Application of Recurrence Quantification Analysis to Power System Dynamic Studies

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Abstract—Recurrence quantification analysis (RQA) is a non-linear data analysis technique that can quantify the number and duration of a dynamical system of being in nearly the same area in phase space trajectory. This paper proposes three applications of RQA to analyse power system dynamics. These are denoising of PMU data, localization of power system events, and transient stability contingency ranking of power system. All three applications have been demonstrated using a multimachine test system.

Index Terms—Denoising, power system disturbance, event location, dynamic event, recurrence quantification analysis, RQA.

I. INTRODUCTION

Signal processing and data mining is a growing area of multidisciplinary research in power system measurement and control. Copious amounts of data are being generated by measurement sensors installed at different power and voltage levels, as part of various worldwide smart-grid initiatives. A substantial amount of data is also generated from repeated system simulations, carried out at grid control centers, to assess the security of the grid at every new operating condition. The data have to be processed in a timely, accurate, and effective manner so as to extract maximum information about currently evolving phenomena as well as future trends in the system behavior. The earlier practice of extracting modal information from this data using parametric statistics is well established [1]. However, power systems exhibit increasingly complicated dynamics, and the data are noisy, nonlinear, and nonstationary; any assumptions otherwise may lead to significant errors of estimation. In this context, this paper explores the application of recurrence quantification analysis (RQA) to study data representing power system behavior [2]. To the best of the authors' knowledge, this is the first application of RQA for this kind of analysis. The implications of the studies reported in the paper are valid for signals originating from other dynamic systems too.

In any sensor data analysis framework, noise poses a key challenge to the application of signal processing techniques. Many mathematical tools have been applied to the problem of denoising of power system sensor data [3]–[6]. Of these, two nonparametric techniques—empirical mode decomposition [7] and wavelet shrinkage [8]—stand out, largely for their ability to adapt to signals with a high degree of nonstationarity as well as nonlinearity. Wavelet was used in [9] to detect ultra high-frequency signals during partial discharge in the presence of high electromagnetic interference. Presently, there is no method available which can denoise signals of any frequency with any noise level. For instance, voltage and power measurements are likely to have sharp changes in magnitude, in contrast to frequency and bus angle measurements which are smoother and more amenable to polynomial-type approximations. Additionally, the signal-to-noise ratio (SNR) index, which is commonly used to tune any denoising strategy for an unknown signal, does not always lead to optimum results [10]. As an alternative, RQA statistics are used in this paper to choose the most effective technique and its parameters for denoising power system measurement data obtained from phasor measurement units (PMUs).

Another major purpose for the analysis of distributed sensor data is the problem of event location [11]. Identifying the approximate location of a disturbance can be automated and used in a near-real-time application to enhance situational awareness and post-mortem analysis of cascading disturbances. A detailed report on FNET implementation in USA has been given in [12] along with analysis of some events observed by FNET. Successful development of such schemes can enable special protection schemes, load shedding and controlled islanding strategies for large grids. The current state-of-the-art techniques for event location mostly rely on the arrival time of the event signal and operate under the basic assumption that the propagation speed remains unchanged for all lines and network conditions [13]–[17]. However, installed frequency monitoring networks have shown that event propagation speeds can vary from 100 to 1000 miles/s depending on the network conditions [11]. Intensity of the frequency perturbation swing or oscillations was used to find the event location in [18]. Maximum absolute value of rate of change of frequency was used in [19] to detect the wave front arrival time at the measurement locations. A method to classify and locate different types of disturbance by matching estimated and measured values of some variables, has been provided in [20]. Last intersection point between denoised frequency waveform and pre-disturbance average frequency data was used in [21] to detect the starting instant of the disturbance. Then, wavefront arrival time was found from the event starting instant. In this paper, RQA statistics are used to identify the location of an event based on distributed synchrophasor measurements that are time-stamped using a universal reference.
The third application of RQA is in online transient stability studies, where the objective is to quickly compare the relative stability of different contingency events such as three-phase faults. The critical clearing time is a well-established index for this purpose, however, its computation remains a key challenge in the context of real-time transient stability assessment. Repeated time-domain simulation of the particular contingency is the most reliable technique to calculate the critical clearing time, but can become computational inefficient in large systems. Lyapunov-based energy function methods have been developed to avoid repeated numerical simulations, but there are practical difficulties to the implementation of such methods, especially when detail models are used [22]–[25]. Modal synchronizing torque coefficient for every line outage was used in [26] to rank contingencies. Performance of different ranking indices was tested in [27]. A coherency index based on maximum difference of maximum and minimum angle for all generators in some short period was used in [28]. Critical trajectory was used to rank contingencies based on critical clearing time [29]. Integral square generator angle index in centre of inertia reference frame, was used along with decision tree to rank contingencies [30]. It is in this context that the RQA statistics have been used as an indicator of relative stability for an event as an alternative to the critical clearing time.

The remainder of this paper is organized as follows. A brief introduction to recurrence plots and RQA is provided in Section II. Sections III–V present the application of RQA to denoising PMU data, localizing power system events, and ranking transient stability contingencies respectively. Section VI is the concluding section.

II. RECURRENCE QUANTIFICATION ANALYSIS

A detailed mathematical exposition of RQA and its several applications are given in [31]. Some essential mathematical results have been included in the Appendix at the end of the paper. Recurrence is a property of any dynamical system that indicates whether the states revisit points in the phase space trajectory. Eckman et al. introduced the recurrence plot (RP) to visualize the periodicity and recurring nature of a dynamical system [2]. Briefly, two points on the phase-space trajectory are considered to be recurrent if the distance between them is less than a threshold value (say, $\epsilon$). Measures of recurrence have been developed to quantify the recurrence nature of a dynamic system [31], [32]. A square matrix called recurrence matrix is used to store the data of recurrence points. Elements of this matrix are given as

$$R_{i,j}(\epsilon) = \theta(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|), \quad i,j = 1 \ldots N$$

where $N$ is the total number of points on the phase trajectory, $\mathbf{x}_i$ and $\mathbf{x}_j$ are two measured state vectors, and $\epsilon$ is the maximum threshold distance for the vectors to be recurrent. Perfect recurrence may not be easily detected for discrete measurements, and it is sufficient if points in a trajectory come adequately close to be called recurrent. $\theta(\cdot)$ is the Heaviside step function given as

$$\theta(y) = \begin{cases} 1, & \text{for } y \geq 0 \\ 0, & \text{for } y < 0. \end{cases}$$

More often, a limited number of measurements are available to observe a multivariable dynamic system. In the case of a single measurement, $x$ is a scalar measurement point rather than a vector.

Fig. 1 shows an RP for a sinusoidal signal and random noise. Each recurrent point is shown with a black dot. As by definition $R_{i,i} = 1$, i.e., any point is necessarily recurrent with itself, RPs are characterized by a solid main diagonal called the line-of-identity (LoI). Further, large diagonals repeating themselves periodically in the RP indicate periodicity of the signal. Non-deterministic behavior or stochastic processes result in very short or absent diagonals. A homogenous plot indicates stationarity, while fading in the upper left and lower right portion of the RP indicates nonstationarity within the data.

Two quantitative measures of recurrence have been used in this paper. A short description of these measures is given below.

Recurrence Rate (RR): RR is the density of recurrent points in the plot. As there is always a diagonal on LoI, this is excluded from the computation as follows:

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}$$

where $R$ is the recurrence matrix and $N$ is length of the data. When used for an infinitely long signal, the limiting case for (2) is the probability that the state/measurement will recur.

Determinism (DET): This quantifies the percentage of recurrent points that form a diagonal of minimum length $i_{\text{min}}$. By default $i_{\text{min}}$ is taken as 2. DET is defined as

$$\text{DET} = \frac{\sum_{i=1}^{N} P(i)}{\sum_{i,j=1}^{N} R_{i,j}}$$

where $P(i)$ is the number of occurrences of a diagonal of length $i$. For fully deterministic signals, DET is close to 1 as most of the recurrent points form diagonal lines. For random white noise, DET is close to 0 as most of the recurrent points in RP are single isolated points and form very few diagonal lines. Some key mathematical results related to determinism in RPs are given in the Appendix.
III. Denoising of PMU Data

A. Proposed Method

In this paper, empirical mode decomposition (EMD) [7] and nonlinear wavelet shrinkage (NWS) [8] are the two techniques used for denoising PMU data. The aim is not to suggest improvement to the techniques, but to develop a strategy to apply them more effectively and appropriately to a wide variety of signal shapes and content at different noise levels. The proposed strategy has been verified on a voltage signal. It is observed that the EMD technique adapts well to slow variations in signal owing to the spline interpolation employed. The NWS performs better for sharp changes in signal owing to its multiresolution capability at different time-scales. More details of denoising application of these two techniques are given in [3]–[6].

The key to separating noise from data in a signal is to quantify the difference between them. The deterministic, as introduced in the previous section, defines the randomness of a signal. Let the probability of finding a recurrent point at time $t$ is $P(t)$. Then the probability of finding two recurrent points consecutively at time $t_i$ and $t_{i+1}$ i.e., a diagonal of size 2, is $P = P(t_i) \cdot P(t_{i+1})$. For random noise, each data point is independent of each other; thus, $P(t) = P(t_{i+1})$ and $P(t_i) = 0$. Similarly, probability of finding a diagonal of length $l$ is $P = P(t_i)^l$. Thus, the probability of getting diagonal lines of length $l_{\text{min}}$ decreases exponentially with increase in $l_{\text{min}}$. As DET defines the percentage of the recurrent points that forms diagonal lines, DET also decreases exponentially with increase in $l_{\text{min}}$. It is close to zero for random noise. From (1), it can be said that, if a state (vector of measurements) is recurrent at time $t_i$ and $t_j$, then the magnitude as well as the slope of the signal at time $j$ and $j$ are similar because $||\overline{x}_j - \overline{x}_j||$ becomes low only when the magnitude as well as the direction both are similar at time $i$ and $j$. For slowly varying deterministic signals, the slope of the signal does not change very fast. Therefore, the probability of $\overline{x}$ being recurrent at time $t_{j+1}$ and $t_{j+1}$ remains high, i.e., that the probability of getting a diagonal line is high for a deterministic signal. From Fig. 1, it may be seen that almost all of the recurrent points have formed diagonal lines in the recurrence plot of a sinusoid. Therefore, determinism which defines the percentage of recurrent points that form diagonal lines is may be used to differentiate between a randomness and information within a signal.

The difference between DET of a denoised signal and the noise extracted $|\Delta\text{DET} = \text{DET}_{\text{signal}} - \text{DET}_{\text{noise}}|$ would be highest when the denoising is perfect. If the extracted noise contains part of a signal, then $|\Delta\text{DET}_{\text{noise}}$ would increase, whereas the $\text{DET}_{\text{signal}}$ would reduce if the denoised signal contains part of the noise. Hence, the maximisation of $|\Delta\text{DET}|$ should be used as a key index while applying any denoising technique.

Fig. 2 shows the flowchart of the proposed method. The noisy signal is first denoised with both techniques—EMD and NWS. The EMD of the noisy signal generates multiple intrinsic mode functions (IMFs), which add up to the original noisy signal. In [3] and [4], the authors proposed to first calculate the frequency of IMFs and then remove the high frequency IMFs. It is expected that the sum of the low-frequency IMFs and residue will give a noise-free signal. This assumption may not hold for the somewhat rare case of white noise, where the noise frequency extends over a wide range. Further, the actual signal may be non-linear and the noise level itself may vary with time. Any IMF may not contain a frequency component of the same order. If some part of an IMF contains high-frequency component and the rest contains a lower frequency component, then removing only the high frequency portion would be more effective than discarding the entire IMF. To do this, an initial data window is selected within the first IMF and a fast Fourier transform (FFT) is performed on that window. The selected window is removed from the IMF (i.e., zeroed) if the FFT yields frequency more than a threshold frequency. The process is repeated by sliding the window until the end of the IMF is reached. Similarly, for other IMFs the high-frequency part can be removed using FFT. Finally, by summing all of the IMFs, the denoised waveform can be obtained. It may be noted here that the FFT is inaccurate for small window sizes and during sharp changes. Thus, the above procedure is followed for a threshold frequency of 2–5 Hz. A higher threshold frequency is also more effective in reducing the overshoot during sharp edges and state transitions. For each threshold frequency, one denoised waveform is obtained and hence total four denoised waveforms are obtained from EMD, corresponding to threshold frequencies of 2, 3, 4, and 5 Hz. Among these four waveforms, some may be more accurate in static part of the waveform and others may be in dynamic part of the waveform. Therefore, taking one window length, $|\Delta\text{DET}|$ is calculated for each of the four waveforms, and the most accurately denoised waveform is accepted that shows highest $|\Delta\text{DET}|$. The same procedure is repeated for subsequent windows and finally one denoised signal is obtained using EMD and stored in a temporary variable “signall.”

For denoising using NWS, the wavelet basis function “db” is varied from “db1” to “db5” to obtain five denoised waveforms.

![Flowchart of the proposed method for denoising.](image-url)
Again, for each data window, the best denoised waveform is selected from these using the $\Delta$DET measure and stored in another variable “signal2.” The $\Delta$DET measure is again used to select between signal1 and signal2 for that particular window.

B. Result

The proposed denoising strategy was tested on a voltage waveform obtained from the nonlinear simulation of a symmetrical 0.15-s ground fault on bus 7 of the IEEE 39-bus test system [33] (cf. Fig. 9). The voltage waveform was chosen because it contains all types of typical power system signal shape, a steady state portion before fault, sharp edges during the fault and post-fault slow electromechanical oscillations. The voltage was measured at bus 7, and then 3% zero mean random noise was added. Fig. 3 shows the voltage waveform after adding random noise.

Waveforms obtained after denoising with EMD and NWS along with the original waveform are shown in Fig. 4. The application of both methods was optimized using $\Delta$DET. The figure shows that NWS performs better during fault portion while the EMD performs better during the electromechanical oscillation portion of the signal.

The $\Delta$DET index was used to select the most suitable technique (between NWS and EMD) to be applied on each 100-point window. The final waveform obtained after assembling the window segments is shown in Fig. 5. The denoised waveform is close to the original one confirming that $\Delta$DET can be used to successfully identify the best technique for a window and can provide the best denoised waveform as a whole.

C. Comparative Analysis

A comparative analysis between the original signal and the denoised signal obtained is presented in Table I. Since the original signal is known, the accuracy of the proposed strategy as well as the original methods used in isolation can be shown as norm of the error between simulated signal and denoised signal. Though EMD performs satisfactorily for the electromechanical oscillation portion of the signal, the error is very high due to its poor performance during the faulted portion. If the proper threshold frequency is selected and the best option is taken for a particular window of the waveform as proposed, the norm of error reduces from 1.335 to 1.0722. Table I shows that though EMD provides worst result, it can be used with NWS to improve the result of both.

The error in case of applying the NWS technique alone for different basis functions is shown in Table II. Even for the best setting, i.e., “db3,” the error is higher than the proposed method. Interestingly, though the error is minimum for the proposed method, the SNR is highest for ‘db4’ and gives unreliable results for the last two rows of Table II. It may be inferred that the maximization of SNR in this case will not always guarantee best possible denoising.
D. Application to Real Measured Data From PMU

The proposed method was also applied to denoise a measured voltage data from PMUs installed in the Indian Grid [34]. The noisy waveform and the denoised waveform have been shown in Fig. 6 for a sampling frequency of 25 Hz. It can be seen from the figure that the proposed method works effectively for every part of the waveform which confirms its ability to denoise any type of signal.

IV. SPATIAL EVENT LOCALISATION

A. Proposed Method

Self-clearing faults, dangerous from the equipment safety point of view, may not always result in radical changes in the system dynamics. However, severe disturbances such as generator outage, load shedding, line outage due to overloading etc. typically initiated in the power system with or without a fault, can significantly affect the system dynamic behavior. In this section, the occurrence of any disturbance event such as a balanced three-phase fault, tripping of a line or generator, is detected from the analysis of voltage measurements collected from all buses of the network. Initial pre-processing of the voltage data involves calculating by subtracting every sample of the voltage data from its subsequent sample. This yields an equivalent of the rate of change of voltage. The recurrence rate (RR) of is used to identify the location of the event accurately. A four second window of RMS voltage data starting from approximately one second before the event, was chosen for each bus. The same starting instant was selected for all the buses. As the exact instant of the event occurrence was not required, the proposed method is independent of threshold selection of event detection.

It is observed from the recurrence rate of calculated for every bus voltage that as an event propagates itself within a network via electromechanical oscillations, the voltage waveform changes its shape as compared to the waveform closest to the event location. For instance, the voltages of the buses closest to the disturbance undergo sharp variations. As the RR is calculated on the change in voltage rather than the voltage itself, the phase space (for calculating the recurrence) increases due to the sharpness of this change in voltage, thereby increasing the neighbourhood threshold which results in increase in RR. The RR of decreases for buses distant from the disturbance.

Thus, the event location is nearer to a bus that shows maximum RR of.

B. Mathematical Justification

The recurrence rate for a signal with probability distribution function, \( \rho(x) \), can be interpreted as the probability of getting a recurrent point in the recurrence plot within a neighbourhood \( \xi \). The convolution of \( \rho(x) \) with respect to itself gives the distribution of distance between points. As two points are considered recurrent only if their separation is below a threshold \( \xi \), the probability of finding a recurrent point is

\[
P_b(\xi) = \int_{-\xi}^{\xi} \rho(x) \ast \rho(x) dx.
\]  

(4)

The recurrence rate depends on the probability density function (PDF) of the signal and the neighbourhood threshold \( \xi \). Therefore, the change in RR of \( \Delta V \) with increase in distance from the event location can be understood by examining the change in the PDF and \( \xi \) at different buses for the same event. The PDF for post disturbance \( \Delta V \) for buses (14, 15, 16, 17, 27, 26) after outage of line 14–15 in the IEEE 39 bus test system is shown in Fig. 7. It can be seen that the probability is higher near 0 and concentrated between \(-0.001\) to \(0.0005\) for all of the buses.

For the outage of line 14–15, the neighborhood threshold selection during calculation of RR and DET for the buses (14, 15, 16, 17, 27, 26) has been shown in Fig. 8.
It can be seen that with an increase in distance from the disturbance location the neighbourhood threshold $\xi$ (calculated by the RQA software automatically depending on the maximum phase space span and standard deviation) decreases. As RR is calculated from convolution integral of the PDF from $-\xi$ to $\xi$, it will decrease with decrease in $\xi$. The results obtained for other types of events agrees with this conclusion.

C. Result

The IEEE 39-bus test system, shown in Fig. 9, is used to demonstrate the performance of the proposed event localization technique. The proposed method was tested for different types of event-bus fault, line outage without fault, line outage after fault, generator outage, and cascaded outages.

1) Event 1: Fault at bus 15: a 0.1-s balanced self-clearing fault at bus 15. Fig. 10 shows that RR is maximum at bus 15 and it reduces as distance increases from the bus.

2) Event 2: Line outage: The line between buses 14–15 is replaced with a double circuit line of same equivalent impedance. One of these two lines was tripped. From Fig. 11, it is seen that RR is maximum at buses 14 and 15.

3) Event 3: Fault on line 7–8 and outage: similar to the previous event, an additional line was added between buses 7 and 8. A balanced earth fault of duration 0.1 s was applied on this line, followed by its tripping. Fig. 12 shows that buses 7 and 8 have maximum RR of voltage.

4) Event 4: Generator outage at bus 30: Fig. 13 shows the RR of all bus voltages, following a generator trip event at bus 30. It may be observed that the RR is maximum at bus 30 and then at the nearest bus 2.

D. Impact of Noise and Cascaded Events

Sometimes, a disturbance can cause steady-state limit violations on other power system equipment and, consequently, cascaded outages may occur. The proposed method was used to detect the evolution of cascaded outages. Further, as PMU data are frequently corrupted with noise, the proposed technique robustness was also evaluated. Two cascaded events were tested. Three-percent zero-mean random noise was added to the simulated voltage signals and the proposed denoising strategy employed.

1) Cascaded Event 1: A balanced earth fault was applied on line 4–14 and disconnected after 0.1 s that resulted in the overloading of line 18–17 and its subsequent outage after 5 s. The first type of event, i.e., line fault has already been discussed in the previous subsection. The RR of all bus voltages for the second
be observed that the RR is higher at bus 30 and its surrounding buses like buses 2, 25, and 37. Fig. 16 shows the correct location of the event which infers that the data window needs to include the starting point of one event only.

It can be concluded that, to get the correct location in case of cascaded outages, the data window should consider only one event at a time for which the event source is needed. The data window considered for the first event can be up to the previous instant of the second event and the window considered for the second event should start after the first event, keeping the window length fixed at four second. Then, only one event will be considered in one window and the event location will be found out for that event.

### E. Limited Measurements

The extent of the PMU deployment in any power system may not be sufficient to make the system completely observable. As the recurrence rate of $\Delta V$ is higher for buses closer to the disturbance, it may be used to approximately locate the event using fewer PMU measurements. The triangulation method [19] commonly used to give the approximate event location in the presence of limited PMU measurements, uses the arrival time of frequency disturbances at various measurement locations. The arrival time is lesser for the measurement points closer to the event and higher for the measurement points at larger distance from the event location. If circles are drawn with centre at the measurement locations and radius proportional to the arrival time of the disturbance, then the common area of all the circles will include the most probable event location. However, due to several assumptions in the method, sometimes the common area is too big and sometimes all of the circles do not have any common intersection zone. Nevertheless, it does give a good indication of the approximate event location. This triangulation technique may be implemented using $[1-RR]$ instead of the arrival time of the disturbance. The recurrence rate of $\Delta V$ being higher at buses closer to the fault means that $[1-RR]$ would be lower for the buses closer to the event. Therefore, circles drawn with centre at the measurement locations and radius proportional to $[1-RR]$ will have a common area that will indicate the event location.

Actual PMU measurements [34] at four locations in the northern grid of India were used to test this methodology. After a disturbance at Rihand, India, PMUs located at Vindhyachal, Kanpur, Dadri, and Moga measured voltages as shown in Fig. 17. The $[1-RR]$ for these four measurement locations were found to be 0.14, 0.37, 0.39, and 0.83 respectively. The geographical distance from Rihand to the measurement locations are 60 km for Vindhyachal, 425 km for Kanpur, 850 km for Dadri, and 1200 km for Moga. It is evident that $[1-RR]$ is smaller when distance from event source to measurement location is also smaller. Therefore, similar to the wave propagation delay time [19], $[1-RR]$ also can be used in the triangulation method. $[1-RR]$ is not as high as expected for Dadri station because there is a separate dedicated HVDC line from Rihand to Dadri parallel to the ac transmission lines, which therefore reduces the electrical distance between Rihand and Dadri. The triangulation method works on the basic assumption that
Electrical and physical distance are equal which causes this “error.”

Currently the disturbance arrival time at different locations of the grid is used by many system operators. Sometimes, it becomes extremely challenging to detect the exact arrival time of the disturbance at different locations of the grid. Last intersection point between the average pre-disturbance frequency and the post disturbance frequency waveform at a particular location was used in [21] as the arrival time of the disturbance at that location. The same technique was applied to the frequency waveform after a generator trip measured at four locations in Indian Grid i.e., Vindhyachal, Kanpur, Dadri and Moga [35]. The technique proposed in [21] indicates that the disturbance reached Moga first, Vindhyachal second, Kanpur third and Dadri at the last which can be seen from Fig. 18.

Maximum absolute value of $(df/dt)$ calculated from the low-pass filtered frequency data at a particular location was used in [19] to detect the disturbance arrival time at that location. Frequency measured at four aforesaid locations were filtered to remove the local and inter-area modes and then frequency derivative was calculated which is shown in Fig. 19.

It can be seen that maximum absolute value of $(df/dt)$ (minimum actual value in this case) occurs at Dadri first, Moga second, Vindhyachal third, and Kanpur at the last.

Disturbance should arrive first at Vindhyachal, second at Kanpur, third at Dadri, and Moga last. Therefore, it can be seen from Figs. 18 and 19 that both of the methods proposed in [21] cannot be applied. As inter-area modes are of very low frequency, it is very difficult to remove the modes perfectly from a changing frequency waveform after a disturbance which creates a problem in applying the method proposed in [19]. Though theoretically it can be proved that disturbance takes time to reach a location at a larger distance using continuum modelling of a power system, finding the propagation delay is a problem in applying the existing methods.

The technique for event location proposed in this paper can successfully locate all types of power system dynamic events without any prior knowledge of network topology and operating condition. It does not require the knowledge of the exact type of disturbance to locate the event. Available methods in the literature use the electromechanical wave propagation delay as the chief input, thus requiring the accurate detection of the exact wave arrival time at different locations. It has been seen that due to the presence of noise and local and inter-area modes in the disturbance signal, it was extremely difficult to detect the arrival time of the disturbance.

V. TRANSIENT STABILITY CONTINGENCY RANKING

The critical clearing time (CCT) of a fault is a reliable indicator of the vulnerability of the system to that fault. It may be significantly affected by changes in system topology and/or operating conditions. Therefore, for every new operating condition, the CCT needs to be re-evaluated, and all possible system contingencies ranked in importance according their CCT. As the focus here is first-swing stability, the likelihood of a generator being stable may be directly linked to the RR of its post-fault voltage excursions. While it is difficult to predict the RR of a swing curve at the CCT, it is clear that the RR of a swing curve decreases, as the fault clearing time increases and approaches the CCT.

Referring to (4), the signal recurrence rate depends on its PDF and the chosen neighborhood threshold. For a fault at a particular location the neighborhood threshold is held fixed for different fault durations. With an increase in fault duration, the magnitude of oscillations increases which results in spreading of the PDF curve and increase in distance in between points in the signal. Therefore with increase in fault duration, the distance distribution function $[\rho(x) \ast \rho(x)]$ increases, integration of
the function within threshold \( \xi \) decreases, and, hence, the RR of \( \Delta V \) also decreases. For a critical contingency, the magnitude of the oscillation increases at a higher rate for increase in fault duration compared to a relatively benign contingency. The decrease in the RR due to increase in clearing time is more pronounced for a critical contingency than the decrease in RR for the same increase in clearing time for a more benign contingency. A critical contingency generally has a low CCT. Thus, the RR decreases at a higher rate for increase in fault duration when CCT is less and decreases very less for increase in fault duration when the CCT for that event is relatively higher.

### A. Proposed Method

Self-clearing faults were created at different buses and the voltage measured at each generator bus. The RR was calculated for changes in measured voltages (\( \Delta V \)), as described in the previous section, considering one second before fault to nine seconds after fault. The generators closer to the faulted bus showed higher values of RR, for reasons described in the previous section. The most disturbed generator was identified as the one that showed the biggest drop in RR with increase in fault duration. For instance, the second and third generators on buses 31 and 32 showed higher RR among all the generators for a 0.05-s fault on bus 12. However, the third generator showed a higher drop in RR if the fault duration was increased to 0.1 s. Therefore, this generator was deemed to be most affected and considered for further observation for a fault at bus 12.

The RR of the \( \Delta V \) of the most disturbed generator is plotted for different faults in Fig. 20. The CCT for balanced self-clearing faults on buses 12, 3, and 28 are 0.44, 0.2, and 0.14 s, respectively. It is seen from the figure that the initial slope is inversely proportional to critical clearing time (CCT). Therefore, if a drop in RR of \( \Delta V \) (i.e., \( \Delta \text{RR}_V \)) for increase in fault duration from 0.05 to 0.1 s is calculated for fault at each bus, then the buses can be ranked as per CCT. A descending order of \( \Delta \text{RR}_V \) will give an ascending order of CCT.

### B. Result

The proposed contingency ranking method was tested on the IEEE 39-bus New England test system, see Fig. 9. Typically, the generator buses always have the least CCT and are almost always ranked on the top. Therefore, all generator buses were excluded this study, and the CCT and \( \Delta \text{RR}_V \) for each of the remaining 29 buses are shown in Fig. 21. In this figure, the buses are shown arranged in an ascending order of CCT, along the x-axis. The \( \Delta \text{RR}_V \) of the corresponding buses are also plotted. It is seen that \( \Delta \text{RR}_V \) decreases when the CCT is in ascending order. As mentioned earlier, the generator which has higher \( \Delta \text{RR}_V \) is chosen for observation. From this analysis not only the ranking of disturbance but also the corresponding generator which is most likely to become unstable first, can be identified. The correlation between the CCT of a contingency and the \( \Delta \text{RR}_V \) was also calculated to be \(-0.9444\).

### VI. Conclusion

Recurrence quantification techniques are useful in analyzing nonlinear, nonstationary signals. They are inherently adaptive in nature, in the sense that their application does not require any prior information about the system from where the signals originate. In this paper, various indices were obtained from applying RQA to power system measurement and simulation data. These indices have been used to perform three main tasks—denoising of data, geographically localizing events within the network, and ranking of contingencies on the basis of their transient stability.

For denoising purposes, no new signal processing technique has been proposed. Rather, the determinism index of RQA has used to evaluate the performance of any denoising technique. It is further used to choose techniques to be applied to different parts of a signal, and thereby the denoised signal obtained is significantly noise free.

The recurrence rate of the change in the bus voltage is used to gauge the proximity of that bus to any disturbance. The recurrence rate is highest at the bus nearest to the disturbance and reduces with distance from the event location. Cascading disturbances have also been studied using this index. This method is independent of the prevalent network topology and operating conditions.

In transient stability studies of various contingencies, the evolution of a generator’s recurrence rate due to a change in the duration of that contingency has also been used to estimate the critical clearing time of the contingency. This technique also leads to a transient stability ranking strategy that does not require the repetitive computational burden associated with calculating the critical clearing time of a fault.
APPENDIX

The probability of a recurrence plot (RP) is given by [36]

\[ P_b(\xi) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j} \tag{5} \]

where \( R_{i,j} \) is a recurrent point, \( N = \text{total} \) is the number of points in recurrence plot, \( \xi = \text{threshold} \) for the neighborhood. If the PDF of a function is \( \rho(x) \), then its distance distribution is found out using the self convolution of the pdf \( \rho(x) \) as

\[ R(x) = \rho(x) * \rho(x). \tag{6} \]

Two points are said to be recurrent only if the distance between the two is below a threshold \( \xi \). Therefore, the probability of getting a recurrent point is

\[ P_b(\xi) = \int_{-\xi}^{\xi} R(x) \cdot d(x) = 2 \int_{0}^{\xi} \rho(x) * \rho(x) \cdot d(x). \tag{7} \]

The probability of finding a diagonal line of length at least \( \ell \) in the RP is defined as

\[ P^\xi_\ell(\ell) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i,j=1}^{N} \prod_{m=0}^{\ell-1} H_{i+j+m,m}. \tag{8} \]

A. Gaussian Noise

For an independent Gaussian noise WN(0, \( \sigma^2 \)), the probability of finding a point in the interval \( |x, x + \Delta x| \) is

\[ P(x + \Delta x) = \int_{x}^{x + \Delta x} \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{x^2}{2\sigma^2} \right) dx. \tag{9} \]

To find a recurrent point at coordinate \((i,j)\), the condition \(|x_i - x_j| < \xi\) must hold as follows:

\[ P_k(\xi) = \frac{1}{(\sqrt{2\pi}\sigma)^2} \int_{-\xi}^{\xi} \int_{-\xi}^{\xi} \exp \left( -\frac{x^2}{2\sigma^2} \right) \cdot \exp \left( -\frac{y^2}{2\sigma^2} \right) dxdy \]

\[ = \text{erf} \left( \frac{\xi}{2\sigma} \right). \tag{10} \]

For \( (\xi/\sigma) = 0.1 \), \( P_k(\xi) \) comes out to 0.056. The probability of finding a recurrent point in the RP is nothing but recurrence rate (RR) of the RP:

\[ P_{RP} = \text{erf} \left( \frac{\xi}{2\sigma} \right). \tag{11} \]

For independent noise, the probability of finding \( l \) recurrent points consecutively to form a diagonal is

\[ P(l) = P_{RP}^l. \tag{12} \]

To find a diagonal line of length exactly equal to \( l \), the points preceding and succeeding the diagonal line must not be recurrent. Hence, the probability of finding a diagonal line of length exactly equal to length \( l \) can be written as

\[ P(l) = P_{RP}^l(1 - P_{RP})^2. \tag{13} \]

The total number \( (N(l)) \) of diagonal lines of lengths \( l \) in a recurrence plot of total length \( L \) can be found out as

\[ N(l) = (L^2 - L)P_{RP}^l(1 - P_{RP})^2. \tag{14} \]

As the main diagonal line is not considered, one \( L \) is subtracted.

B. Uniformly Distributed Noise

Similarly, the RR and total number of diagonal lines of length equal to \( l \) can be found out for uniformly distributed noise. The distance distribution function for uniformly distributed noise can be written as

\[ R(x) = \begin{cases} 1 - |x|, & \text{if } |x| < 1 \\ 0, & \text{else} \end{cases}. \tag{15} \]

The probability of getting a recurrent point in the RP of uniformly distributed noise can be obtained by the convolution integral of the probability distribution function

\[ P_b(\xi) = 2\xi - \xi^2 + \theta(\xi - 1)[1 - 2\xi + \xi^2]. \tag{16} \]

The probability of getting a diagonal of length \( l \) is

\[ P(l) = (2\xi - \xi^2)^l. \tag{17} \]

This probability decreases exponentially. The determinism of a signal is directly proportional to the probability of getting a diagonal of length at least \( l \). The probability of getting a recurrent point and a diagonal line of length \( l \) is very low when recurrence is checked between two points and it will further reduce if recurrence is checked between two \( m \) dimensional vectors rather than two scalar points. That is why RR of uniformly distributed noise and Gaussian noise is extremely small and the determinism of noise is close to zero.

C. Straight Lines or Slowly Varying Signal

The PDF of a straight line of magnitude “k” is

\[ \rho(x) = \begin{cases} \delta(x) = 1, & \text{if } x = k \\ 0, & \text{else} \end{cases}. \tag{18} \]

The probability of getting a recurrent point in the RP

\[ P_b(\xi) = \int_{-\xi}^{\xi} R(x) \cdot d(x) = 2 \int_{0}^{\xi} \rho(x) * \rho(x) \cdot d(x) - 1. \tag{19} \]

As the RR is 1, all the points in the RP will be recurrent. The probability of getting a diagonal line of length \( l \) will also be 1 because the probability of getting a recurrent point next to a recurrent point is also 1. So, the RR and the determinism of the signal will also be 1.

\[ P(l) = P_{RP}^l = 1 \tag{20} \]

D. Sinusoids

The probability distribution function of a sinusoid of magnitude 1 can be written as

\[ \rho(x) = \frac{1}{\pi\sqrt{1 - x^2}}, \quad -1 \leq x \leq 1. \tag{21} \]
The probability of getting a recurrent point in the RP can be found using (19). Normally, recurrence is checked between two vectors. Therefore if any two vectors at two time instant are recurrent, then it can be said that the slope of the waveform at that instants are also nearly same. For a slowly varying signal, the slope does not change very fast. Hence, if two vectors at two instants are recurrent, then it can be said that the slope of the waveform at these vectors. Therefore if any two vectors at two time instant are recurrent, then the waveform is most probably the slope does not change very fast. Hence, if two vectors at two time instants are recurrent, then it can be said that the slope of the waveform at these vectors. Therefore if any two vectors at two time instant are recurrent, then the waveform is most probably recurrent at next instant also. For a sinusoid, suppose the data point at 0° is recurrent with the data point at 360°. If the sampling interval is θ, then data points at θ and (θ + 360°) will also be recurrent. As most of the recurrent points are followed by other recurrent points which forms a diagonal of length at least 2, determinism becomes close to 1.

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REFERENCES


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1MATLAB codes are available at https://toscy.pik-potsdam.de/CRPtoolbox/