Complex Network Approach for Recurrence Analysis of Time Series

Norbert Marwan
Jonathan Donges, Reik Donner, Yong Zou, Jürgen Kurths
Outline

• Recurrence plots
• Complex networks
• Complex networks from time series
• Applications
Recurrence

• fundamental characteristic of many dynamical systems

• recurrences in real life:

Milankovich cycles, weather after storm, El Niño phenomenon, heart beat after exertion, Maya calendar etc.
Recurrence

• Anaxagoras, approx. 450 BC:
  perichoresis: chaotic circular movement
Recurrence

- Poincaré, 1890:

  “a system recurs infinitely many times as close as one wishes to its initial state”
Investigating Recurrence

- Poincaré map
- Recurrence time statistics
- First return map
- Recurrence plot
Investigating Recurrence

• Poincaré map
• Recurrence time statistics
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• Recurrence plot

• **Recurrence network**
Recurrence Plots
Recurrence Plot
Recurrence Plot

Euclidean Norm
Recurrence Plot

- to visualise the phase space trajectory by its recurrences

\[ R_{i,j} = \begin{cases} 
1 : \vec{x}_i \approx \vec{x}_j \\
0 : \vec{x}_i \not\approx \vec{x}_j 
\end{cases} \quad i, j = 1, \ldots, N \]

- formal

\[ R_{i,j}(\varepsilon) = \Theta \left( \varepsilon - \| \vec{x}_i - \vec{x}_j \| \right), \quad i, j = 1, \ldots, N \]

N. Marwan et al., Physics Reports, 438, 2007
Recurrence Plot

- to visualise the phase space trajectory by its recurrences

- recurrence matrix:
  - binary
  - symmetric

\[
R_{i,j} = \begin{array}{ccccccc}
1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 \\
\end{array}
\]

N. Marwan et al., Physics Reports, 438, 2007
Recurrence Plot Typology

- homogeneous
- periodic
- drifty
- disrupted
Recurrence Plot Quantification

- Line structures related to dynamical properties
- Measures of complexity: quantify line length distribution (recurrence quantification analysis)

Recurrence Plot

- Transition detection
- Differentiate dynamics
- Finding time scales
- Interrelation detection
- Synchronisation analysis
- Surrogates
- Recurrence time statistics
- etc.

N. Marwan et al., Physics Reports, 438, 2007
Complex Networks
Complex Networks

• link matrix (undirected, unweighted network):
  ‣ binary
  ‣ symmetric

\[ A_{i,j} = \]

\[
\begin{array}{cccccc}
0 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 \\
\end{array}
\]
Complex Networks

- link matrix (undirected, unweighted network):
  - binary
  - symmetric

\[ A_{i,j} = \]

- link matrix: similar to recurrence plot
Time Series Analysis using Complex Networks

- Link matrix = recurrence matrix of time series
- Nodes: states in phase space
- Links: local neighbours of states (i.e. recurrence)
- Path: connected neighbourhoods

Time Series Analysis using Complex Networks

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Time Series Analysis using Complex Networks

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Time Series Analysis using Complex Networks

- Complex network measures applied to recurrence plot
  - measures of complexity explaining dynamical properties complex systems
- „recurrence network“

Clustering Coefficient
Clustering Coefficient

- probability that neighbours of a node are also connected
Clustering Coefficient

\[ C_v = \frac{\sum_{i,j} A_{v,i} A_{i,j} A_{j,v}}{k_v (k_v - 1)} \]

- probability that neighbours of a node are also connected
Clustering Coefficient in Phase Space

Regular/periodic

Diverging/chaotic

- clustering coefficient: regularity of dynamics
Example: Logistic Map

- Logistic map:

\[ x_{i+1} = ax_i(1 - x_i) \]
Example: Logistic Map

- Periodic windows

Control parameter $a$
Example: Logistic Map

Control parameter $a$

Clustering coefficient

Intermittent phases
Time Series Analysis using Complex Networks

- Degree centrality (recurrence probability)
- Clustering coefficient (regularity)
- Betweenness centrality (attractor fractionation)
- Average shortest path length (mean phase space separation)
- Matching index ("twinness" of states)
- etc.

Donner et al., New J. Phys 12, 2010
Applications
Asian Monsoon
Asian Monsoon
Asian Monsoon

$\delta^{18}O$

Age (yr BP)

-7  -6  -5  -4  0  1000  2000  3000  4000  5000  6000  7000  8000  9000  10000  11000
3900-3700 yr BP
Harappan culture vanished
Early Detection of Preeclampsia in Pregnancy

- Life-threatening cramps for mother and fetus
- Under-supply of the fetus
- Growth retardation

- Positive predictive value appr. 20-30%
Early Detection of Preeclampsia in Pregnancy

- 20th week of gestation
- Systolic and diastolic blood pressure (S, D)
- Heart rate variability (H)

Walther et al., J Hypertens 24, 2006
Malberg et al., Chaos 17, 2007
Early Detection of Preeclampsia in Pregnancy

\[ \vec{x} = \begin{pmatrix} H \\ D \\ S \end{pmatrix} \]
Early Detection of Preeclampsia in Pregnancy
Early Detection of Preeclampsia in Pregnancy

<table>
<thead>
<tr>
<th></th>
<th>Preeclampsia</th>
<th>Control</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (ms)</td>
<td>734.5 (±110.8)</td>
<td>760.5 (±111.7)</td>
<td>n.s.</td>
</tr>
<tr>
<td>S (mmHg)</td>
<td>123.0 (±15.4)</td>
<td>123.5 (±20.0)</td>
<td>n.s.</td>
</tr>
<tr>
<td>D (mmHg)</td>
<td>75.5 (±10.4)</td>
<td>66.6 (±13.9)</td>
<td>n.s.</td>
</tr>
<tr>
<td>recurrence rate</td>
<td>0.14 (±0.04)</td>
<td>0.16 (±0.05)</td>
<td>0.0024</td>
</tr>
<tr>
<td>laminarity</td>
<td>0.80 (±0.10)</td>
<td>0.83 (±0.08)</td>
<td>n.s.</td>
</tr>
<tr>
<td>clustering</td>
<td>0.60 (±0.03)</td>
<td>0.62 (±0.04)</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

positive accuracy value: 60%  negative accuracy value: 80%
Summary

• Complex networks from time series
• Recurrence analysis using complex network statistics
• Complementary analysis to traditional recurrence measures
Publications


