \hspace{1cm} (22)

where \( x \) in our case are the successive values in the time of the mean field of the network. \( \Theta \) is the Heaviside function, \( \delta \) is the recurrence threshold and \( S \) is the size of time series analyzed. If a state \( x_b \) is (not) recurrent to \( x_b \), \( R_{ab} \) receives (zero) one.

The determinism is a recurrence quantifier that measures the ratio of recurrent points belonging to diagonal structures over all recurrent points in the recurrence matrix [31], and distinguishes the level of synchronization [9]

\[ \Delta(\ell_{\min}, \delta) = \frac{\sum_{\ell=\ell_{\min}}^{S} \ell P(\ell, \delta)}{\sum_{\ell=1}^{S} \ell P(\ell, \delta)}. \]  \hspace{1cm} (23)

where \( \ell \) are the diagonal sizes, \( \ell_{\min} \) is the minimum diagonal size considered and \( P(\ell, \delta) \) is the probability distribution function (PDF) of diagonal lines computed over the mean field of the neural network

\[ \overline{V}(t) = \frac{1}{N} \sum_{i=1}^{N} V_i(t). \]  \hspace{1cm} (24)

In a similar approach as used to Kuramoto order parameter, we define the mean value of the determinism as

\[ \langle \Delta \rangle = \frac{1}{A} \sum_{\ell=1}^{S} \Delta(\ell). \]  \hspace{1cm} (25)

Here, the determinism \( \langle \Delta \rangle \) of the mean field potential is evaluated through Eq. (23) using an overlapped moving window. Through this procedure a single value of \( \Delta \) for each window is obtained. Considering the total time of simulation, a time series for the determinism is obtained and a mean value in time is computed, \( \langle \Delta \rangle \).

The recurrence threshold is chosen using the maximum sensitiveness of \( \Delta \) to capture small changes in the dynamics as proposed in [43]. Using this procedure, phase synchronized network results in a mean field that leads to \( \langle \Delta \rangle \sim 1 \). Due to the spike nature of the mean field signal, spike synchronization results in small values of \( \langle \Delta \rangle \), associated with \( R \sim 1 \). The standard deviation as a function of time computed over the neuron membrane potential signals

\[ \sigma(t) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (V_i(t) - \overline{V}(t))^2}, \]  \hspace{1cm} (26)

is used to corroborates the results of \( \Delta \) and \( R \) [17], since the temporal coincidence of all membrane potentials leads to \( \sigma \to 0 \) quantifying a complete synchronization. On the other hand, higher values of \( \sigma \) indicate unsynchronized states.

### 4. Results and discussions

Fig. 2(a) and (b) are a summary of our findings and depict color coded mean values of the Kuramoto order parameter and the determinism in the parameter space \( \varepsilon \times T \). The results are based on time series of \( t_f = 150 \) s and the transient time is given by \( t_0 = 120 \) s. Here, are considered 15 different initializations of the system. The recurrence threshold parameter \( \delta = 0.11 \) is chosen from the condition that \( d\langle\Delta(\delta)\rangle/d\delta \) assumes a maximum [43], which results in the highest sensitiveness of the quantifier. The minimum diagonal size is set to \( \ell_{\min} = 35 \) ms to avoid small diagonals [9,44] and the determinism is evaluated using a moving window of 10 s (10,000 points) using an overlapping of 9.99 s.
In general the system is not sensitive to temperature for coupling strength larger than 0.025 where a (partially) phase synchronized regime (PS2) is observed for all range of $T$. In this case, the collective behavior imposed by the coupling overrides the influences of the temperature over the local dynamics of the neurons. On the other hand, for small coupling strength the increase of the temperature results in a very complex dynamical behavior due to the interplay between temperature and coupling effects. For $T < 38.0^\circ C$ and coupling smaller than 0.01, the neural network displays unsynchronized state (dark areas in both panels in Fig. 2) followed by a monotonic transition to phase synchronization as the coupling strength increases. For temperatures higher than $38.0^\circ C$ and coupling strength $0.005 < \varepsilon < 0.015$, a non-monotonic synchronization is observed where a local maximum is noticed in both panels for $\langle R \rangle$ and $\langle \Delta \rangle$ denoting some level of phase synchronization even for low coupling strengths (PS1). This first phase synchronization regime is strongly related to the small interval of frequencies experienced by the neurons, resulting in a small variability of the neuron frequencies, as we will go to show later on in this text. In this region of the parameter space, a rising coupling strength makes the system to lose synchronization characteristic. Similar behavior was reported as an anomalous phase synchronization regime [14,16,17,25,45]. The unsynchronized regime extends further for higher values of the temperature and coupling parameter.

For temperatures above $39^\circ C$ and small coupling strengths $0.001 < \varepsilon < 0.01$, the neural system displays high levels of synchronization, indicating that higher temperatures can even induce spike synchronization (SS). Such the regime of SS is related to an extreme smallvariability of neurons resulting in a spike synchronization of the neurons. PS1, SS and PS2 regimes will be investigated in details along this text.

Differently from other situations where high values of the deterministic and the Kuramoto order parameter are observed simultaneously for the synchronized regime, what in fact, characterizes phase synchronization [14], for $T > 39.0^\circ C$ and for $0.001 < \varepsilon < 0.01$ it is noticed that $\langle R \rangle$ displays a local maximum but $\langle \Delta \rangle$ shows a local minimum, denouncing a distinct synchronization regime in this region, depicted as “SS” in Fig. 2.

A better view of this region is depicted in Fig. 3 where mean values of the Kuramoto order parameter ($R$) and the determinism ($\Delta$) are plotted as a function of the temperature for a fixed coupling parameter $\varepsilon = 0.005$. An initial increase of the temperature makes the neural system to transit from unsynchronized to phase synchronized states, however, for $T > 39.0^\circ C$, $\langle R \rangle$ reaches almost 1 at the same time that $\langle \Delta \rangle$ decreases. So, we say that the simultaneous observations of $\langle R \rangle$ and $\langle \Delta \rangle$ make possible the distinction among two distinct synchronization regimes, namely the phase and spike synchronizations. It is possible since differently from the order parameter, $\langle \Delta \rangle$ is computed based on the network mean field and it shows to be sensitive enough to capture distinct behaviors of the mean field of the membrane potential of the neural network (Eq. (24)) of both regimes.

Fig. 4 (black lines) depicts details of $\overline{V}(t)$ (Eq. (24)) and its standard deviation $\sigma(t)$ (magenta lines) of the neuron membrane potential (Eq. (26)) for unsynchronized regime occurring for $\varepsilon = 0.005$ and $T = 37.0^\circ C$ (a), phase synchronized regime for $\varepsilon = 0.0040$ and $T = 40.0^\circ C$ (b) and spike synchronized regime for $\varepsilon = 0.005$ and $T = 40.0^\circ C$ (c). For panel (a), the amplitude of oscillation of $\overline{V}(t)$ is almost null and $\sigma(t)$ depicts just random fluctuations around a large value, corroborating the idea of an unsynchronized network for these coupling and temperature values. Panels (b) and (c) depict higher synchronized regimes and for both cases, the Kuramoto order parameter shows high and similar mean values (see Fig. 3), however for the panel (b) case, the determinism follows the Kuramoto order parameter trend, while for panel (c) the Kuramoto order parameter presents a local maximum and the determinism a local minimum. The mean field helps to understand the real situation since for both cases there are a high amplitude of both quantifiers indicating the existence of synchronization, but the case (c) shows two time scales, a first one related to phase synchronization (slow time scale) and a second one related to spike synchronization.
synchronization (fast time scale). A similar scenario is found in [18] where the spike synchronization regime is reached after the occurrence of a burst regime. This fact explains the antagonistic behavior of \( \langle R \rangle \) and \( \langle \Delta \rangle \) since the determinism is evaluated from mean field signal. Corroborating this scenario, \( \sigma(t) \) in panel (b) depicts large slow time oscillations related to the phase synchronization process. When the network is in spike regime, \( \sigma(t) \) displays values larger than those expected for an unsynchronized regime while smaller values are expected for the quiescent one. Accordingly \( \sigma(t) \) decays when temporal coherence of the membrane potential is observed. Therefore, panel (c) depicts smaller oscillations of \( \sigma(t) \) resulted of a more synchronized spike regime occurring in the fast time scale. Even slower values of \( \sigma(t) \) is observed for the quiescent regime pointing out for a very (spike) synchronized network. In fact, an absolute level of synchronization is obtained by the mean value of \( \sigma(t) \)

\[
\langle \sigma \rangle = \frac{1}{T} \int_{0}^{T} \sigma(t)\,dt. \tag{27}
\]

resulting in \( \langle \sigma \rangle = 10.44 \) for unsynchronized case of Fig. 4(a), \( \langle \sigma \rangle = 9.25 \) for phase synchronized case of Fig. 4(b) and \( \langle \sigma \rangle = 5.94 \) for spike synchronized case of Fig. 4(c).

Fig. 5(a), (b) and (c) depict examples of recurrence plots evaluated through Eq. (22) from the mean field potential time series, as described by Eq. (24) for unsynchronized states (panel (a)) where \( \varepsilon = 0.005 \) and \( T = 37.0^\circ \)C, phase synchronized states (panel (b)) where \( \varepsilon = 0.040 \) and \( T = 40.0^\circ \)C and spike synchronized states (panel (c)) where \( \varepsilon = 0.005 \) and \( T = 40.0^\circ \)C. The difference between the synchronized and unsynchronized case is very clear since in the panels (b) and (c) it is possible to observe diagonal structures while in the panel (a) no structure is noticed [9,44]. On the other hand, the differences between burst and spike synchronization are subtler. Panels (b) and (c) depicts similar diagonal lines, however, as before mentioned the phase synchronization is related to the slow time scale while the spike synchronization to the fast one, resulting in smaller structures in the fast time scale of the panel (c), which denounces the partial spike synchronization.

Fig. 6 shows a qualitatively way to observe distinct network synchronization phenomena induced by temperature increases, namely the Poincarè surface at \( V = -20 \) mV for all neurons in the network. Panel (a) represents the case where \( \varepsilon = 0.005 \) and \( T = 37.0^\circ \)C, panel (b) \( \varepsilon = 0.040 \) and \( T = 40.0^\circ \)C and panel (c) \( \varepsilon = 0.005 \) and \( T = 40.0^\circ \)C. For the first panel, there is not spatial-temporal coherence of membrane potential and it is representative of an unsynchronized state. Panels (b) and (c) depicts spatial patterns observed for synchronized states, however, panel (c) displays synchronization even for the fast time scale denouncing spike synchronization. The synchronization on the slow time scale or phase synchronization is very similar in both panels (b,c), which is expected since the \( \langle R \rangle \) is almost the same value for each case. On the other hand, spike synchronization should be not expected to occur due to small values of the coupling parameter. In this case, we conclude that spike synchronization is induced just by the increase of the temperature.

In some cases, the global behavior of a network synchronization can be associated to the individual neuron variability [25]. To investigate the correlation between individual variability of the neurons and the global synchronization states of the network, Fig. 7 depicts bifurcation diagrams for the inter-spike (ISI) and inter-burst (IBI) intervals for an uncoupled neuron, panels (a) and (d); for a random chosen neuron of the network, panels (b) and (e), for a coupling strength of \( \varepsilon = 0.0050 \) as an example of weak coupling regime; and for a random chosen neuron of the network, panels (c) and (f) for a coupling strength of \( \varepsilon = 0.040 \) a strong coupling regime. For ISI, we compute just time intervals belonging to the same burst, discharging the time interval between the last spike of one burst and the first spike of the next.

For \( 37.5^\circ \leq T \leq 40^\circ \) the ISI for an uncoupled neuron suffers a double period bifurcation process, going from a great variably chaotic ISI to a single value of ISI diminishing drastically its variability as \( T \) grows as observed in Fig. 7(a). Despite the individual behavior of the uncoupled neurons, a weakly network coupling imposes an almost constant variability of ISI of the coupled neurons for the interval \( 37^\circ \leq T \leq 38.5^\circ \) as observed in panels (b). We conclude that even a weak coupling regime results to be strong enough to destroy the stability of all sequence of periodic
behavior depicted by ISI. Nevertheless, the strong stability of the period “one” orbit of the individual dynamics of the (uncoupled) neurons, starting in $T \approx 39.25 \, ^\circ C$ influences heavily the network neuron behavior, resulting in a broader ISI but centered on its former uncoupled behavior. For greater temperature, the progressive increasing stability of the period “one” orbit for the uncoupled neuron makes the variability of the coupled neurons to diminish as seen in Fig. 7(b). Based on Figs. 2 and 3, such a behavior can be associated with the appearance of spike synchronization of the network occurring for a weak coupling regime.

On the other hand, for the strong coupling as observed in panel (c), the coupling effect destroys all influences of the individual behavior of the neurons, producing a large variability for all intervals of the temperature. So, for strong coupling, the influence of the natural spike frequencies of the neurons is lost and spike synchronization is not induced, leading the network to display only phase synchronization as shown in Figs. 2 and 3 and detailed next.

Fig. 7. Inter-spike intervals (ISI) and inter-bursting intervals (IBI) as a function of the temperature of the system ($T$). Panels (a,d) depict the ISI and IBI for an uncoupled neuron, respectively, panels (b,e) depict the ISI and IBI for a weakly coupled neuron where $\varepsilon = 0.0050$. Panels (c,f) display the ISI and IBI for a strongly coupled neuron where $\varepsilon = 0.040$. The increase of the temperature results in a decrease of the variability of a single neuron, however, the variability of the weakly coupled neuron is almost constant for large intervals suffering subtle changes related to the bifurcations occurring in the individual dynamics of the neurons (panels b,e). For strong coupling (panels c,f) the coupling destroys all influence of the individual dynamics of the neurons.

Fig. 6. The Poincare surface in $V = -20 \, mV$ for the neural network. Panel (a) depicts an unsynchronized state, where $\varepsilon = 0.005$ and $T = 37.0 \, ^\circ C$. Panel (b) depicts a phase synchronized state, where $\varepsilon = 0.040$ and $T = 40.0 \, ^\circ C$ and panel (c) depicts a synchronization in the two time scales, the fast and the slow ones, which suggests a burst and spike synchronization level ($\varepsilon = 0.005$ and $T = 40.0 \, ^\circ C$).

Fig. 8. Power spectrum of a representative neuron and $T = 38.5 \, ^\circ C$ for (a) An uncoupled neuron $\varepsilon = 0.0$, showing an intrinsic (natural) period width of 53.9 ms. (b) A weakly coupled neuron, $\varepsilon = 0.006$ and period width slightly increased to 65.8 ms. (c) A strongly coupled neuron, $\varepsilon = 0.040$ and period width increased to 141.2 ms as the effect of the strong coupling.

Fig. 7(d), (e) and (f) depict the behavior of the intervals between consecutive bursts (IBI) as a function of the temperature for an uncoupled (d) and a weakly (e) and strongly (f) coupled network neurons. Again, the variability of IBI for an uncoupled neuron decreases as the temperature increases, panel (d). The network coupling makes the variability of IBI of a network neuron almost constant for a large part of the period doubling cascade $37^\circ C < T < 38^\circ C$, and again, it seems to be not strong enough to destroy completely the stability properties of the period “one” orbit that starts at $T = 37.9^\circ C$, and even high temperature period “two” ($T \approx 38.5^\circ C$, resulting in a drastic diminishing of the network neuron variability, panel (e). The result of this small variability in IBI is the transition to phase synchronization observed in Figs. 2 and 3 even for weak coupling regime. So the phase synchronization regimes occurring for weak couplings are strongly related to the individual dynamics of the neurons, in contrast to the phase synchronization occurring for larger values of the coupling. For strong coupling, panel (f), the presence of phase synchronization is not a function of the individual dynamics of the neurons. In this case, the strong coupling imposes a collective dynamics and large variability neurons can also be synchronized.

A view of the neuron dynamics for $T = 38^\circ C$ in the period space ($P(\xi) \times \xi$) is plotted in Fig. 8. Panel (a) depicts results for an isolated neuron. It is characterized by a discrete spike periods for
\( \zeta < 100 \text{ ms} \) (cyan dashed area) and a clear burst period around 1000 ms with a variability period band of 53.9 ms (orange dashed area). Fig 8(b) depicts results for a representative network neuron for the same temperature and considering a phase synchronized state occurring for a weak coupling regime \( \epsilon = 0.006 \). The discrete nature of the spike periods is lost due to the coupling effect as suggested by the broader band of periods for \( \zeta < 200 \text{ ms} \) but due to the weak influence of the coupling in the burst dynamics, as pointed in Fig. 7(e), the burst period around 1000 ms is still present and just the variability is slightly increased to 65.8 ms. So we conclude that the phase synchronization occurring for weak coupling has a strong influence of the individual dynamics of the neurons, the network will display a synchronous behavior almost at the same bursting frequency depicted by the neurons. Finally Fig. 8(c) shows results for a phase synchronized state and a strong coupling regime (\( \epsilon = 0.04 \)) for the same temperature. Now we observe a large broad band in the spike period regime, and the burst period regime is characterized by a broader peak around 1200 ms showing a variability of 141.2 ms. The large variability corroborates our results depicted in Fig. 7(f) and suggests that the phase synchronization occurring for strong coupling destroys (at least partially) the influence of any particular neuron dynamics.

5. Conclusions

We have shown that a neural network composed of 2000 thermally sensitive Hodgkin–Huxley like neurons can present rich dynamical characteristics as a function of the temperature of the neurons and coupling strength. The network displays a non-monotonous synchronization scenario for weak coupling regime and temperature greater than 38°C, where phase and spike synchronizations are observed. For these synchronization regimes, the presence of a small coupling parameter results in a highly synchronized network. Further increases of the coupling parameter destroy the phase and spike synchronizations and after a recovery region, as the coupling parameter is continuously increased, an asymptotic globally stable phase synchronization is acquired.

Corroborating reference [25], we have shown that for the weak coupling regime, the phase synchronization observed in the network is strongly related to the individual variability of the neuron. The increase of the temperature promotes a decrease of the variability of the neurons and even weak couplings between the neurons lead the network to display slow time scale synchronization, a phase synchronization regime. Further increases of the temperature promote also a decrease in the variability of the spike time intervals (a fast time scale regime). The result of this reduction of variability is the emergence of a spike synchronization regime occurring for weak coupling parameters. For both synchronization regimes, further increases of the coupling parameter destroy the synchronization regimes.

It is known that neural diseases can be associated to an excess of synchronization [19,20] and that high body temperature can be related to epilepsy phenomenon [22,23,46] or even to Parkinson’s diseases [24]. Here, it is shown that the neural system can present a high level (excess) of synchronization just increasing the temperature of the system. We have shown that such an increase of the synchronization level is associated to individual variability of the neurons and the correct understanding of this dependence relation can improve the understanding of the scenario of phase and spike synchronization associated to important deceases.

Declaration of Interest Statement

None.

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