Phase synchronization between solar radiation and wind speed data from some locations across Nigeria via nonlinear recurrence measures

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ABSTRACT

In this paper, we investigate phase synchronization between the solar radiation and wind speed data from different stations across Nigeria, located within latitude 3° and 14°N. The linear correlation and recurrence techniques are used to investigate the linear and nonlinear relationship, respectively. The results of the nonlinear relationship, using Cross recurrence plot (CRP), show that the underlying dynamics of the two sets of data are characterized by nonlinearity, non-stationarity and deterministic chaos. Furthermore, Correlation between probabilities of recurrence (CPR) is used to quantify the degree of synchronization between the two meteorological parameters during dry and wet seasons. High [low] CPR values, which indicate strong [weak] synchronization are obtained for dry [wet] season in both the northern and southern regions. However, for each season the CPR values for northern region are higher than the corresponding values for the southern region. This may be due to strong [weak] coupling between the two meteorological parameters which is attributed to strong [weak] effect of west African monsoon during dry [wet] season.

1. Introduction

Due to growing population and infrastructural development in Nigeria, the energy sector has not been able to meet the increasing demand for electricity. Also, the environment and human health have been deteriorating due to the emission of fossil fuel from electricity generators. Therefore, there is need to embrace alternative energy sources. Currently, the country is endowed with renewable energy resources like biomass, solar, wind and hydropower [1].

The major source of electricity supply in Nigeria is hydropower because the country is blessed with waterfalls, dams and large rivers except for the low water levels during the dry season. The electric power generation from hydropower depends largely on the amount of annual rainfall and its distribution as well as the river systems which is subject to seasonal drought [2]. The extent of availability of water is controlled by seasonality at the different hydropower stations, which might lead to intermittent supply during dry season at low water levels [3].

Hydropower has the utmost potential, which amounts to 10,000 MW for large hydropower and 734 MW for small hydropower. Moreover, renewable energy resources are estimated at 3.5–7.0 kWh/m² for solar radiation, 150,000 terra joule per year of 2.0–4.0 m/s wind speed and 144 million ton per year of biomass [4]. However, these resources are still in nascent stages and are yet

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to be maximally explored.

The combination of a hybrid electric system such as solar electric (photovoltaic or PV) and wind electric technologies provide unique possibility in generating electricity across the country, either as grid-connected or as stand-alone which offer numerous advantages over single electric system.

The hybridization of solar PV and wind energy sources for power supply in particular, is becoming very attractive solution to single electric system. These renewable energy sources offer better reliability, preserve the environment and become more economical to run since the strength of one system can be used to complement the weakness of the other [5].

Weather events have great influence on wind and solar power which may ramp up or down abruptly. These events affect power production thereby causing not only non-availability of energy but also the instability of the entire power grid [6].

However, unpredictable factors such as weather and climatic conditions have been major shortcomings for all renewable energy sources, but the complementary nature of the wind and solar energy sources can reduce non availability of energy associated with each of these energy sources and this concept will help in the development of hybrid solar-wind power plant [7]. Therefore, there is need to understand the degree of synchronization of the dynamics of both solar radiation and wind speed data.

The idea of synchronization was first proposed in 1673 by Christian Huygens when he studied two coupled pendulum clocks, which become synchronized in phase as time $t \to \infty$ [8]. The concept of Synchronization in nonlinear sciences is a universal phenomenon which has been studied extensively during the last three decades [9–11] and also found numerous applications in natural systems like (geology [12], tropospheric radio refractivity and rainfall [13], El Niño-Monsoon [14], multichannel seizure EEG [15], neuronal systems [16–21], magnetoencephalography [22], cardiorespiratory systems [23], ecological systems [24,25], electroencephalographic activity of Parkinsonian patients [22]), and engineering (i.e. design of communication devices [26–28], electronic circuits [29–34], electrochemical oscillators [35–37], plasma [38,39], and lasers [40–45]).

The different types of synchronization states that have been studied over the years, include generalized synchronization [46,47], intermittent lag synchronization [48,49], imperfect phase synchronization [50], complete or identical synchronization [51,52], phase synchronization [9,53], lag synchronization [48], and almost synchronization [54]. For a comprehensive description of different types of synchronization, interested readers are referred to the book by Pikovsky et al. [55] which gives excellent review of the subject.

In natural systems, complete synchronization may be exceptional because systems are generically heterogeneous and coupling is mostly weak and complicated. Due to these effects, a relation may only be established between phases but not in amplitudes of the systems which makes the phase analysis to be completely insensitive to amplitude and may considerably reduce the error from uncorrelated amplitudes, thus detecting successfully a hidden synchronization. [35,40]. Phase synchronization takes place between two signals when their respective frequencies and phases are locked.

In this work, we examine linear and nonlinear relationship between solar radiation and wind speed data from some locations in Nigeria. The linear correlation can detect zero-lag first order phase synchronization but cannot detect higher order and lag synchronization associated with nonlinear correlation [56,57]. Therefore, the method based on nonlinear time series analysis is suitable to characterize degree of phase synchronization during the wet and dry seasons between solar radiation and wind speed using the concept of probability of recurrences in phase space which is based on correlation between probability of recurrence (CPR). This method has advantage over other data analysis methods due to its ability to analyse non-stationary data and rather short time series data [58,59]. The aim of this study is to (1) characterise the underlying dynamics of the combined solar radiation and wind speed data for the purpose of modelling and prediction; (2) investigate the best season and region suitable for optimum power generation from synergy of solar and wind power for the purpose of sustainability of energy supply and economic development for both urban and rural areas across the country from available data.

The rest of this paper is arranged as follows: Section 2 describes the study area and methods, Section 3 deals with the analysis, results and discussion, while conclusive remarks are given in Section 4.

2. Study area and methods of analysis

The study area is Nigeria located in the tropical zone and lies between latitudes 3° and 14° N and longitudes 3° and 15° E as shown in Fig. 1. The solar radiation and wind speed data used were collected from National Space Research and Development Agency (NASRDA) which covered the period of two years for five different stations located at Abuja (9°40.12° N and 7°28′59.88° E), Akungba (6°59′05.40°N and 5°35′52.23°E), Nsukka (6°51′28.14°N and 7°24′28.15°E), Port Harcourt (4°47′05.41°N and 6°59′30.62°E), and Yola (9°17′33.58°N and 12°23′26.69°E).

In this work, the $p$-value and Pearson’s cross correlation coefficient are used to investigate linear correlation between the solar radiation and wind speed data. The threshold value for $p$ called the significance level of the test, traditionally 5% or 1% and the null hypothesis is rejected when $p < 0.05$ and accepted when $p > 0.05$ which measures the level of relationship between the two variables.

Pearson’s cross correlation coefficient between solar radiation and wind speed data is evaluated at 95% confidence interval using the Eq. (1).

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}}$$  \tag{1}

Where $n$ is the sample size, $x_i, y_i$ are the individual sample points indexed with $i$ and $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$ is the sample mean and
The correlation coefficient that determines whether two paired sets of data are correlated was found to lie between 1 and $-1$. The closer to 1 $[-1]$ the more 'confident' we are of a positive [negative] linear correlation. There is no evidence of any correlation when the correlation coefficient is close to zero. Pearson's cross correlation coefficient is arguably the most widely used measure of linear correlation, and also an indicator of phase synchronization since it takes only relative phases between the signals into consideration and not the amplitudes [60].

2.1. Recurrence plot

The dynamics of the single-dimensional series $x_1, x_2, x_3, \ldots, x_{n-1}, x_n$, can be reconstructed in a multi-dimensional phase-space via the use of a time delay $\tau$ according to Takens theorem [61]. The phase space trajectories $\vec{x}_i$ are defined as:

$$\vec{x}_i = [x_i, x_{i+\tau}, x_{i+2\tau}, \ldots, x_{i+(m-1)\tau}], \quad i = 1, 2, \ldots, N$$

(2)

Where $\vec{x}_i$ denotes the $i^{th}$ state, $N = n - (m-1)\tau$ is the number of states, $\tau$ is the time delay and $m$ is the embedding dimension. The time delay $\tau$ and embedding dimension $m$ are essential parameters that are necessary in phase space reconstruction and recurrence analysis. These embedding parameters are quantified by the mutual information function for the time-delay and the false nearest neighbours for the embedding dimension [62].

The recurrent behaviour of dynamical systems in phase space can be visualized by the trajectories of the system's dynamics whose structures can be quantified by a plot called recurrence plot (RP) [63]. In recurrence plot, the main step of the visualization of recurrence in time series data is the calculation of the $N \times N$ matrix whose elements take values 0 or 1 and can be expressed mathematically as follows:

$$R_{ij} = 1(|\vec{x}_i - \vec{x}_j|)$$

(3)
where \( R_0 \) is known as recurrence matrix, \( \varepsilon_i \) is a threshold distance, \( \| \cdot \| \) is a norm (specifically the Euclidean norm), \( \Theta \) is the Heaviside function forcing \( R_0 \) to be either 1 which corresponds to a black dot on the recurrence plot or 0 for white dot, \( \vec{x}_i \) and \( \vec{y}_j \) define the point in phase space at which the system is situated at times \( i \) and \( j \) respectively. A black point in the RP signifies that the system returns to an \( \varepsilon \)-neighbourhood of the corresponding point in phase space [64]. The choice of recurrence threshold \( \varepsilon \) is made not to exceed 10% of the mean or the maximum phase space diameter [65,66]. Euclidean norm is used throughout this study.

2.2. Cross recurrence plot

The cross recurrence plot (CRP) was introduced to analyse the similarity and synchronization between the underlying dynamics of two different systems by comparing their states. It is a bivariate extension of the recurrence plot (RP) [57,67]. Suppose there exist the trajectories \( x_i \) and \( y_j \) in \( d \)-dimensional phase space, each trajectory corresponding to each of the two dynamical systems whose CRP shows all the times when the phase-space trajectory of the first system visits roughly the same state in the phase-space closer to the trajectory of the second system. Therefore, the cross recurrence matrix is defined by

\[
CR_{ij}(\varepsilon) = \Theta(\varepsilon - \|\vec{x}_i - \vec{y}_j\|), \quad i = 1, \ldots, N, \quad j = 1, \ldots, M.
\]  

(4)

The notations are analogous to the definition of RPs but length of the trajectories of \( \vec{x}_i \) and \( \vec{y}_j \) do not need to be equal which implies that the matrix CR is not necessarily square. If the embedding parameters are estimated from both series, but are not equal, the higher embedding should be chosen.

In recurrence analysis, when \( R(i,j) = 1 \) with \( j = i + \tau \) it implies that the trajectory of the dynamical system recurs to the \( \varepsilon \)-neighbourhood of the former state after \( \tau \) time steps. If the probability of occurrence of the first system visiting \( \varepsilon \)-neighbourhood of the former state after \( \tau \) time steps is high, then the probability that the second system recurs after the same time interval will be also high, and vice versa [67]. Therefore, detecting and quantifying the degree of phase synchronization between two systems can be done properly by comparing their \( \hat{p}(\tau) \). The probability \( \hat{p}(\tau) \) of the system returning to a pre-defined state after \( \tau \) time steps can be computed using Eq. (5)

\[
\hat{p}(\tau) = \frac{1}{N - \tau} \sum_{i=1}^{N-\tau} R_{i,i+\tau} = \frac{1}{N - \tau} \sum_{i=1}^{N-\tau} \Theta(\varepsilon - \|\vec{x}_i - \vec{y}_{i+\tau}\|)
\]

(5)

The coincidence of the positions of the maxima of \( p(\tau) \) for any two dynamical systems allows to quantify the degree of phase synchronization via a measure called Correlation coefficient between Probabilities of Recurrence (CPR) [57]. The probabilities of recurrence \( \hat{p}^x(\tau) \) and \( \hat{p}^y(\tau) \) for the first and second systems are calculated and thereafter the correlation coefficient between probabilities of recurrence is computed where these probabilities are normalised to zero mean and standard deviation of one. The correlation between \( \hat{p}^x(\tau) \) and \( \hat{p}^y(\tau) \) are estimated using Eq. (6).

\[
CPR = \langle \hat{p}^x(\tau) \hat{p}^y(\tau) \rangle
\]

(6)

The probability of recurrence will be maximal at the same time when two dynamical systems are in phase synchronization and CPR = 1 while CPR has low values when both systems are not in phase synchronization meaning that the maxima of the probability of recurrence will not occur simultaneously and a drift will be observed.

3. Results and discussion

The embedding parameters for both solar radiation and wind speed data are chosen to be \( \tau = 4 \) and \( m = 8 \) for the time delay and embedding dimension, respectively. The choice of these parameters are based on the concept in [68] in which a single value is picked that gives good characterization across all the stations. The embedding parameters estimated from both time series are not equal in all the stations and in order not to under-embed any of the systems, the higher embedding parameter should be chosen [69].

Fig. 2(a) shows the normalised time series plot of solar radiation and wind speed data while Fig. 2(b) shows its CRP for the dry season period specifically December 2010. Similarly, Fig. 3(a) shows the normalised time series plot of solar radiation and wind speed data while Fig. 3(b) shows its CRP for the wet season period specifically June 2010. We notice that for Akungba, Nsukka, Port Harcourt and Yola similar features are observed for each season (supplementary Figs. S1–S4 for dry season and supplementary Figs. S5–S8 for wet season). The CRPs in Figs. 2b and 3b (as well as supplementary Figs. S1(b)–S4(b) for dry season and Figs. S5(b)–S8(b) for wet season) show increased proportion of short line segments along the main diagonal which are clearly visible in both dry and wet seasons suggesting deterministic structures. It is also evident from the plot that the short lines are interrupted by isolated points (suggesting chaotic behaviour) as well as white horizontal and vertical spaces (suggesting laminar states). There is evidence of synchronization between the two systems with short diagonal lines along and in the direction of the main diagonal which are interrupted by isolated points because of variations in the amplitudes of both systems which was observed and reported by Marwan, et al. [12]. These features show that the underlying dynamics of the two systems is deterministic chaos. It is observed that there are more deterministic structures in the dry season than in the wet season which depict the hourly variation in the natural time series of both processes. The variability of solar radiation and wind speed is ultimately driven by the rotation of the Earth under the Sun, but they exhibit different variability characteristics [70]. The distribution of recurrence points on the CRPs shows similarities between both systems which suggest Synchronization behaviour.

The white horizontal and vertical spaces depict laminar states in the CRPs which suggest intermittency in the dynamics of both
processes. It is observed that the degree of intermittency in dynamics of both processes is higher in the wet season than in the dry season. The variability and intermittency observed in solar radiation and wind speed are attributed to changing West Africa monsoon cloud cover [71] and switching from Harmattan (strong) to monsoon (weak) conditions around the monsoon trough [72], respectively. These effects are less pronounced in the northern part (Yola and Abuja) of the country than the southern part for both dry and

Fig. 2. Normalized time series plot of wind speed and solar radiation data (a) and its Cross Recurrence Plot (b) for dry season for the month of December 2010 for Abuja station. Normalized time series plot shows the monthly variability of the solar radiation and wind speed data. The Short line segments beside single isolated points, in CRPs indicate a positive maximal Lyapunov exponent.

Fig. 3. Normalized time series plot of wind speed and solar radiation data (a) and its Cross Recurrence Plot (b) for wet season for the month of June 2010 for Abuja station. Normalized time series plot shows the monthly variability of the solar radiation and wind speed data. The Short line segments beside single isolated points, in CRPs indicate a positive maximal Lyapunov exponent.
wet seasons, which may be due to the short wet season experienced by the northern region.

The values of phase synchronization index or Correlation coefficient between Probabilities of Recurrence (CPR) vary for all the stations for the period of two years (see Supplemental Table S1). The Correlation coefficient between Probabilities of Recurrence (CPR) ranges from 0.35 to 0.96 for Abuja, 0.39 to 0.97 for Akungba, 0.14 to 0.85 for Nsukka, 0.5 to 0.96 for Port Harcourt and 0.61 to 0.95 for Yola stations. It is observed that the lowest values are recorded at Nsukka, Akungba and Port Harcourt which might be attributed to long wet season that exists between March and October in the region while high values are observed at Abuja and Yola due to short wet season between June and September experienced in the region. Notably, the coupling strength between the two natural time series data varies from region to region which suggests the varying degree of synchronization, as evident from our results (see Supplementary Table S1).

During the dry season, Yola exhibits longer months of high synchronization followed by Abuja, Port Harcourt, Akungba and Nsukka in decreasing order of phase synchronization index. It is clearly evident that CPR values increase considerably during dry season as a result of increased coupling strength between wind speed and solar radiation data during this season.

The probability of recurrence curves for solar radiation and wind speed during dry and wet seasons are shown in Figs. 4 and 5, Supplementary Figs. S9 and S10. It is obvious from Fig. 4 and Supplemental Fig. S9 that the positions of the local maxima of $p(\tau)$ coincide and clearly indicate that the frequencies are locked which implies phase synchronization has occurred. This corresponds to high values of phase synchronization index (CPR) during dry season across the study area. Fig. 5 and Supplemental Fig. S10 show that

![CPR: 0.78](image1)

**Fig. 4.** Probability of Recurrence curves during dry season for the month of December 2010 for Abuja station. The position of the local maxima of $p(\tau)$ coincide and clearly indicate that the frequencies are locked which implies phase synchronization has occurred.

![CPR: 0.76](image2)

**Fig. 5.** Probability of Recurrence curves during wet season for the month of June 2010 for Abuja station. The position of the local maxima of $p(\tau)$ do not coincide perfectly, indicating quasi phase synchronization. The drift of the corresponding $p(\tau)$ shows low values of Phase synchronization index (CPR).
the positions of the local maxima of $p(r)$ do not coincide perfectly, indicating quasi phase synchronization. The drift of the corresponding $p(r)$ shows low values of Phase synchronization index. Phase synchronization index for the wet season is observed to be lower in the Southern part of the country than in the northern part.

The Pearson correlation coefficient ($r$) and statistically significant measure $p$-value are used to investigate linear relationship between the solar radiation and wind speed data whose results are displayed in Supplementary Table S1. The Pearson correlation coefficient ($r$) [$p$-value] obtained range from 0.3115 to 0.8048 [5.87E-154 to 3.32E-18] for Abuja, 0.4775 to 0.8161 [7.37E-179 to 1.44E-39] for Akungba, 0.1350 to 0.6326 [2.07E-84 to 8.80E-03] for Nsukka, 0.3285 to 0.7002 [1.11E-110 to 1.42E-19] for Port Harcourt and 0.5052 to 0.8393 [3.22E-192 to 6.71E-48] for Yola stations indicating positive correlation in all the stations under investigation. It is observed that strong linear correlation exists during dry and wet seasons in all the stations except at Nsukka where we have weak linear correlation. The linear correlation between the two meteorological parameters during the dry season in all the stations is more significant at $p$-values < 0.05 than wet season.

The analysis of the results above, clearly shows that the two renewable energy sources have more potential to complement each other in the dry season than in the wet season. In diurnal time scale, wind reaches its maximum at night while solar radiation reaches its maximum during the day time. In seasonal time scale, wind reaches its maximum in wet season while solar radiation reaches its maximum during dry season [73]. Currently, Nigeria generates her electricity mainly from hydro- and thermal power which cannot meet the demand of the teeming population given that its effective operation is seasonal and requires high maintenance cost.

4. Conclusion

We have investigated the linear and nonlinear relationship between solar radiation and wind speed data from five stations across Nigeria using $p$-values, Pearson’s cross correlation coefficient and recurrence based techniques. The results of the statistically significant measures comprising $p$-values and Pearson’s cross correlation coefficients are consistent but cannot fully reveal the linear relationship between the two meteorological parameters during the dry and wet seasons. The result from the nonlinear relationship using CRP, has shown that the dynamics of the two sets of data are characterized by nonlinearity, nonstationarity and deterministic chaos. There was an increase in the distribution of deterministic structure on CRP in the dry season which implies higher predictability than in the wet season. The effects of intermittency, which is a natural phenomenon associated with solar radiation and wind speed were more pronounced in the wet season than dry season. The degree of synchronization was studied using phase synchronization index (CPR) between the two meteorological parameters during the dry and wet seasons. High [low] CPR values, which indicate strong [weak] synchronization were obtained for dry [wet] season in both the northern and southern regions. However, for each season the CPR values for northern region were found to be higher than the corresponding values for the southern region. This is due to strong [weak] coupling between the two meteorological parameters which is attributed to strong [weak] effect of west African monsoon during dry [wet] season. Therefore, the nonlinear relationship of these meteorological parameters must be considered in the implementation of the renewable energy and energy efficiency programs in order to ensure stable energy supply for economic development in both urban and rural areas across the country.

Declaration of Competing Interest

None.

Acknowledgements

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.cjph.2019.08.015.

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