Recurrence analysis of surface pressure characteristics over symmetrical aerofoil

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ABSTRACT

We study the surface pressure data exhibiting the underlying dynamical behavior of the flow transition over the upper surface of the aerofoil by using recurrence quantification analysis (RQA). In this study, NACA 2415 aerofoil subjected to a turbulent inflow of $TI = 8.46\%$ at various angles of attack ranging from $\alpha = 0^\circ$ to $20^\circ$ with an increment of $5^\circ$ corresponding to $Re = 2.0 \times 10^5$ is considered. We show that the values of recurrence quantification measures effectively distinguish the underlying dynamics of time series surface pressure data at each port, which proves RQA as an effective tool in accurately predicting the flow transitions.

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The quantification of recurrent characteristics in phase space has drawn significant interest as a specific idea, the fundamental concepts of which originated in the pioneering work of Poincaré in the late 19th century. In this study, we explore the nonlinear surface flow behavior on a two-dimensional NACA 2415 aerofoil experiencing turbulent inflow by using recurrence quantification analysis. The change in the general surface flow and the transition of surface pressure from periodic to chaotic characteristics are well predicted using recurrence quantification measures such as the recurrence rate, determinism, and entropy.

I. INTRODUCTION

Aerfoils are unique curved surfaces designed to produce favorable lift to drag ratios in several engineering applications like airplane wings, wind turbine blades, propellers, marine rudders, etc.¹⁻³ Generally, with the increase in the angle of attack, aerfoils produce complex flow patterns caused by flow separation with variations in parameters like Reynolds number ($Re$), Turbulence intensity ($TI$), etc. In the process of flow separation, the attached laminar flow over the aerofoil tends to transition to an unsteady flow due to the formation of an adverse pressure gradient. Meanwhile, the prediction of the onset of transition from periodic to chaotic behavior of flow parameter over the surface of the aerofoil is a great challenge.⁴ Recent studies reported that the transition of the flow behavior of the aerofoil was predominantly focused on the surface pressure characteristics alone. Consequently, only very few works address this problem by considering nonlinear methods to predict the flow transition induced over the aerofoil. Qian and Li suggest that the local birth and death of the periodic solution, i.e., Hopf bifurcation over the surface of the aerofoil caused by the viscous effects and the flow separation can be effectively used to predict the flow transition over the aerofoil.⁵ Since the extended region of flow attachment over the aerofoil helps increasing the operational capability, working range, and aerodynamic efficiency, the evaluation of nonlinearities associated with flow separation has become one of the most important facets of the aerofoil utilized in the field of aerodynamics and wind engineering.

Assuming the flow over the aerofoil as a dynamical system, incorporating turbulence in the flow makes the system no longer linear. Solving such problems using analytical techniques assuming a priori information of periodicity is inapplicable. Studies claim that nonlinearity associated aerodynamic effects are more difficult to solve, since their analytical solutions are practically nonexistent.⁶⁻¹³ Under such circumstances, the ability to identify the flow transition over the aerofoil is most often related to physical experiments. Over the past few years, wind tunnel testing is considered as one of the most generalized ways of measuring the flow field characteristics over the aerofoil.¹⁴⁻¹⁷ Several studies on aerfoils have been performed on wind tunnels to investigate both the qualitative and quantitative flow field changes over the aerofoil under different test...
conditions. Based on the framework of the previous researchers, it becomes clear that flow transition is one of the important criteria in design. Therefore, the assessment of the nonlinear flow behavior associated with the flow transition over the aerofoil at different operating conditions is deemed important for aircraft and wind turbine blades, where safety is of paramount importance. However, observational data of these systems are limited. Linear approaches to time series analysis are often inadequate and most nonlinear techniques such as fractal dimension or Lyapunov analysis suffer from the curse of dimensionality and require rather long time series. For instance, Lyapunov analysis is usually performed over the trajectory of a single fluid particle in time domain to quantify the nature of the flow. One of the principal disadvantages of using Lyapunov exponent to identify the periodic to chaotic transition is that the fluid particle spends only a finite time in the chaotic regime, making it difficult to distinguish between the periodic and chaotic behavior. Hobbs and Ord also reported that the goal of any nonlinear time series analysis is to extract the features of the dynamics of the underlying signal in terms of quantitative measures. It should, however, be noted that, whenever there is a physical experiment, noise present in the data poses additional challenges to understand the underlying flow behavior. Therefore, preprocessing requisites to remove the noise may be required, and they all come at the expense of cost.

Gottwald and Melbourne developed 0-1 test to effectively distinguish between the periodic and chaotic behavior of any dynamical system from the time series data without involving any preprocessing requisites. Similarly, recurrence quantification analysis (RQA) is used to quantify the periodic to chaotic transition by visually identifying the temporal features on the recurrences obtained from trajectory of the dynamical system. Recent studies suggest that more tools from nonlinear dynamics are required to identify the flow transition. Further, conclusive tests to characterize the flow transitions over the aerofoil not only helps revealing the underlying flow dynamics at each port but also can directly help enhance the design of efficient wings, wind turbine blades, propellers, and marine rudders with better aerodynamic performance capabilities by applying corresponding control measures.

Out of the many methods that arise in the past, recurrence plot (RP) and the associated RQA have been intensively studied in nonlinear data analysis. However, the use of RQA over aerofoils subjected to flow nonlinearity is still left unexplored. Therefore, the focus of the present work is to use RQA to identify the flow transition from the time series surface pressure data measured over the test aerofoil subjected to a flow characterized by a turbulence intensity of TI = 8.46%. The time series surface pressure data over the surface of the aerofoil were measured from wind tunnel testing. Quantitative measures like recurrence rate (RR), determinism (DET), and entropy (ENT) obtained from the RQA are then used as a signature to predict the periodic to chaotic transition.

The paper is organized as follows. The experimental methodologies used to measure the time series surface pressure data are described in Sec. II, following which Sec. III explains the fundamental equations and the introduction of the RQA. The results of the RQA in terms of RR, DET, and ENT to predict the flow transition were then discussed in Sec. IV for test aerofoil at various angles of attack ranging from α = 0° to 20° in increment of 5°. Based on the results, the conclusions are drawn and are presented in Sec. V.

II. EXPERIMENTAL DESIGN

All the wind tunnel tests were carried out in an open-circuit low-speed subsonic wind tunnel of rectangular cross section of 300 × 300 × 1500 mm. The wind tunnel can reach a maximum velocity of about 60 m/s controlled by Emotron variable frequency drive (VFD) and the rpm controller. The freestream turbulence intensity of the wind tunnel is 0.51%. A self-developed passive grid made up of parallel arrays of round bars was used to generate desired turbulence in the wind tunnel. The test model is mounted at 595 mm downstream of the turbulence grid as outlined in Ref. 14. The aerofoil model considered for measurement is an infinite model. A rectangular planform wing test model with a maximum camber of 2% having its maximum thickness lying at 15% of the chord location based on unsymmetrical profile has been considered. Flow field measurements over the test aerofoil were carried out at a mean velocity of 30 m/s corresponding to Re = 2.0 × 10^7 at TI = 8.46%. In order to measure the aerodynamic forces and the surface pressure distribution acting over the model, the test aerofoil model is housed with 21 pressure ports uniformly distributed over both the upper and the lower surface of the aerofoil. The diameter of each pressure port is approximately 1 mm. The surface pressure ports are then pneumatically connected to the MPS4264 model simultaneous pressure scanner of M/s Scanivalve make. The time-series surface pressure data were collected at a sampling frequency of 700 Hz corresponding to 10,000 data samples. Using the pressure integration technique, the aerodynamic coefficients like coefficient of lift (C_l) and the coefficient of drag (C_d) can be obtained (see Fig. 1).

III. RECURRENCE QUANTIFICATION ANALYSIS FOR SURFACE PRESSURE DATA

The RP was first introduced to visualize the time dependent behavior of the dynamics of systems, which can be identified in the phase space. The RP analysis is found to be efficient in the identification of system dynamics that cannot be observed using other types of conventional linear and nonlinear approaches. Moreover, this RP analysis has been found very useful for nonstationary time series and noisy time series.

For a given time series x_i of N datasets, first the phase-space reconstruction is constructed using vectors u_i = x_i, x_{i+1}, ..., x_{i+(d-1)τ},

where \( d \) is the embedding dimension and \( τ \) is the time delay, which can be estimated through autocorrelation function. The main step is then to calculate the N x N matrix and can be determined by the expression

\[
R_{ij} = \Theta(\xi - \|x_i - x_j\|), \quad i, j = 1, 2, \ldots, N, \tag{1}
\]

where \( ξ \) is a threshold value, \( \| \cdot \| \) denotes the Euclidean norm, and \( Θ(\cdot) \) is the Heaviside step function. \( x_i \) defines a sphere centered at \( x_i \). If \( x_i \) falls within the sphere, the state will be close to \( x_i \) and the value of \( R_{ij} = 1 \). In this paper, a fixed value of \( d \), \( τ \), and Euclidean norm are used for the purpose of the resulting symmetric RP. Finally, the binary values can be simply observed by using matrix plot (i,j) with the colors black (1) and white (0). However, the above visual inspection of the RPs is not that straightforward to conclude the dynamical transitions when the time series are drawn from the systems. Zbilut and Webber have introduced the RQA to identify the system dynamics and their transitions through the diagonal
structures available in the RPs. In the framework of RQA, different quantitative measures were introduced such as the recurrence rate (RR), determinism (DET), and entropy (ENT).

The recurrence rate $\text{RR}(\xi)$ is the probability of finding a black recurrence point in recurrence matrix $(R_{ij} = 1)$, or

$$\text{RR}(\xi) = \frac{1}{N^2} \sum_{i,j=1,i\neq j}^{N} R_{ij}(\xi),$$

where $N^2$ is the total number of black points in a RP. We also note that main diagonal lines are excluded from the double sum because each point is recurrent with itself.

The presence of diagonal lines in RP is usually related to deterministic behavior. Thus, the percentage of recurrence points belonging to diagonal lines of at least $l_{\text{min}}$ is called determinism (DET), defined as

$$\text{DET} = \frac{\sum_{l=l_{\text{min}}}^{l_{\text{max}}} P(l)}{\sum_{i,j=1}^{N} R_{ij}},$$

where $P(l)$ is the frequency distribution of the lengths $l$ of diagonal lines and $l_{\text{min}} = 2$ is the minimum length allowed for a diagonal line, whereas the maximum diagonal length is $l_{\text{max}} = \max(l_i, i = 1, 2, \ldots, N)$. Here, $N$ denotes absolute number of diagonal lines. Thus, the determinism measures the percentage of points in a RP belonging to diagonal lines. Further, we also measure the Shannon entropy based on the relative frequency of the occurrence of the diagonal segments of nonrecurrent points.\cite{38,39} It is defined as

$$\text{ENT} = - \sum_{l=\text{min}}^{N} E(l) \log E(l),$$

where $E(l) = \frac{Q(l)}{\sum_{l=\text{min}}^{N} Q(l)}$ and $Q(l)$ denotes relative frequency of diagonal occurrence of segments with the length $l$ of nonrecurring points and $N$ with the maximum length of nonrecurring segments as suggested in Refs. 38 and 39.

The aforementioned variables are used to detect the transitions in the time series and have been applied to diverse fields in the recent past.\cite{29-37} Here, we extend the applications of RPs and RQA to study the dynamical transitions of the time series surface pressure data. In the present study, the free stream velocity is turbulent, the stagnation pressure measured at port 1 for any angle of attack is chaotic, and this value is considered as a reference for the classification of periodic and chaotic surface pressures. The main goal is to demonstrate the ability of RQA to detect the unique crossover phenomenon observed in the surface pressure data in the aerosoil.

### IV. RESULTS AND DISCUSSION

We consider a time series of length $N = 10000$ and compute the RP after embedding the time series with a dimension of $m = 1$, a delay of $\tau = 1$ and the value of $\varepsilon$ is chosen as $0.1\sigma$, which is based on the previous studies,\cite{19} where $\sigma$ denotes standard deviation of the...
the time series, which is automatically calculated from the surface pressure data. Since our pressure time series is relatively low dimensional, $d = 1$ is considered. The time series of the surface pressure data for two different angles of attack ($\alpha$) as shown in Figs. 2(a) and 2(b) is hardly indistinguishable. On the other hand, from their corresponding visual inspection of RPs, the changes of dynamical behavior of the surface pressure data with respect to $\alpha$ are evident [see Figs. 2(c) and 2(d)]. However, it is important to use RQA to identify the dynamical transitions in the surface pressure data over the aerofoil with respect to different angles of attack. The detailed dynamical transitions and changes observed over the aerofoil are discussed with the help of RQA as follows. In order to show the reliability of RQA test for wind tunnel data, the time series velocity and the turbulence intensity (TI) from a low value of 0.51 to 8.46% as mentioned in Ref. 15 have been measured and the corresponding RR and ENT are shown in Table 1. The TI at 0.51% is a periodic flow with a very less significant chaotic flow. But for the case of 8.46% TI, the flow is highly chaotic. The corresponding RR and ENT are evident from low TI to high TI which in turn corresponds to periodic flow to chaotic flow.

A. Recurrence rate-based signatures of the periodic to chaotic flow transition

The simultaneous surface pressure measurements obtained over the test aerofoil subjected to turbulent inflow have been investigated using RQA, and their results are discussed in detail in this section. To characterize the observed dynamical states from RQA, emphasis has been given to quantitative measures, namely, RR, DET, and ENT. The values of RR obtained for all the pressure taps located in the upper surface of an aerofoil ranging from $\alpha = 5^\circ$ to $20^\circ$ in increment of $5^\circ$ are shown in Fig. 3. At $\alpha = 5^\circ$, the time series pressure data processed at port 1 exhibit a RR of 0.26, indicating chaotic behavior [see Fig. 3(a)]. Since the freestream flow itself is turbulent featuring a turbulence intensity of 8.46%, the flow at port 1 is chaotic. Based on the framework of the previous research studies related to aerofoil, it is well known that the freestream flow tends to bifurcate at the leading-edge of the aerofoil followed by flow acceleration over the upper surface of the aerofoil. Subsequently, it can also be seen that the value of RR gradually increases with the increase in the chordwise location toward the vicinity of the trailing edge. For instance, the value of RR at port 2 = 0.28 is comparatively lower than the value of RR at port 9 which is 0.37. It is worth noting that the accelerated flow over the upper surface of the aerofoil starts decreasing gradually due to the formation of adverse pressure gradient toward the vicinity of the trailing edge. At $\alpha = 5^\circ$, the flow undergoes transition from periodic to chaotic between ports 9 and 10. The values of RR further ascertain this transition by a sudden departure from the increasing trend line. The value of RR at port 9, which is 0.37 reduces to 0.25 at port 10, indicates the flow transition from periodic to chaotic behavior. Following which at port 11, the value of RR further reduces to 0.22, indicating that the separated shear layer arising out of flow separation at port 10 tends to make the flow unstable leads to a relatively more chaotic behavior. Similarly at $\alpha = 10^\circ$, the bifurcated streamwise flow moving over the upper surface of the aerofoil begins to undergo transition from chaotic to periodic between ports 8 and 9. It is illustrious from Fig. 3(b) that the value of RR increases with the increase in the flow acceleration from 0.24 at port 1 to 0.28 at port 7, followed by a sudden drop to 0.25 at port 8. It can be seen that with the increase in the angle of attack ($\alpha$), the separation point, i.e., point at which flow transition occurs, tends to gradually move toward the vicinity of the leading-edge. This can be further substantiated by the value of RR that at $\alpha = 5^\circ$, the sudden rise in the trend line is seen between ports 9 and 10, whereas at $\alpha = 10^\circ$, the sudden rise in the trend line is seen between ports 8 and 9, thus confirming the onset of earlier flow separation with the increase in the angle of attack ($\alpha$).

Figure 3(c) represents the variation of RR over the static pressure taps of the aerofoil inclined at $\alpha = 15^\circ$. The streamwise bifurcated flow over the upper surface of the aerofoil at $\alpha = 15^\circ$ tends to accelerate from port 1 and reaches the peak acceleration at port 8. RR confirms the flow transition from periodic to chaotic over the upper surface of the aerofoil beyond port 8 resulting in a trailing edge stall. Once the aerofoil undergoes trailing edge stall, then the separated shear layer, arising from the point of separation tends to form a separation bubble extending till the surface of an aerofoil, makes the flow more unstable resulting in chaotic dynamics between ports 8 and 11. For instance, at $\alpha = 15^\circ$, the flow classification based on the RR values corresponding to pressure ports is shown in Fig. 4.

Studies suggest that when the attached flow over the upper surface of the aerofoil is forced to undergo separation by gradually increasing the angle of attack, it may result in a combination of leading-edge and trailing edge stall. The trailing edge stall is

<table>
<thead>
<tr>
<th>Grid location</th>
<th>TI%</th>
<th>RR</th>
<th>ENT</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.3863</td>
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<tr>
<td>Location 4</td>
<td>8.46</td>
<td>0.2408</td>
<td>1.6478</td>
</tr>
</tbody>
</table>
FIG. 3. The variation of recurrence rate (RR) for the surface pressure ports at various angles of attack $\alpha$ (a) 5°, (b) 10°, (c) 15°, and (d) 20°; (e) location of the pressure port.

B. Determinism based signatures of the periodic to chaotic flow transition

As discussed in Sec. IV A, the periodic to chaotic flow transition can be further verified using the DET obtained through RQA. Figure 5 shows the variation of DET along each pressure ports located over the upper surface of the aerofoil from $\alpha = 5^\circ$ to $20^\circ$. From the visual inspection of Fig. 5, it is easy to understand that the DET value changes with the change in the flow behavior over the aerofoils at different chordwise distances over the upper surface of the aerofoil. Previous literature studies suggest that the low DET value signifies the chaotic zone and the high DET signifies the periodic zone. The crossover region or transition region is identified by an abrupt jump in the trend line. We observe a sudden decrease in the value of DET toward the vicinity of the trailing edge, indicating the periodic to chaotic transition or crossover phenomenon over the aerofoil surface. Based on the values of DET shown in Figs. 5(a)–5(d), it becomes clear that the RQA quantified in terms of DET is also able to predict the periodic, chaotic, and the periodic to chaotic transition or crossover phenomenon successfully for the aerofoils tested under various angles of attack. For instance, at $\alpha = 5^\circ$, where port 1 has low value of DET, following which the streamwise flow experiences an acceleration over the upper surface.
of the aerofoil, gradually loses its chaotic behavior, and becomes periodic resulting in the increase in the value of DET. This acceleration happens between ports 2 and 9. The sudden decrease in the value of the DET observed between ports 9 and 11 can be attributed to the flow separation associated with the trailing edge stall behavior. These chaotic to periodic and periodic to chaotic transitions observed over the upper surface of the aerofoil at $\alpha = 5^\circ$ based on the value of DET coincide with the value of RR, thus confirming the flow behavior. Similarly, the value of DET coincides with the value of RR for the aerofoils tested at different angles of attack ($\alpha$) as shown in Figs. 5(b)–5(d), thus confirming the ability of the RQA to predict the flow transitions over the aerofoil.

C. Entropy-based signatures of the periodic to chaotic flow transition

Entropy is a third invariant of considerable importance in the RQA and is a measure of disorderness, therefore, the greater the disorderliness in the time series surface pressure data, the higher will be the entropy (ENT). The variation of ENT over the upper surface of the aerofoil at different angles ranging from $\alpha = 5^\circ$ to $20^\circ$ shown in Figs. 6(a)–6(d) can enable us to identify the underlying flow transitions. Generally, when the system approaches a periodic behavior, it is believed that there is an order in the system which is implied by the decrease in the ENT, whereas in the chaotic zone, the increase in the disorderliness causes increase in the ENT. The variation of the ENT is quite appealing in the sense that the ENT value is low in the periodic zone and high in the chaotic zone. To further ascertain this behavior, time histories of the pressure data measured at ports experiencing the minimum and maximum ENT values are plotted at $\alpha = 15^\circ$. From Fig. 6(c), it is clearly seen that port 1 experiences the maximum value of ENT around 1.45, while port 8 experiences the minimum ENT value of 0.98.

Additionally, at $\alpha = 15^\circ$, the maximum value of ENT measured near the vicinity of the trailing edge at ports 9 and 10 is also plotted. Therefore, it becomes clear that the ENT is a useful measure that can portray the underlying dynamics associated with each port over the surface of the aerofoil, which helps predicting the flow transition phenomenon over the aerofoils.

V. CONCLUSION

Prediction of flow transition from periodic to chaotic and chaotic to periodic behavior from the time series surface pressure data measured over the upper surface of the aerofoil subjected to the turbulent intensity of $\text{Tl} = 8.46\%$ is investigated through recurrence quantification analysis (RQA). The qualitative and quantitative prediction of flow transition along with the underlying dynamics of each port over the aerofoil is estimated. The onset of flow transition found in the present work is in good agreement with the 0-1 test. The quantitative measures like recurrence rate (RR), determinism (DET), and entropy (ENT) from RQA were utilized effectively to predict the flow field over upper surface of the aerofoil. Identification of surface pressure behavior will be useful in understanding the time series force acting on the aerodynamic body and, in turn, helps the structural designers to design the fail-safe structure. Transition behavior observed in this study can be applicable to wind turbine blades, which experience a free stream velocity with high TI, airplane’s take-off and landing in the airport, and vertical axis wind turbine, which is in general located inside the atmospheric boundary layer.

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