Potential Pitfalls in Recurrence Plot Analysis

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Recurrence

- fundamental characteristic of many dynamical systems
- recurrences in real life:
  Milankovich cycles, El Niño phenomenon, extreme floods, heart beat after exertion, predator prey cycles, metal cutting processes, etc.
Pitfalls

- Choice of parameters (embedding, recurrence plot calculation, RQA parameters)

- Interpretation:
  - visual patterns
  - indicators of determinism, chaos, periodicities, nonstationarity
  - significance

- Further pitfalls (e.g., dynamical invariants, coupling detection, twin surrogates)
Roessler system

Embedding Parameters

x-variable, $m = 3$, $\tau = 6$

- diagonal lines parallel to LOI
- no perpendicular lines
- continuous lines

Marwan, Int J Bif Chaos 21, 2011
Embedding Parameters

Optimal RP

x-variable, $m = 1, \tau = 1$

Insufficient embedding dimension:
- perpendicular lines

Marwan, Int J Bif Chaos 21, 2011
Embedding Parameters

Optimal RP

\(x\)-variable, \(m = 3, \tau = 12\)

Insufficient time delay:
- interrupted lines
- wobbly lines

→ check appearance of RP

Marwan, Int J Bif Chaos 21, 2011
Recurrence Threshold

- no general rule
  -> choice depends on application
Recurrence Threshold

- dynamical invariants
- threshold as small as possible

Thiel et al, Chaos 14, 2004
Recurrence Threshold

- signal detection from noise
- receiver operator characteristics (ROC) and area under curve (AUC)

range of optimal thresholds $\varepsilon = [0.2 \ldots 0.8] \sigma$

Schinkel et al., European Physical Journal ST 164, 2008
• observational noise

threshold $\epsilon > 5\sigma$

correctly detected recurrence points (%)

correctly detected non-recurrence points (%)
Macro Structures and Sampling

- interference effect of sampling frequency and signal frequency

Nyquist-Shannon sampling theorem not enough!
Macro Structures and Sampling

- RPs of Roessler with different sampling

→ many diagonal lines vanish

\[ \Delta t = 0.05s \]

\[ \Delta t = 1.0s \]
Macro Structures and Sampling

- interference effect of sampling frequency and signal frequency

Facchini & Kantz, Phys Rev E 75, 2007
Marwan, Int J Bif Chaos 21, 2011
Macro Structures and Sampling

- very sensitive to slight frequency variations

⇒ magnification lens to detect tiny frequency modulations

\[
\sin(2\pi f_c t + 2\pi \sin(2\pi f_m t))
\]

\[
\sin(2\pi f_c t + 2\pi \sin(2\pi f_m t)^3)
\]
Macro Structures and Sampling

- large RPs (larger than screen resolution)

be aware of optimal sampling and size of RP
Indicators of Determinism and Chaos

- "close-by" states, divergence behaviour
- Heuristic measure for determinism:

$$DET = \frac{\sum_{l=l_{\text{min}}}^{N} l P(l)}{\sum_{l=1}^{N} l P(l)}$$

not determinism in mathematical sense!
Indicators of Determinism and Chaos

Marwan, Int J Bif Chaos 21, 2011
Indicators of Determinism and Chaos

- embedding: spurious correlations
- many long diagonal lines
- $L_{\text{max}} = 16$

Gaussian white noise, $m = 6$, $\tau = 1$

> alone: not a test for chaos! (apply surrogate test)
> use low embedding dimension!

Marwan, Int J Bif Chaos 21, 2011
Indicators of Periodicities

- high determinism: periodic windows

Logistic map

Trulla et al, Phys Lett A 223, 1996
Marwan et al, Phys Rev E 66, 2002
Indicators of Periodicities

Roessler system

- periodic window not revealed
- continuous system

better measures: $K_2$ entropy or transitivity coefficient

Marwan, Int J Bif Chaos 21, 2011
Zou et al, Chaos 20, 2010
Indicators of Periodicities

- periodic window not revealed
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Roessler system

better measures: $K_2$ entropy or transitivity coefficient

Marwan, Int J Bif Chaos 21, 2011
Zou et al, Chaos 20, 2010
Indicators of Transitions and Nonstationarities

Time series

Recurrence plot

depends on the application and question
Indicators of Transitions and Nonstationarities

- AR(1): stationary process
Indicators of Transitions and Nonstationarities

• RQA measures: nonstationarity?
Indicators of Transitions and Nonstationarities

Marwan et al, EPL 101, 2013
Marwan, Int J Bif Chaos 21, 2011

95% confidence interval

significance test!
Indicators of Transitions and Nonstationarities

- sensitivity on window size
- some RQA measures very sensitive (e.g. TREND)

Marwan, Int J Bif Chaos 21, 2011
Summary

• Careful selection of parameters (embedding, recurrence plot calculation, RQA parameters)

• Careful interpretation:
  • visual patterns
  • indicators of determinism, chaos, periodicities, nonstationarity
  • apply significance test

• Further pitfalls possible!