Chaotic behavior of price in the power markets with pay-as-bid payment mechanism

N. Bigdeli a,*, K. Afshar b

a Linear Systems Control Lab., EE Department, Imam Khomeini International University, Qazvin, Iran
b Electrical Machinery Lab., EE Department, Imam Khomeini International University, Qazvin, Iran

A R T I C L E   I N F O

Article info
Article history:
Accepted 31 March 2009

A B S T R A C T

Price forecasting in the current deregulated power markets is an important requirement for deriving proper bidding strategy and profit maximization of producers. On the other hand, the energy price in the power market experiences lots of fluctuations which may affect the accuracy of the price forecasting seriously. Seeking for predictability, in this paper, the characteristics of these fluctuations are investigated through time series analysis methods. The results of analyses are representative of the existence of a deterministic chaos in the system with a mimic predictability. Besides, it is observed that because of existing the seasonality and non-stationarity in the system dynamics, a fixed model cannot perform properly even in case of normalized input data, but the developed models should be updated regularly.

© 2009 Elsevier Ltd. All rights reserved.

1. Introduction

One of the most important issues in the last decade in power system area is deregulation and restructuring. Deregulation and restructuring have recently taken place in most countries in the world, and is increasing more and more. In the new environment, a vertically integrated utility (VIU) is divided into its three main components of Generation companies (Genco’s), Transmission companies (Transco’s), and Distribution companies (Disco’s). Increasing efficiency using creation of competition is one of the most important objectives of deregulation, as competition can facilitate efficiency, price transparency and also supply–demand satisfaction [1].

In most commodity markets, the effects of production or supply chain on prices are damped by surplus storage. By contrast, the electricity market lacks storage for practical purposes (all generated electricity must be consumed), which is an intrinsic source of volatility. As a result, both producers and consumers need accurate price forecasts in order to establish their own strategies for benefit or utility maximization [2]. In fact, millions of dollars of utilities could be lost by misjudging future prices. Therefore, accuracy of electricity price forecasting is very critical in the deregulated environment and more accuracy in forecasting reduces the risk of under/over estimating the revenue from the generators for the power companies and provides better risk management [3]. Similar situation exist in different economic markets. Therefore, characterizing economic markets properties has been a matter of interest in many researches [4–7].

Up to now, several methods have been proposed to forecast prices in the electricity markets which their differences are mainly due to their employed model structure. For example, time series models [8], jump diffusion/mean reversion models [9], neural networks [10,11], GARCH model [12], wavelet transform [13], and ARIMA models [14]. In order for determining the model order, generally the autocorrelation function of the price time series as well as it cross correlation of with other factors and especially the required load (RL) are employed. Afterwards, model identifying stages are correspondingly...
employed for determining the model parameters. But, as mentioned earlier, there is lots of volatility in the energy market price which are due many known and unknown factors which imply lots fluctuations in the energy price as well. These fluctuations, however, may be the main reason of lack of accuracy in price forecasting and so characterizing the nature of them are worthwhile to determine the degree of predictability of the price.

In this context, the fundamental question is: does these fluctuations originate from a stochastic system or come from a deterministic, say chaotic, system? One of the most common approaches for responding to this question is time series analysis. Different time series approaches have been widely used in the literature for characterizing the properties of economic markets as well as other phenomena in the nature [15–17]. Based on these arguments, in this paper, we try to investigate the properties of the weighted average energy price (WAP) of Iran’s electricity market based on a one year price time series. For this purpose, the properties of the data has been examined through different methods such as power spectral density (PSD) analysis, phase space reconstruction and test of surrogates, Poincaré and return maps, the fractional dimension and the slope of integral sums, stochastic versus deterministic plots, $\delta-e$ method, and the recurrence plots. The results of these analyses not only emphasize the existence of a deterministic chaos with a mimic predictability but also show that there exists some type of the seasonality and non-stationarity in the system dynamics. This result implies that a fixed model cannot perform properly, even in case of normalized input data, and so, any developed model should be updated regularly.

In continue the employed methods and the result of applying them on the time series will be described in more details. This paper is organized as follows: in Section 2, the Iran's electricity market will be introduced briefly and in Section 3, the chaos and time series analysis methods will be dealt with. The results of time series analysis of the experimental data will be stated in Section 4 and conclusive remarks are brought in Section 5.

2. Iran’s electricity market

In this section, the Iran’s electricity market is briefly introduced. For more details one may refer to [18]. Iran’s electricity market was launched at 23 October, 2003. The heart of the market is a mandatory pool. All producers and consumers have to send their bids before 10 am every day to the market. Once the electricity purchase and sales offers have been received, checked and accepted, they are matched by market operator, who calculates the marginal price of each hour, weighted average price (WAP) of each hour, and shares out production and demand amongst parties involved in the auction. The results from this process are sent to the system operator, who draws up the provisional feasible daily schedule at 2 pm. System operator is responsible for secure, safe and reliable operation of interconnected grid [18].

Payments for generators are divided to two parts: capacity payment and energy payment. Capacity payment is the main component of the payment in the Iran power market. All the available capacities in the market receive a certain hourly fixed payment, which is set annually by the market regulatory board. Energy payment is the other part of generators payments. The payment mechanism of energy in the Iran power market is pay-as-bid [18]. Therefore, an exact estimate of WAP is necessary for producers and customers to offer their bids through proper bidding strategies. Seeking for predictability, in this paper, we will analyze the properties of the WAP time series of Iran electricity market. The main attitude in these analyses is toward characterizing the fluctuations in WAP to infer that if they are due to either stochastic, deterministic nonlinear effects (chaos) or non-stationary nature of system. In following sections, the methodology and the results will be described.

3. Chaos and time series analysis

Chaotic behavior has been reported in a broad range of scientific disciplines including astronomy, biology, chemistry, ecology, engineering, and physics [19,20]. Based on the chaos theory, it is well known that a random-like series or fluctuated behavior can be attributed to deterministic rules [19]. From such random-like signals, chaotic techniques are capable of exploring the inherent nonlinear dynamic and deterministic nature of that system which allows one to distinguish between random and deterministic processes. One major result of such determinism in experimental systems is, however, that they are predictable. Therefore, it is reasonable to characterize the nature of existing fluctuations in complex systems where any kind of forecasting is applicable.

There are different hallmarks which are representative of chaos in a system. Based on these hallmarks, different methods have been developed to investigate the chaotic properties of a system both from its dynamical model and/or from the system output (states) time series [20–22]. Positive Lyaponov exponent, bifurcation via period-doubling, sensitivity with respect to initial conditions, low fractal dimension and existence of strange attractors are the main features of chaotic systems [23]. When the system model is in hand, the above properties can be checked easily through developed methods in the literature. But, in experimental systems, generally just limited time series from the system output (states) are available. Besides, these types of data are commonly noisy. Therefore, to extract the overall properties of the system, more complex numerical time series analysis methods should be applied to data to: (1) derive the embedding dimension to reconstruct the phase space and describe the attractor of system. The embedding dimension is related to the minimum number of independent variables necessary to describe the system [24] and (2) analyze the reconstructed phase space to study the system dynamics, nonlinearity, stationarity, and stochastic versus chaotic behavior of system [19,22].
To distinguish between these types of processes several methods such as power spectrum analysis, phase space reconstruction, surrogate testing, slope of integral sums, Poincaré and return maps, and recurrence plots can be applied to the data using proper tools such as the TISEAN package [25].

As stated earlier, the existence of low fractal dimension and a strange attractor are the two important features that signify the existence of chaos in a system [19]. The fractal dimension of the attractor provides information on how much of the phase space is filled by the system. One interpretation of the fractal dimension (i.e., non-integer exponent) is that it measures how many degrees of freedom are significant [19]. To determine the fractal dimension of a dynamical system, one should first decide on the embedding delay and embedding dimension for the correlation dimension analysis. The embedding delay can be obtained through evaluation of the autocorrelation function and/or average mutual information [26,27]. The embedding dimension is related to the minimum number of independent variables necessary to describe the system. Correlation dimension can be used to estimate the sufficient embedding dimension and the estimated fractal dimension [24]. A strange attractor could also be revealed in a chaotic system under phase space reconstruction [24,28]. A broadband dense power spectrum is another hallmark of chaos [29]. Poincaré and return maps [30] and the recurrence plots [31–33] visually highlight the properties of a dynamic system and surrogate data [34,35] testing can identify the existence of pure random process and thus help to reject the presence a chaotic process. Finally, the existence of positive Lyaponuv exponents can be investigated through analysis of the slope of correlation sums [22,24,28].

As mentioned earlier, many known and unknown different factors affect the properties of the WAP which their nature and exact relationships are really unknown. On the other hand, precise forecasting of this parameter is vital for bidding strategy in the well-known pay-as-bid power markets. Based on these facts, in the following section, we try to investigate the properties of WAP data of Iran electricity market based on a limited price time series. In continue the employed methods and the result of applying them on the time series will be described.

4. Experimental data and time series analysis results

Seeking for the nature of the price index in Iran electricity market, in this section the experimental WAP data from Iran electricity market will be examined closely. The available data are the required load (RL) as well as weighted average price (WAP) for about 3 years where the results for about one solar year from March 2006 to March 2007 for every hours of each day will be brought, in this section. The mentioned data have been shown in Fig. 1, where Fig. 1(a) shows the required load and Fig. 1(b) shows the weighted average price of the interested period versus time in hours. From the above graphs, a quasi-periodic behavior (with period 24 h) is observed for daily variations in required load whose variance is almost constant and its average changes continuously. The pattern in the second graph is however completely different as the periodicity disappears for example in summer (2000–5500 h) and in other days the daily pattern is different in different days. From one point of view, RL and WAP can be considered as system parameter and system operating point, respectively. Based on this interpretation, the WAP values versus RL have been plotted in Fig. 2. From this figure, by decreasing RL the WAP value bifurcates. This bifurcation diagram may, however, be due to a chaotic or stochastic nature of a nonlinear process [20,22]. To distinguish between these two types of processes several methods such as power spectrum analysis, phase space reconstruction, surrogate testing, slope of integral sums, Poincaré maps, and recurrence plots will be examined using the TISEAN package [25] as our tool.

4.1. The power spectral density

Regarding the behavior of price and load indices Figs. 1 and 2, what is important for us is the answer to this question: "what is the nature of such complex behavior in the market, deterministic chaos or stochastic nature?" To answer this
question first of all we analyze the power spectral density (PSD) of the delivered data as shown in Fig. 3(a). The graph has been derived based on periodogram PSD estimation method. In this figure, sharp peaks are representative of 24-h periodic nature of the signal. These sharp peaks have been superimposed on the background broadband spectrum. To analyze the results of Fig. 3(a), first of all one should note that existence of the higher harmonics in the spectra indicates that the processes underlying the time series are not linear processes, but there is some kind of nonlinearity [22–24]. As another point, consider the frequency content of the plots. A broadband dense spectrum which also preserves these properties in small frequency ranges is often considered as hallmark of chaos. Spectrum of a chaotic system is not solely comprised of discrete frequencies, but has a continuous broadband nature [29]. In case of our data, the periodic part of signal occurs on very large scale with high power, while chaos only occurs on a smaller low power scale. To show that the spectrum preserves its properties in small frequencies as well, the graph in Fig. 3(a) has been widened in Fig. 3(b). However, one should note that the occurrence of broadband spectrum is also possible in stochastic/noisy systems as well. Thus, a definitive conclusion of the existence of chaos is reasonable if other characteristics of chaos are also observed [19]. Based on these discussions, the existence of other hallmarks of chaos will be investigated in the following subsections.

4.2. Embedding delay determination

The first step in analyzing a time series is to reconstruct its embedded phase space via method of delays [25]. In this method, vectors in a new space, the embedding space, are formed from time delayed values of the scalar measurements [25]. For implementing this method, two important parameters should be determined firstly: the embedding delay and embedding dimension. In order for determining the embedding delay, two methods may be employed. In the first approach, the first zero cross or first cutoff (corresponding to 95% confidence level) of autocorrelation function (ACF) is considered as the embedding delay [26]. In the second approach, the first minimum of mutual information (MI) is taken as the embedding delay [27]. In this paper, both approaches have been examined and the resulting plots are shown in Fig. 4. As expected, there are cyclical peaks in the AFC plot which represent the existence of the 24-h periodic signal in the time series. It should be noted that due to the variations in the original time series, the shape and value of these peaks varies by increasing the time lag (not shown here). However, the existence of periodicity is not a sufficient reason for rejection of existence of chaotic signal in the original data set [19].

In Fig. 4(a), the first cross of the ACF plot with the shown confidence interval does not reveal an integer lag. Therefore, the selection of embedding delay is inconclusive because the results of ACF plot only approximates the location of the linearly
uncorrelated embedding delay \[19\]. For the approximation of embedding delay from a nonlinear perspective, the plot of AMI has been also shown in Fig. 4(b). From this figure, the embedding delay has been determined as the first minimum of the AM plot which is \( \tau = 6 \) h. This embedding delay will be used in following sections for phase space reconstruction.

4.3. Embedding dimension determination

The other parameter to be determined for phase space reconstruction is the embedding dimension. A well-known method to determine the minimal sufficient embedding dimension \( m \) is the false nearest neighbor (FNN) method proposed by Kennel et al. \[28\]. The idea behind this method is that in with a right embedding dimension the neighborhoods in a reconstructed phase space are mapped onto neighbors again. But, for wrong embedding delay the topological structure is no longer preserved and the neighbor points are projected into neighborhoods of other points. These points are called false neighbors. Based on this idea, of the algorithm false nearest is the following: for each point in the \( m \)-dimensional reconstructed phase space look for its nearest neighbor and calculate the distance between these two neighbors. Iterate both points and compute distance of the iterated neighbors. If the ratio of these two distance are less than a threshold, this point is marked as having a false nearest neighbor \[25,28\]. The criterion that the embedding dimension is high enough is that the fraction of points for which false nearest neighbors occur is zero or at least sufficiently small. Fig. 5 shows the fraction of false nearest neighbors for the considered WAP time series. As seen in this figure, this fraction falls to zero at \( m = 3 \). Therefore, the correlation dimension is considered as \( m = 3 \) in the rest of this paper.

4.4. Reconstructed phase space and surrogated data

Mapping time series data into a phase space allows one to view the temporal series in a spatial manner. The distinguishing feature of chaotic processes is their sensitive dependence on initial conditions and highly irregular behavior that makes prediction difficult except in short term. This feature is the so-called “strange attractors” associated with chaotic processes which often have a complex, fractal structure \[36\]. Based on these properties, one of the most interesting procedures for checking the presence of chaos is based on the ability of recovering the strange attractor of a system in the phase space and especially observation of the so-called butterfly effect \[29\].

Three-dimensional phase space plots of the data for three time delay with a delay time of \( \tau = 6 \) h have been shown in Fig. 6. Fig. 6(a) belongs to first 2500 h, while Fig. 6(b) is related to 2501–5500 h and Fig. 6(c) is due to the rest of the data. This classification has been derived considering different operating modes of the data from Fig. 1. As seen, the butterfly effect is observed in Fig. 6(a) and existence of the strange attractors is obvious in the reconstructed phase spaces of Fig. 6(b) and (c).
In order for removing the non-stationary phenomenon of the data, in Fig. 7, the same three-dimensional phase spaces of Fig. 6 is plotted for the same data where each 24-h data set has been normalized by its average and variance. As seen, the butterfly effect is now observable in the three subplots of Fig. 7. This is one important hallmark of chaos in our data set.

To develop a better appreciation whether the data set is chaotic or stochastic, one can comparatively assess the phase space maps of the original and/or the daily normalized data (Fig. 6 or 7) with their corresponding surrogated data sets. Surrogate data sets have Fourier decompositions with the same amplitude of the original data set but with random phase components. The method of surrogate data serves as a null hypothesis whose objective is to reject the hypothesis that the original data have come from a random process [19,30–32]. The method may be used as a reference for visual comparison between the original and random data sets’ phase space. These two phase spaces would not look like if the data is a randomly spreading out cloud [19]. The phase space map of surrogate data for the daily normalized data (Fig. 7) has been shown in Fig. 8. As it is observed, the surrogated phase space maps look like the corresponding daily normalized data phase space plots in Fig. 7. This observation emphasizes the chaotic nature of our data, once again.
4.5. Poincaré map

Poincaré and return maps can be applied to reduce the dimension of the phase space and allow for an easier determination of a chaotic dynamics. A horseshoe behavior of the resulting map is a sufficient criterion to infer a chaotic behavior of the system [22,33]. For a Poincaré map, here, we choose the values of successive maxima \( \max_p \) of the time series Fig. 9(a). Return maps are formed by consecutive periods \( \Delta t_i \) of the time series as the time intervals between the successive measured maxima in hours Fig. 9(b). The results in Fig. 9 are representative of the horseshoe behavior and so the chaotic nature of our system.

4.6. The slopes of integral sums

A fractal dimension of the attractor is one of the fascinating features of chaotic processes. One measure for this dimension is the correlation dimension \( D_2 [24,34] \), which is defined by:

\[
D_2 = \lim_{r \to 0} \frac{d \log C(m, r)}{d \log(r)}
\]

where \( C(r) \), the correlation integral, is given by:

\[
C(m, r) = \text{constant} \times \sum_{i=1}^{N} \sum_{j=i+\mu}^{N} \Theta(r - |\tilde{x}(i) - \tilde{x}(j)|)
\]

where \( \tilde{x}(i), \tilde{x}(j) \) denote the states embedded in reconstructed phase space with embedding dimension \( m [26] \), \( \Theta(\cdot) \) is the Heaviside step function applied to count the number of pair of points within radius \( r \) and \( \mu \) is the Theiler correction employed to exclude temporally correlated points [35]. This method has frequently been applied to measured data, but some of the results were a matter of debate. A serious problem is that the existence of a scaling region for small radii \( r \) where Eq. (1) holds may not be assumed but has to be established [22]. Therefore, plots of local slopes of the logarithm of the correlation integrals with respect to the logarithm of \( r \) can be investigated. In a chaotic system, there is a plateau where several higher embedding dimensions curves are saturated. Such a plateau is known to exist only over a short period where a proper scale region has been considered. The level at which most of the curves settle down defines the fractal dimension. The minimum embedding dimension, from which the correlation plots converge to one, is the proper system embedding dimension for reconstructing phase space vectors [19].

The plots in Fig. 10(a) show the local slopes of the correlation integral versus \( r \) for the embedding delay \( \tau = 6 \) h (as found in last subsections) and for embedding dimension from 1 to 10 from bottom to top graphs, respectively. As seen in this figure, from \( m = 3 \) the right tails of the graphs have been saturated to one graph with a low fractional number. This observation has been highlighted in Fig. 10(b), where the corresponding fractional dimensions have been shown versus \( m \). From this figure a fractional dimension of 1.32 and a minimum embedding dimension of 3 is obvious. The existence of the bounded fractional dimension emphasizes the hypothesis of existing chaos in our system.

4.7. Deterministic versus stochastic plots

Deterministic versus stochastic (DVS) plots were introduced in [36] and further discussed in [37]. For different embedding dimensions, this method applies local linear predictions of the time series using different large neighborhoods for the local linear modeling and evaluates the mean prediction error. The information provided by this method is twofold. First, for sufficiently high embedding dimensions, nonlinear deterministic, nonlinear stochastic, and linear stochastic processes result in different appearances of the DVS plots. For nonlinear deterministic processes the prediction error approaches zero for small...
neighborhoods and increases monotonically. For linear stochastic processes, the prediction errors should be largest for the smallest neighborhoods and decrease monotonically. Depending on the amount of dynamical noise and the degree of non-linearity, nonlinear stochastic processes show either a minimum error in an intermediate size of neighborhoods or a monotonically increasing prediction error that does not reach zero for small neighborhoods. Second, the order of the process can be inferred. For embedding dimensions larger than the true order, the prediction errors should not decrease any further and the functional form of the prediction errors in dependence on the size of the neighborhoods should not change anymore. Fig. 11 displays the results for our data for embedding dimension of 1–5. The nonlinearity of the processes is reflected by the increase of the prediction error for larger $e$. For small $e$ the prediction error shows a tendency to decrease to zero. Thus, the DVS plots give evidence for a chaotic process. A third order process is suggested by the invariance of the functional form of the prediction error curves for higher embedding dimensions. From these observations, it is concluded that the process can be described by a nonlinear third order chaotic dynamics.

4.8. $\delta-e$ method

The $\delta-e$ method was first introduced in [38], where a detailed discussion of the method is given. The basic idea is that a deterministic dynamics embedded in a sufficiently high-dimensional state space should induce a continuous mapping from past to present states. Similarly to the DVS plots the size of the neighborhoods is increased to investigate the continuity. The size of the neighborhood of past states ($\delta$) and the size of the resulting neighborhood of present states ($e$) are plotted. For deterministic processes $e$ is expected to decrease to zero for decreasing $\delta$ for sufficiently high embedding dimensions. For stochastic processes and processes which are covered by a significant amount of additive observational noise a non-zero intercept for $e$ is expected.

Fig. 12 displays the results for embedding dimension from 1 to 8 for our data. Based on the above discussions, these results confirm the results of the DVS plots in the previous section that a third order chaotic dynamics can describe the behavior of the price index time series.

4.9. Recurrence plots

Recurrence plots (RPs) were firstly introduced by Eckman et al. [39] as a tool for analyzing experimental time series data, especially useful for finding hidden correlations in highly complicated data and to determine the stationarity of the time series [29]. With RP, one can graphically detect hidden patterns and structural changes in data or see similarities in patterns across the time series under study [40]. RPs visualize the behavior of trajectories in phase space [39,41] via a graphical representation of the matrix:

$$R_{ij} = \Theta(e - \|\bar{x}_i - \bar{x}_j\|), \quad i, j = 1, ..., N$$

(3)

where $\bar{x}_i$ stands for the point in the reconstructed phase space at time $i$, and $e$ is a predefined threshold and $\Theta(\cdot)$ is the Heaviside function. One assigns a “black” dot to the value one and a “white” dot to the value zero. The two-dimensional graphical
representation of \( R_{ij} \) then is called RP \([40]\). This plot then can be used to distinguish between different dynamic systems. In this context, three main types of systems can be considered which are the periodic systems, stochastic random systems, and chaotic ones. In all of these systems recurrences can be observed, but the patterns of the plots are rather different. The periodic motion is reflected by long and non-interrupted diagonals. The vertical distance between these lines corresponds to the period of the oscillation. The chaotic systems also lead to diagonals which are seemingly shorter. There are also certain vertical/horizontal distances, which are not as regular as in the case of the periodic motion. The RP of the uncorrelated stochastic signal consists of many single black points. The distribution of the points in these RPs look rather erratic. Reconsidering all three cases, we might conjecture that the shorter the diagonals in the RP, the less predictable the system \([42]\). This conjecture was already made by Eckmann et al. \([39]\) who suggested that the inverse of the longest diagonal (except the main diagonal) for which \( i = j \) is proportional to the largest Lyapunov exponent of the system \([42]\).

The RP of the WAP data set has been shown in Fig. 13. In this figure, the plot in Fig. 13(a) illustrates the RP for whole data with embedding delay \( \tau = 6 \) and \( m = 3 \), where Fig. 13(b) zoom outs a smaller region of the plot in Fig. 13(a) to have a better sense about the system properties. As the first feature of system one can observe some type of seasonality of system dynamics, which is illustrated by the four independent region of the RP in Fig. 13(a). Based on this observation, it is concluded that a fixed model cannot perform properly for price forecasting, but the developed models should be updated regularly. The white vertical and horizontal lines in the RP are representative of non-stationarity and system dynamic variations by time \([42]\).

Now, look at the Fig. 13(b). In this figure, the aperiodic black dot patterns and short, non-identical diagonal lines are once again representative of a chaotic behavior in the system.

5. Conclusions

Price forecasting in the current deregulated power markets is an important requirement for proper bidding strategy and profit maximization of the market attending selling and purchasing units. On the other hand, experimental observations show that the energy price show aperiodic complex fluctuations. This so-called volatility is due to variety of factors which can neither be completely known nor be modeled, but have a great effect on the predictability of the WAP. Seeking for predictability, in this paper, the characteristics of these fluctuations has been investigated via time series analysis methods which are very common in investigating the nature of complex processes. The main goal of the research is to distinguish between the determinism (possibly chaotic) and the randomness of system. For this purpose, the properties of the WAP time series of Iran electricity market has been examined through different methods such as power spectral density (PSD) analysis, phase space reconstruction and test of surrogates, Poincaré and return maps, the fractional dimension and the slope of integral sums, DVS plots, \( \delta-e \) method and the recurrence plots. The derived PSD graph contains a few very sharp peaks superimposed on a dense broadband background spectrum. It, however, is representative of a high power 24-h quasi-periodic signal superimposed on a smaller low power chaotic signal. Observation of strange attractor and butterfly effect in the reconstructed phase space of the data are the most important hallmarks of chaos in this process and the test of surrogates rejects...
the null hypothesis of randomness in the system. The horseshoe behavior in the Poincaré and return maps are the other reasons for chaos in system. Investigation of correlation sums have resulted in a fractional dimension of 1.32 and an embedding dimension of 3. Existence of such small fractional and embedding dimensions is another strong reason for conclusion of chaos in the system. DVS plots and the results of \( \delta - e \) method also emphasize the above-mentioned conclusions. As the final part and seeking for the stationarity and the predictability of this research we have studied the recurrence plots of the WAP data set. However, the results of these analyses not only emphasize the existence of a deterministic chaos with a mimic predictability but also show that because of existing the seasonality and non-stationarity in the system dynamics, a fixed model cannot perform properly even in case of normalized input data, but the developed models should be updated regularly.

References