DETECTING CHAOS IN KUWAIT EXCHANGE RATE DATA

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1. Introduction

Stock market and finance experts in exchange rates have always been keen to study the behavior of the market, and much research has been done in this area. Research is still going on concerning efficient market hypotheses and the dynamic behavior of market patterns. There has been a long-term debate on whether it is market sentiment or some rationality that dominates. Various AI techniques as well as statistical techniques were tried to handle the nonlinearity (ANN, genetic algorithm, expert systems, and nonlinear time series model). This is not the proper time to go into the debate to prove the efficiency of one tool over another in a particular area, rather integrating all the tools so that we can freeze all of their efficiency into one.

We follow the procedure proposed by Tan in [10]. According to this procedure, processing starts with the nonlinear system detector part, consisting of various functional criteria. One of the functions is to measure the Hurst exponent value to determine whether the number of data sets is adequate for measuring the Hurst exponent, and then calculating mean orbital period and the overall nature of the time series, in other words, its persistency and anti-persistency nature. We discuss the way of measuring the Hurst value in a way such as choosing a good compressor (e.g., log return is a very high compressor) for the time series, averaging for different $n$, thus removing the AR residual part, and the first peak from the data set of $\log(R/S)$ and $\log(N)$. From the value of $H$, we fit a short-term (e.g., ARIMA) or long-term (e.g., FARIMA) model. The other functions are average mutual information, calculating minimum embedding dimension and time delay to reconstruct the phase space. The simplest way of measuring the time delay is from the peak-to-peak analysis or time domain auto-correlation function. The system predictability is measured from recurrent analysis rather than the time delay plot, on the basis of a parameter called spectral entropy. This is a very useful measure for choosing a financial time series to invest in from many. We use a model which preprocesses the data in varying time window sizes and window gaps and measures the dynamic nature of the system and then fits the appropriate model to that particular time frame. The idea is based on the concept that a time series changes its basic nature in mainly three forms (trend, chaotic, and random). The change of the dynamic nature of the system is detected by calculating the largest Lyapunov exponent for different time windows. We also discuss how the prediction error can be guessed from the value of the Lyapunov exponent.

Finally, we compare the differences between two approaches of forecasting, using ANN and the statistical method. As Hurst’s exponent value is used to determine the long or short-term memory effect, we calculate the value of the Hurst exponent first. There are various methods of finding the value of the Hurst exponent. The method we used here is known as rescale range analysis ($R/S$ analysis), suggested by Peter in [2]. The interpretation is as follows: $0 \leq H < 0.5$ means the series is persistent, $H = 0.5$ means the series is random, $0.5 < H \leq 1$ means the series is persistent.

But many studies suggest (see [2]) that a Hurst value greater than 0.7 is a cause of very long-term memory effect, and then we need the fractional difference modeling technique rather than the short-term Markovian technique. There is still no valid agreement on the choice of the compressor that is the transformation of the series. In [1], Trippi suggests a detrending method before considering the analysis of dynamics of the series. As we followed the method suggested by Peter, detrending will be our next step to consider. We compared the method that takes the first log difference of the series with the method suggested by Peter, but the simulation shows the method suggested by Peter is a better one. The following table shows all the Hurst exponent values.

<table>
<thead>
<tr>
<th>Exchange Rate</th>
<th>Hurst’s Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kuwait–GBP exchange rate</td>
<td>0.557</td>
</tr>
<tr>
<td>Kuwait–US exchange rate</td>
<td>0.678</td>
</tr>
</tbody>
</table>

It can be observed that the Hurst exponent for the Kuwait–US exchange rate is higher than the Kuwait–GBP exchange rate, which suggests that the Kuwait–US exchange rate is more predictable than the Kuwait–GBP exchange rate, which shows a more or less random pattern.

The considered time series, i.e., one-dimensional random variables $X_1, X_2, \ldots, X_n$, can represent either a nonlinear stochastic or nonlinear deterministic process (see [4]). We are interested here in the deterministic version of the process. If we assume that the process underlying the observation is dynamic and can be modeled by some nth-order ordinary differential equation, then we need to reconstruct the state space concerning all the information about the data. We use $x(t)$ to calculate two new phase-space variables $y(x(t))$ and $z(X(t))$ defined by

$$
Y(t) = \frac{dX(t)}{dt}, \quad Z(t) = \frac{dY(t)}{dt}.
$$

There are two ways to proceed further. One way is to process the original series with a differentiator, and another way is to compute the derivatives manually. But each problem is fraught with experimental difficulties, because differentiation is an inherently noisy process.

The numerical derivatives of a digitized time series are defined by $Y_t = (X_t - X_{t-1})/(T_t - T_{t-1})$. If $X_{t-1} = 1.0 \pm 0.1$, $X_t = 1.1 \pm 0.1$, and $T_t - T_{t-1} = 1$, then $Y_t = 0.1 \pm 0.2$; thus the problem get even worse while taking higher order derivatives. If the sampling time is constant, then the derivatives are contained in a variable constructed by the difference of two points. So we can recover almost all of the same information. The only problem that remains is calculating the time delay and the estimate of the embedding dimension.

1.1. Phase-space reconstruction. The phase space is reconstructed using Taken’s theorem

$$
X^n(t_i) = [X(t_i), X(t_i + \tau), \ldots, X(t_i + (n-1)\tau)]^T,
$$

where $t_1 = i\delta t, I = 1, \ldots, N, \delta t$ is the sampling time, and $n$ is the embedding dimension.

For an infinite noise-free data the choice of time delay is arbitrary, but for a finite amount of data it affects the quality of the reconstructed trajectory. First, we used two methods to calculate the time delay:

(i) peak-to-peak analysis;

(ii) auto-correlation plot.

Then we compared their results by finding the average mutual information. The threshold value for peak-to-peak analysis is slightly greater than the mean value of the signal. The average of the maximum and minimum value is taken from the peak-to-peak analysis to compute the delay. The significant value for the autocorrelation function is determined by $2/\sqrt{n}$, where $n$ is the number of observations.

<table>
<thead>
<tr>
<th>Exchange Rate</th>
<th>Peak-To-Peak Analysis</th>
<th>Auto-Correlation Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kuwait–GBP</td>
<td>1000</td>
<td>650</td>
</tr>
<tr>
<td>Kuwait–US</td>
<td>700</td>
<td>1000</td>
</tr>
</tbody>
</table>

1.2. Average mutual information. The average mutual information (AMI) establishes a nonlinear relationship between measurements of one distributed set of variables $a_i$ and another set $b_j$. The algorithm evaluates the normalized histogram of variables $a_i$, $p(a_i)$, the normalized histogram of variables $b_j$, $p(b_j)$, and the joint normalized histogram of variables $a_ib_j p(a_ib_j)$. The average mutual information in bits is then evaluated by averaging the mutual information $\log_2((p(a_i) \ast p(b_i))/(p(a_ib_j)))$ over all measurements to arrive at $\sum p(a_ib_j) \ast (\log_2((p(a_i) \ast p(b_j))/(p(a_ib_j)))$ for empirical time series; we consider $a_i$ as the original signal and construct $b_j$ as the delay signal $a_i+\tau$. The AMI is thus the function of time $I(t)$, and the first minimum of $I(t)$ is then selected as the time delay to reconstruct the phase space. We assume a three-dimensional attractor, which can be plotted now in three-dimensional space using time delay vectors as

$$
g(t) = \lceil g(t), t(t + \tau), y(t + 2\tau) \rceil.
$$

The assumption made here is that the attractor requires dimension five or less, otherwise, it does not produce any significant output. Most people think that AMI does not do any better than the linear correlation other than taking more CPU time (see [9]). Taken’s theorem guarantees the local invertibility. Thus, we need an embedding mapping that has a Jacobian of rank equal to the dimension of the manifold the attractor resides on. Obviously, if the delay is small, the delay-vector components are almost linearly proportional and that rank condition would not hold. Of course, just because the linear correlation is small does not mean there is no functional relation between delay coordinates. So it would seem that mutual information would show this.

As the only difference between the above definition and the time-dependent transinformation (which is a special case of Kullback information) is the base of the logarithm (which is 10 in the case of time-dependent transinformation),
we derive the value using this transinformation. The assumption that there exists some kind of informational flow between the two systems is not needed; rather there exists a mechanism leading to the predictability of one observable when knowing the other observable.

1.3. Fractional dimension for the attractor. Fractional dimension gives us a good idea of the underlying attractor. It has been shown that the next higher integer of the fractional dimension gives the true value of the dimension of the dynamic system (see [1]). If the process underlying the system is stochastic, then it fills up the entire phase space because whatever dimension we consider the series remains uncorrelated. In other words, we get some dense points if the attractor is chaotic. Grassberger and Procaccia [5] estimated the fractional dimension as the correlation dimension $D$; $D$ measures the density of the attractor finding the probability of one point within a certain distance $R$ from another point. The correlation integral $C_m(R)$ is the number of pairs of points that are within the limit of distance $R$. For a chaotic attractor $C_m$ stabilizes to a certain value as $m$ reaches its true value. Thus, by calculating the slope from the log/log plot for different values of $m$, we can get the correlation dimension. If stabilization does not occur, then we can conclude that the underlying process is stochastic.

The graph shows the correlation integral $C_m(R)$ for different embedding dimensions (minimum 2 and maximum 8 for AMI maximum considered as 3). It is clear that there is no stabilization.

1.4. Checking for stationarity. Most of the time we assume that the time series belongs to an autonomous dynamical system, i.e., the evolution equations do not contain them explicitly. Firstly, we check stationarity by calculating the auto-correlation and partial auto-correlation functions. In the case of auto-correlation and partial auto-correlation functions, both of them should not decay exponentially. Secondly, we do peak-to-peak analysis, which should not appear in the period of oscillation and, thirdly, we construct the recurrence plot. For the time series of a stationary autonomous system the recurrence plot should be more or less homogeneous along the diagonal. If the plot is nonuniform and boxes of dense points appear along the diagonal, then nonstationarity is indicated. After computing all three methods we compare the result with the local Lyapunov exponent to observe the change of system dynamics.

<table>
<thead>
<tr>
<th>Exchange Rate</th>
<th>Peak-To-Peak</th>
<th>Recurrence Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kuwait–GBP</td>
<td>Shows no period of oscillation</td>
<td>Overall plot is homogeneous except first and last few points</td>
</tr>
<tr>
<td>Kuwait–USD</td>
<td>Shows no period of oscillation</td>
<td>Overall plot is homogeneous but there are some points of time when dense points are detected</td>
</tr>
</tbody>
</table>

1.5. Largest Lyapunov exponent. The Lyapunov exponent (LE) measures the loss in predictive power experimented by nonlinear models. A positive exponent measures expansion and a negative one measures contraction. We get different Lyapunov exponent values for each dimension. The most important Lyapunov exponent to consider is the largest one.

Though the largest Lyapunov exponent gives us some idea about the overall system, whether its trajectory diverges or not, normally in a stock market there are different phases that we come across (for, e.g., trend, chaotic, and random). So the main thing to consider here is not only the overall system dynamics but how it changes with time. When we simulate the Lyapunov exponent for different known equations, it seems that for an equation like $y = a + bx$, where the Lyapunov exponent is $\log |b|$, for values $|b| < 1$ the attractor is a point. Thus, for a bull market the Lyapunov exponent
will give a positive value, the meaning of which is its trajectories diverge with time, but we cannot relate that with the price values.

Figures 1–12 show for the different series whether their trajectories diverge with time or not with evolution of time. One hindrance here to find the window size, because the current software restricts us to using a constant window size. So we consider here the minimum window size for the whole series. But, in reality, we should consider a variable window size. One more problem we found here is window overlapping, because in reality the windows can overlap. So the overlapping size is also very important. We are interested here not only in the change of the Lyapunov exponents but in the change of their values in different phases, e.g., from negative to positive or vice versa.

2. Kuwait–GBP Exchange Rate

The time series plot is shown in Fig. 1.

The descriptive statistics of the signal is as follows:

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<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Number of data points</td>
<td>1150</td>
</tr>
<tr>
<td>Sample Distance</td>
<td>1</td>
</tr>
<tr>
<td>Min Value</td>
<td>1.926</td>
</tr>
<tr>
<td>Max Value</td>
<td>2.2268</td>
</tr>
<tr>
<td>Mean</td>
<td>2.04</td>
</tr>
<tr>
<td>Median</td>
<td>2.013</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.08</td>
</tr>
<tr>
<td>Linear Regression (Y’ offset)</td>
<td>2.153853</td>
</tr>
<tr>
<td>Linear Regression (Slope)</td>
<td>−0.0001</td>
</tr>
</tbody>
</table>

As the average mutual information and the global embedding dimension were not used, the optimal time delay and embedding dimension cannot be used. So, a time delay of 1 and an embedding dimension of 3 are used to reconstruct the time series.

Figure 2 shows the correlation integral of the time series for the reconstructed parameters as stated earlier.

The diagram (see Fig. 2) shows an almost linear pattern and a slow convergence. Figure 3 shows the correlation dimension when the dimension reaches the minimum embedding dimension.

The recurrence plot diagram is shown in Fig. 4.

The diagram (see Fig. 4) shows very much random behavior in the data set. The spectral entropy is calculated from the diagram (see Fig. 5), which shows the amount of randomness as 77%.

The largest Lyapunov exponent is found to be 0.09 (for embedding dimension 3 and delay 1); now, for a time window of 100 days, the LE is shown in Fig. 6.

So, the Lyapunov exponent value changes from one phase to another over the time horizon.

3. Kuwait–US Exchange Rate

The time series plot is shown in Fig. 7.

The descriptive statistics of the signal is as follows:
As the average mutual information and the global embedding dimension were not used, the optimal time delay and embedding dimension cannot be used. So, a time delay of 1 and an embedding dimension of 3 are used to reconstruct the time series.

Figure 8 shows the correlation integral of the time series for the reconstructed parameters as stated earlier.

The diagram (see Fig. 8) shows an almost linear pattern and a slow convergence. Figure 9 shows the correlation dimension when the dimension reaches the minimum embedding dimension.

The recurrence plot diagram is shown in Fig. 10.

The diagram (see Fig. 10) shows less random behavior than the GBP in the data set. The spectral entropy is calculated from the diagram (see Fig. 11), which shows the amount of randomness as 64%.

The largest Lyapunov exponent is found to be 0.07 (for embedding dimension 3 and delay 1); now, for a time window of 100 days, the LE is shown in Fig. 12.

All the time the LE is in the same phase, which is chaotic.
Conclusion

From the analysis given above, it is clear that US versus Kuwait exchange rate is more predictable as compared to the Kuwait-GBP exchange rate, which is more random. The above analysis could be helpful to investors as well as to the Government for planning future resources.

REFERENCES

7. Sinha, Tapen, and Tan, Using Artificial Neural Networks, Professional Investor, United Kingdom (1994).

Paacet,
Kuwait