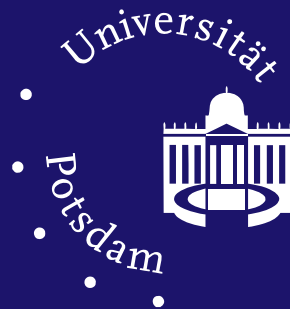


# 3D Measures of Complexity for the Assessment of Complex Trabecular Bone Structures

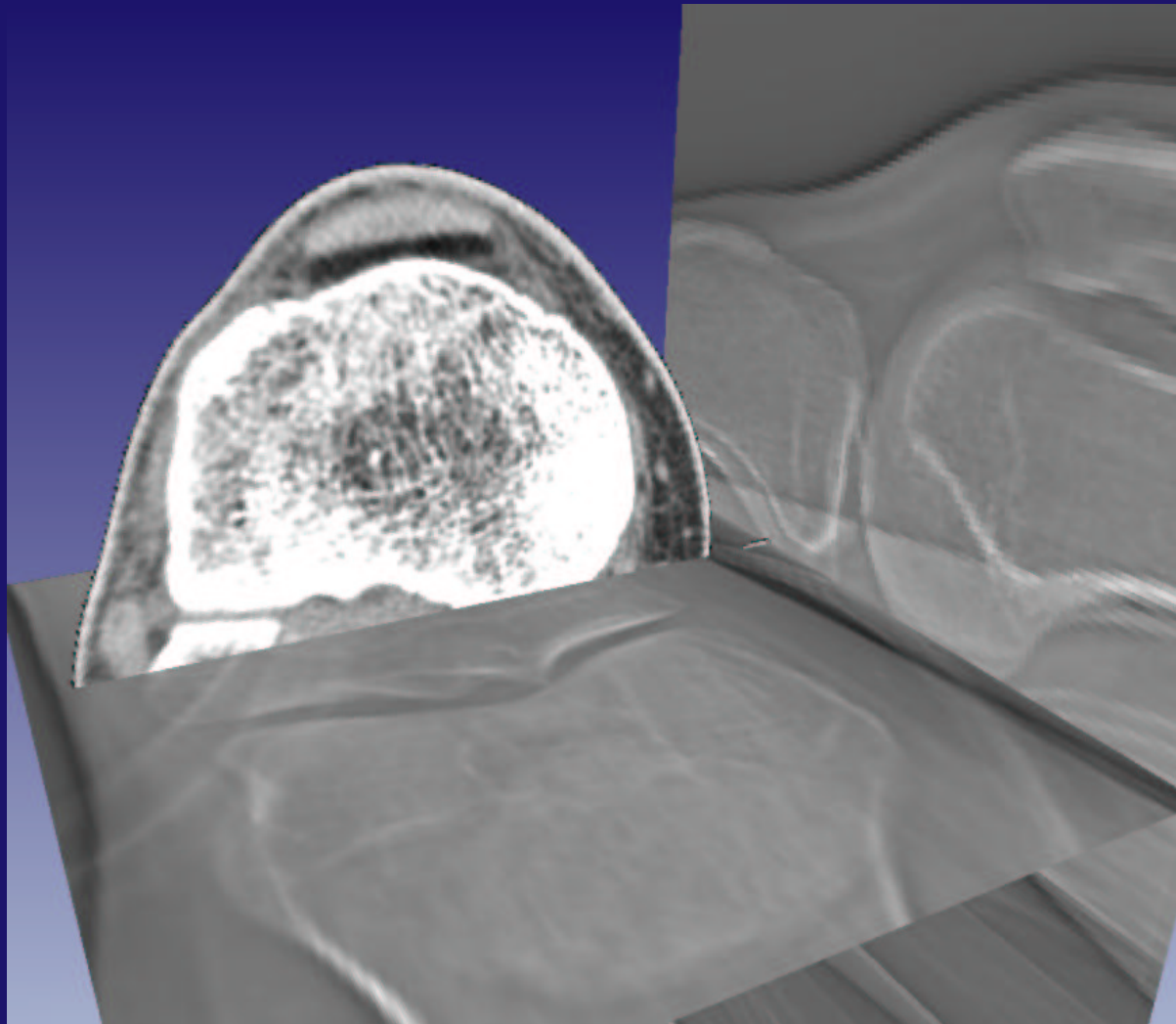
Norbert Marwan



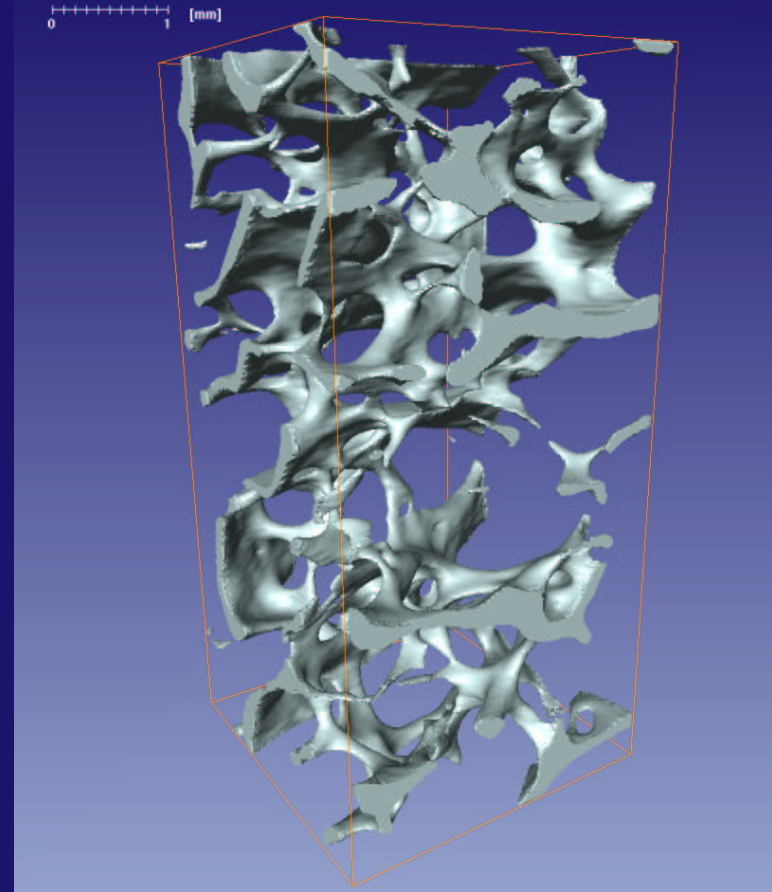
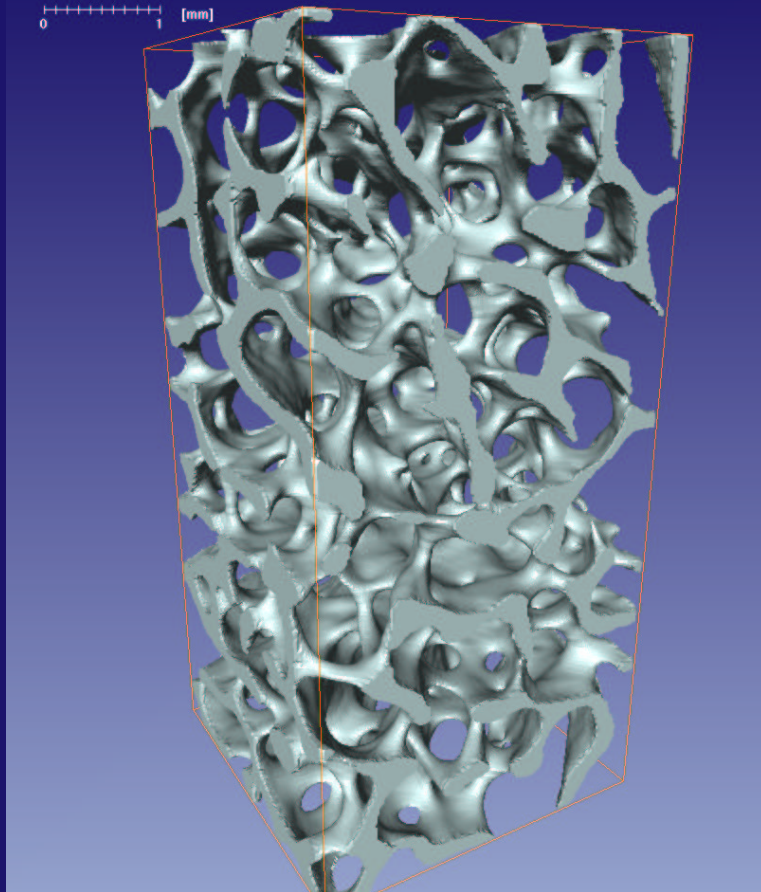
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## Trabecular Bone (Proximal Tibia)



## Effects of Osteoporosis and Microgravity



Healthy (left) and osteoporotic (right).

**How to describe the changes in the complex structure of trabecular bone?**

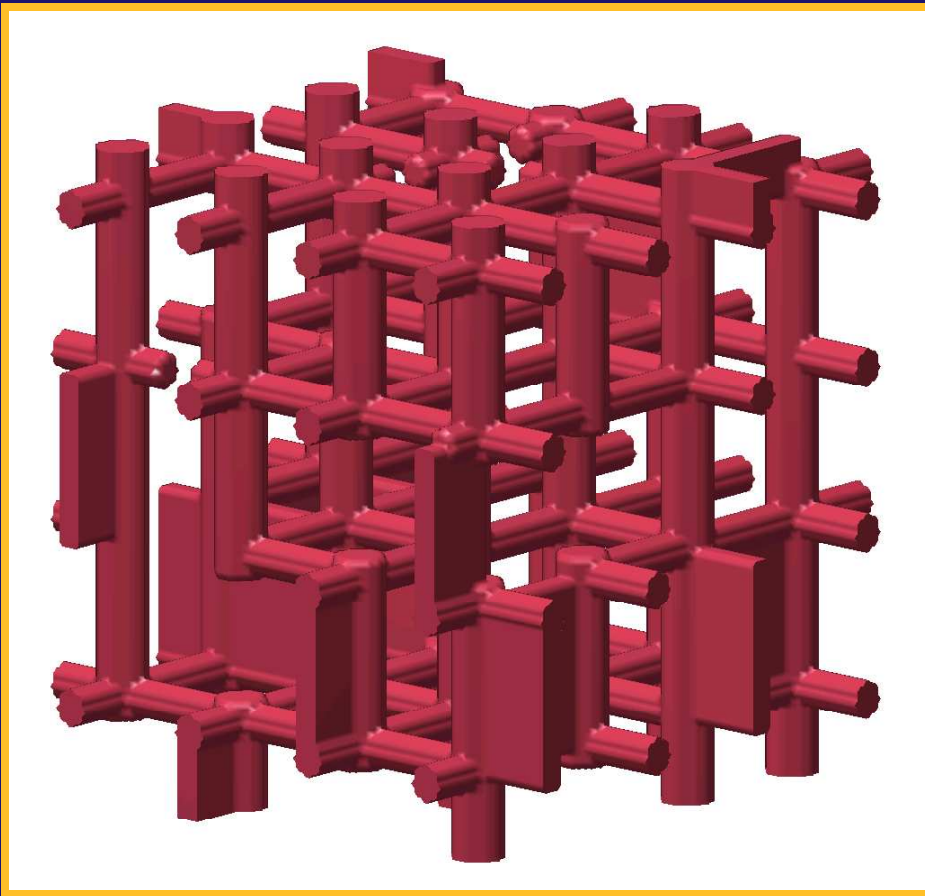
## How to describe the changes in the complex structure of trabecular bone?

- statistical evaluation of local correlations and changes in the bone shape  
→ **Measures of Complexity:**
  1. Lacunarity
  2. Moran's/ Geary's Index
  3. Shape Index

## How to describe the changes in the complex structure of trabecular bone?

- statistical evaluation of local correlations and changes in the bone shape  
→ **Measures of Complexity:**
  1. Lacunarity
  2. Moran's/ Geary's Index
  3. Shape Index
  
- Problems: 3D, large data sets

## Bone Model for Verification



Age related trabecular dimensions (in  $\mu\text{m}$ ):  
L – distance between nodes; R – radius of rods (Jensen, 1990).

| Age | $L_v$ | $L_h$ | $R_v$ | $R_h$ |
|-----|-------|-------|-------|-------|
| 40  | 770   | 720   | 105   | 75    |
| 60  | 1110  | 870   | 105   | 62    |
| 80  | 1450  | 990   | 105   | 47    |

→ Simulation of removed rods (**gaps**), broken rods (**terminii**) and **plates**

# Moran's Index

Spatial auto-correlation test of 2D images (Chuang and Huang, 1992)

$$I = \frac{N \sum_{j=1}^{d_1 \times d_2} \sum_{i=1}^{d_1 \times d_2} \delta_{ij} (x_i - \bar{x}) (x_j - \bar{x})}{S_0 \sum_{i=1}^{d_1 \times d_2} (x_i - \bar{x})^2},$$

$d_1$  and  $d_2$  – geometric size of the image,  $x_i$  – value at the specified position,  $\bar{x}$  – mean of the image,  $\delta_{ij} = 1$  if pixel  $i$  and  $j$  are adjacent (else 0),  $N = d_1 d_2$  – total number of pixels and  $S_0 = \sum \sum \delta_{ij}$  – total number of contiguous pairs ( $S_0 = 4d_1 d_2 - 3(d_1 + d_2) + 2$ ).

→  $I \in [-1, 1]$

→  $-1$  – high correlated,  $0$  – uncorrelated,  $+1$  – high anti-correlated



## Geary's Index

Spatial auto-correlation test – similar to the Moran's Index

$$C = \frac{(N - 1) \sum_{j=1}^{d_1 \times d_2} \sum_{i=1}^{d_1 \times d_2} \delta_{ij} (x_i - x_j)^2}{2 S_0 \sum_{i=1}^{d_1 \times d_2} (x_i - \bar{x})^2}$$

→  $C \in [0, 2]$  (inversely related to the Moran's index)

→ 0 – anti-correlated, 1 – uncorrelated, 2 – correlated

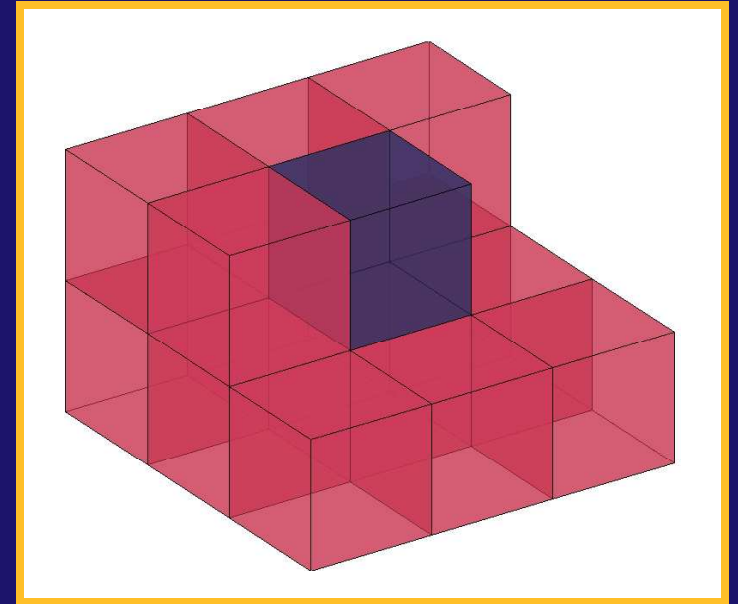
## 3D Extension

Extension to 3D , e. g. Moran's Index:

$$I = \frac{N \sum_{j=1}^{d_1 \times d_2 \times d_3} \sum_{i=1}^{d_1 \times d_2 \times d_3} \delta_{ij} (x_i - \bar{x}) (x_j - \bar{x})}{S_0 \sum_{i=1}^{d_1 \times d_2 \times d_3} (x_i - \bar{x})^2}$$

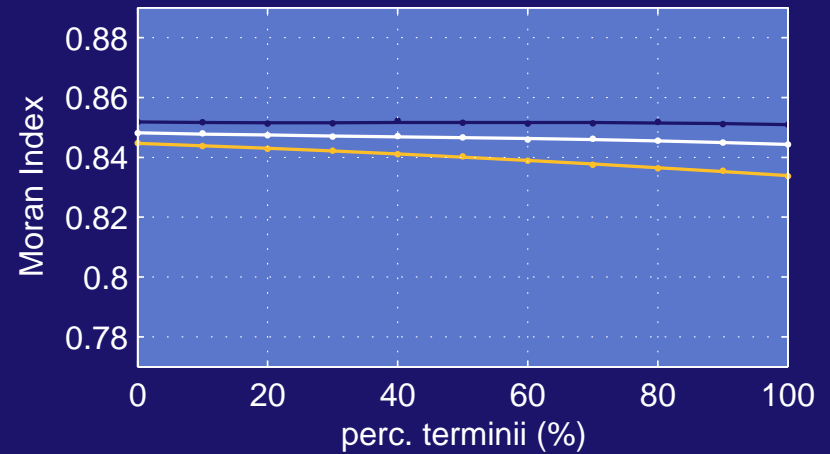
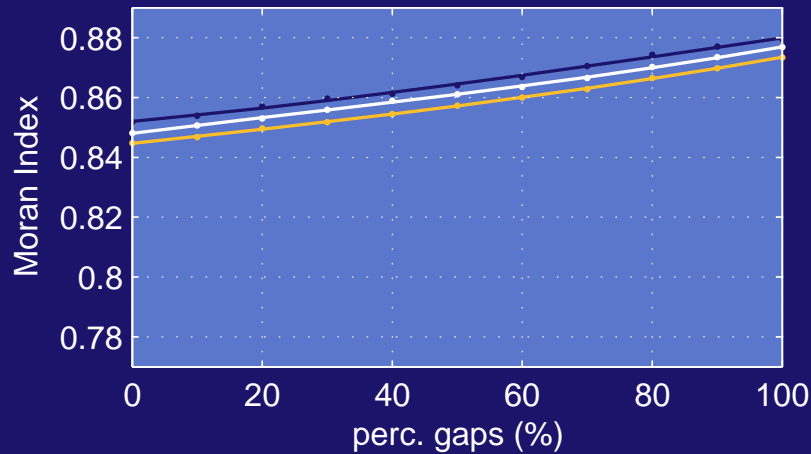
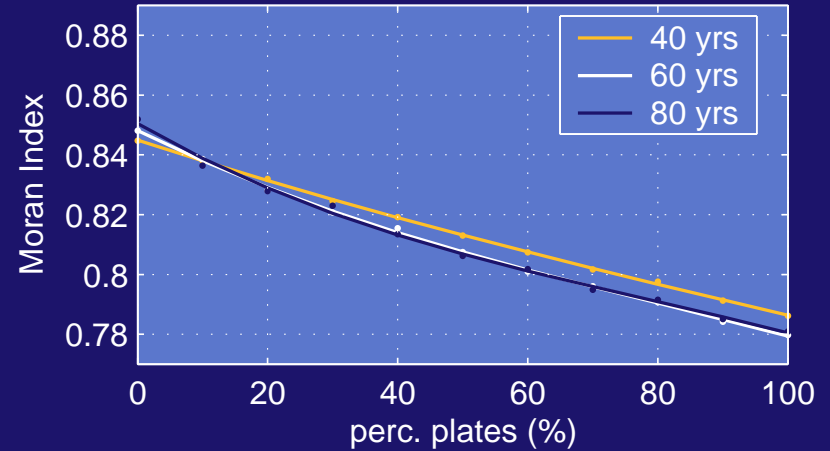
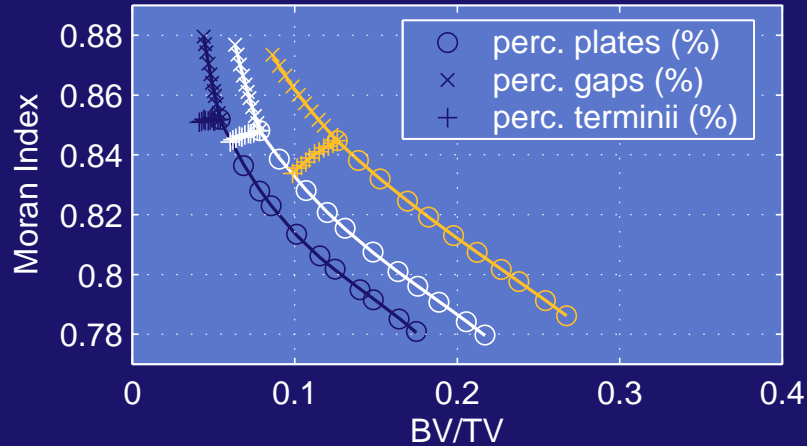
Number of contiguous neighbours  $S_0$  is now

$$S_0 = 13 d_1 d_2 d_3 - 9 (d_1 d_2 + d_2 d_3 + d_3 d_1) + 6 (d_1 + d_2 + d_3) - 4$$



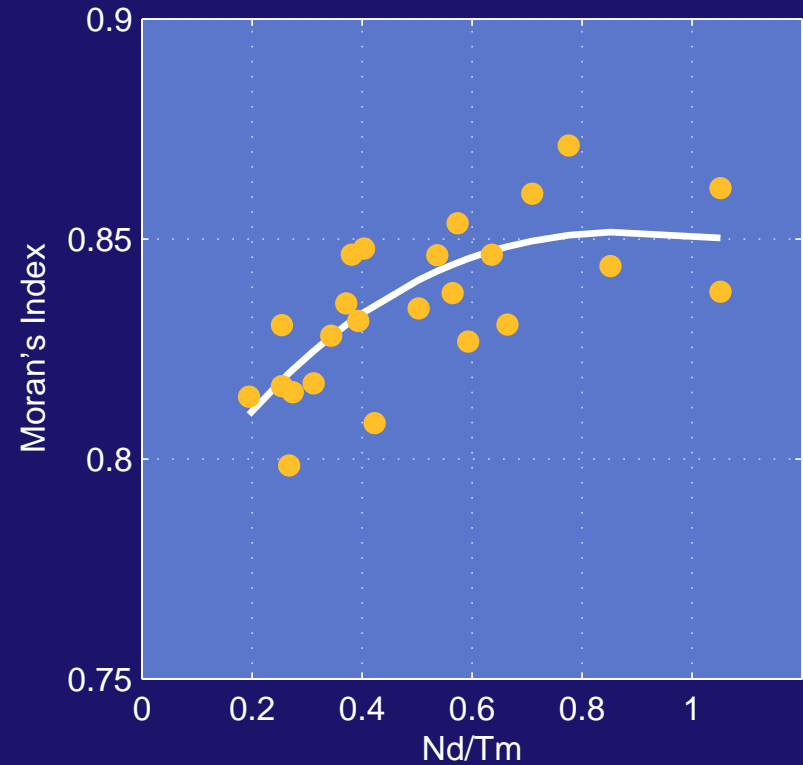
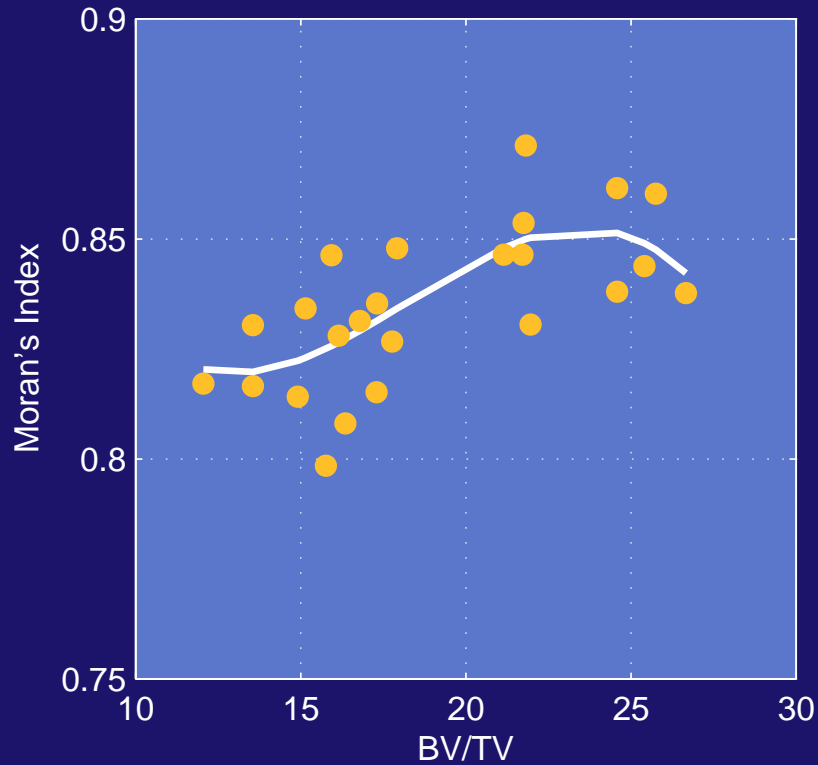
Contiguous neighbours (red) of a voxel (blue) in 3D in order to analyse contiguous neighbours only once.

# Moran's Index Applied to Bone Models



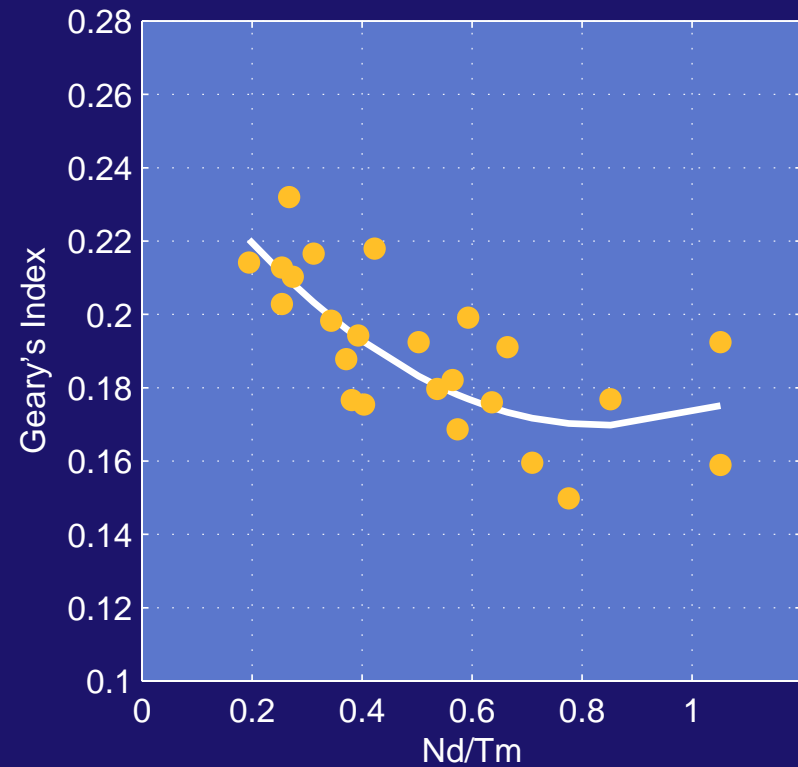
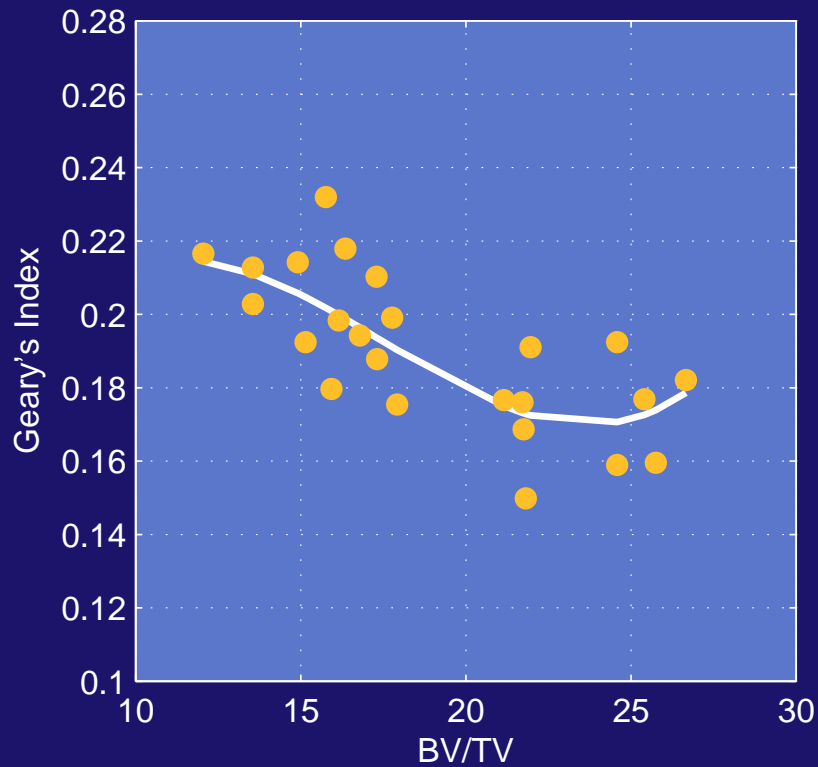
Strong dependence on percentage of plates and gaps.

# Moran's Index of Proximal Tibia



→ **Spatial correlation** is related with connectivity

# Geary's Index Applied to Proximal Tibia Biopsies

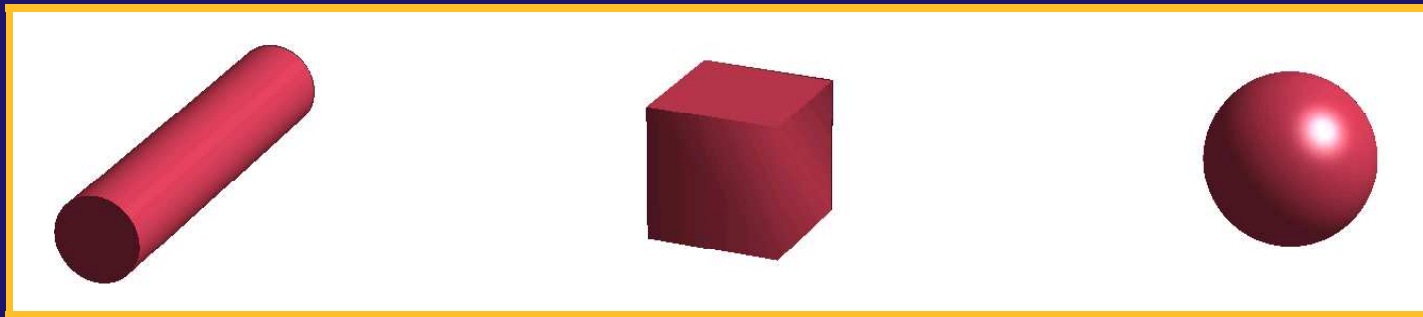


→ **Spatial correlation** is related with connectivity

# Shape Index

Quantification of the **geometrical shape**

$$SHI = \frac{S_{bone}}{S_{sphere}} = \frac{S_{bone}}{\sqrt[3]{36\pi V_{bone}^2}} \quad S - \text{surface}, \quad V_{bone} - \text{volume}$$

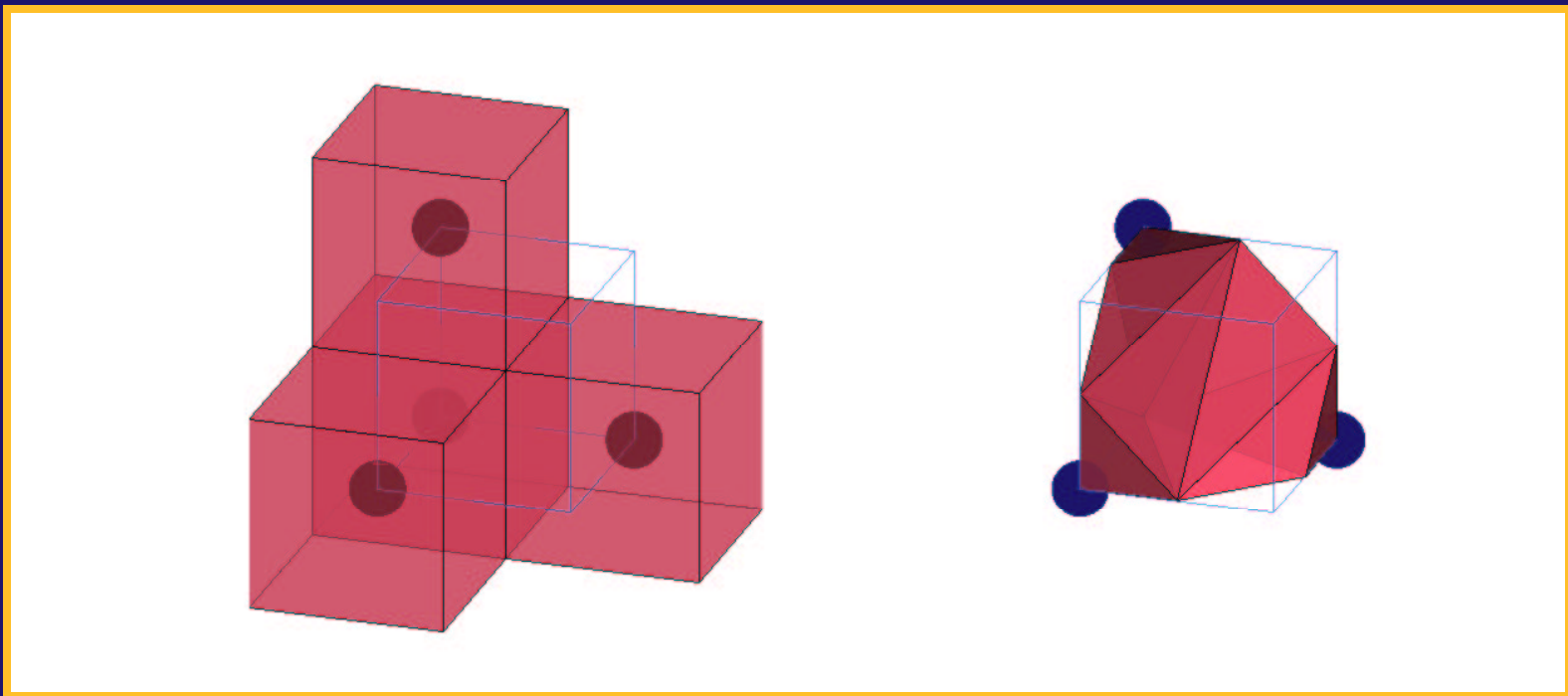


Cylinder, cube and sphere of same volume ( $V = 1000$ ) have different surface ( $S_{cylinder} = 694$ ,  $S_{cube} = 600$ ,  $S_{sphere} = 484$ ).

→ **Statistics** of the distribution of SHI (e. g. entropy)

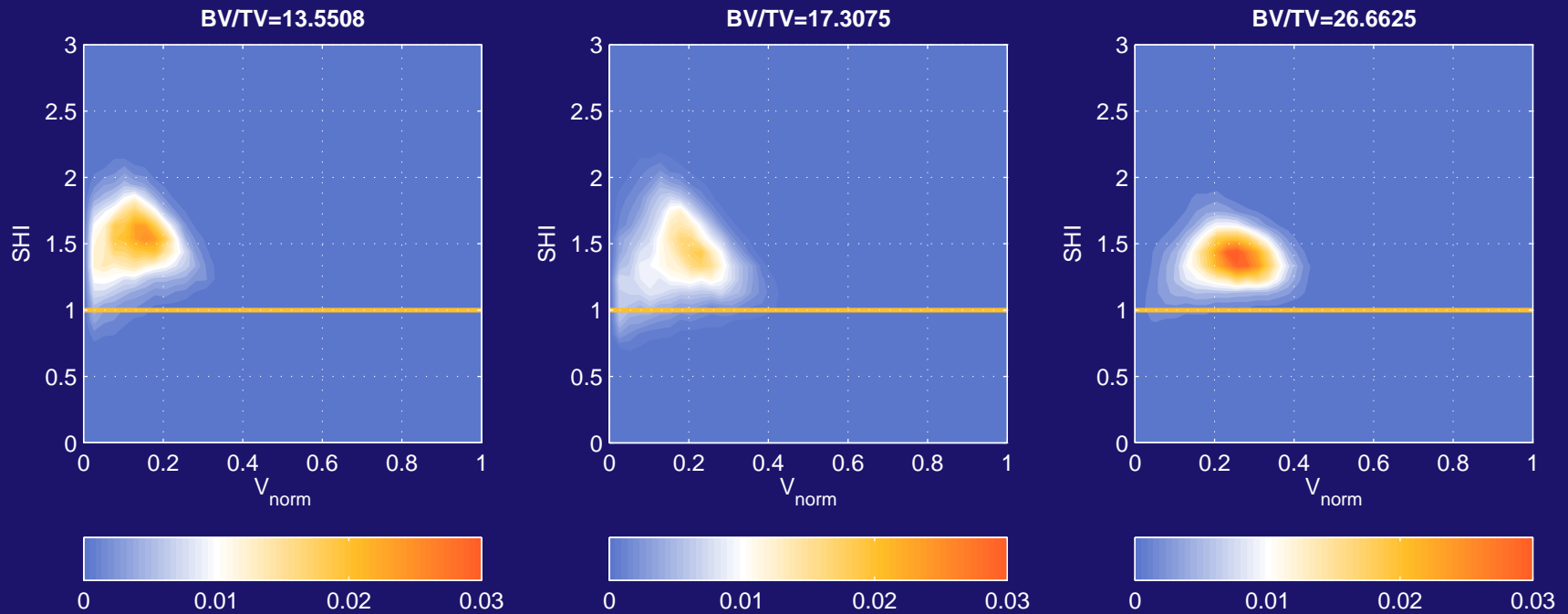
# Shape Index

Volume and surface estimation: **iso-surface algorithm**



Voxel-based estimation (left) and iso-surface algorithm (right).

# Shape Index of Proximal Tibia

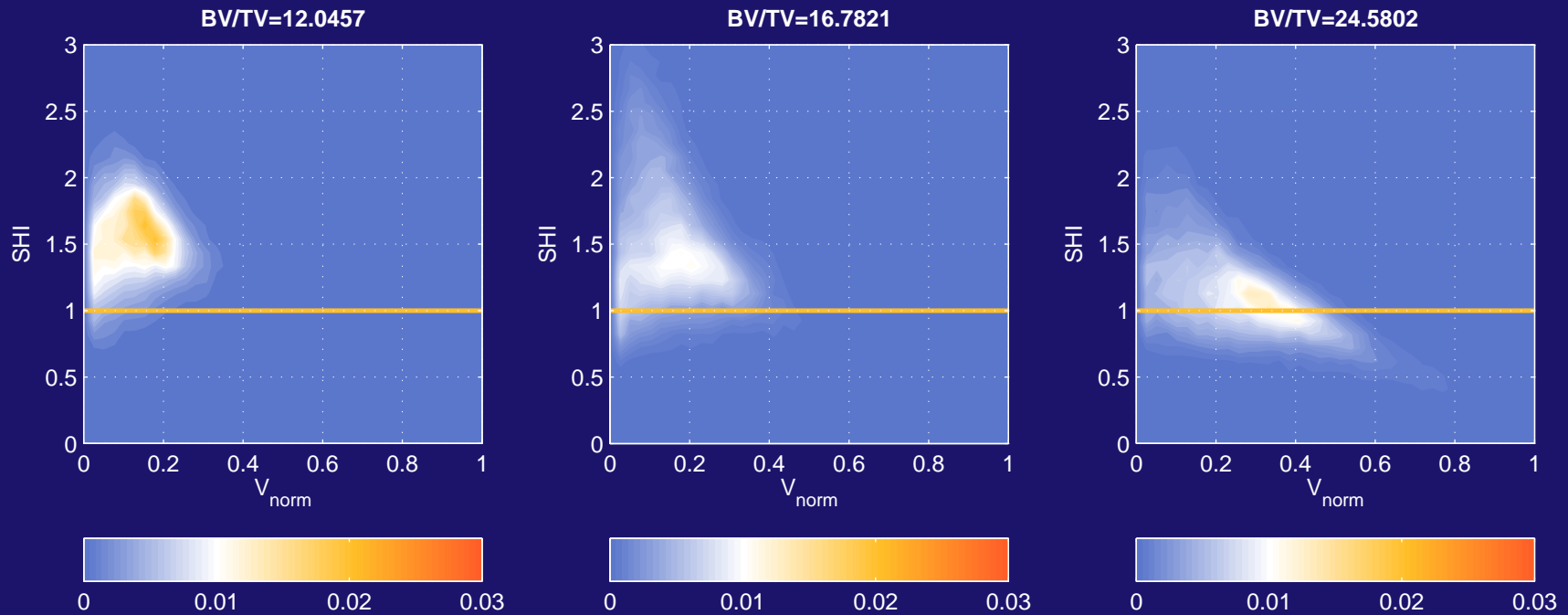


Distributions of SHI and normalized volumes.

→ **Distribution** moves for changes in bone



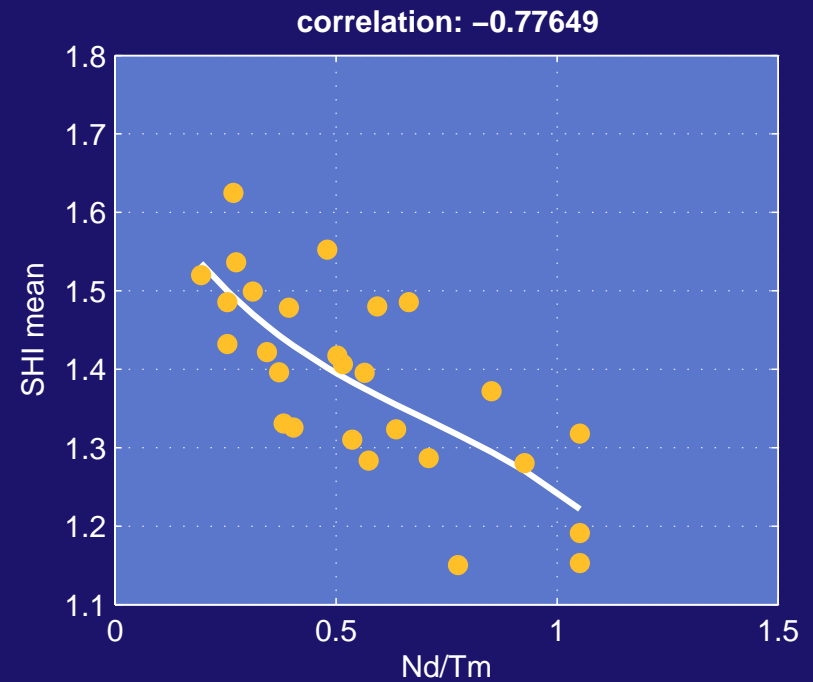
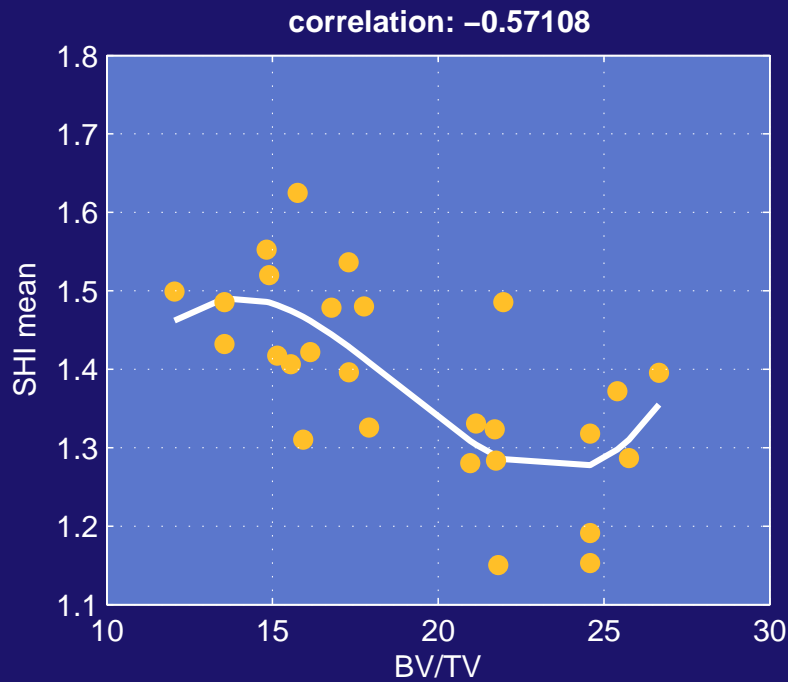
# Shape Index of Proximal Tibia



Distributions of SHI and normalized volumes.

→ **Intermediate** stages have spreaded distributions

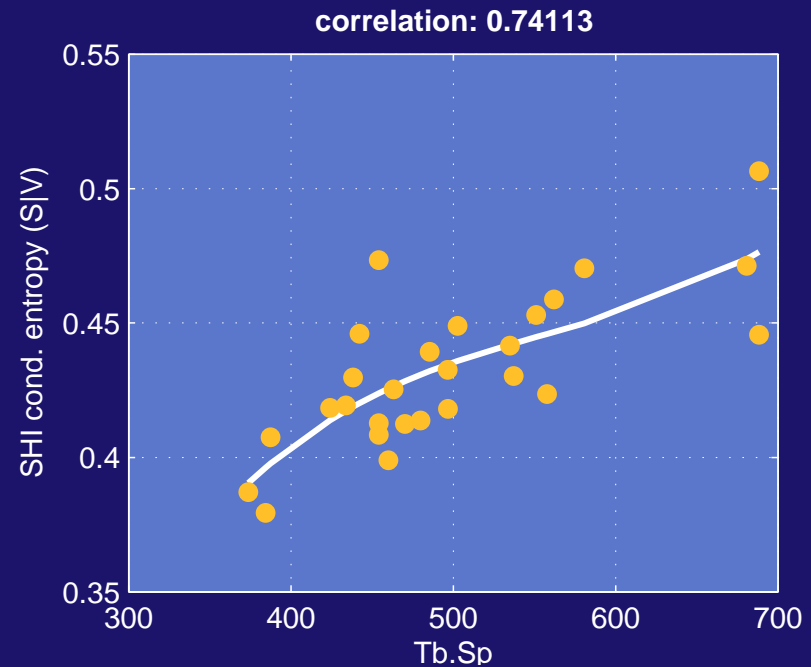
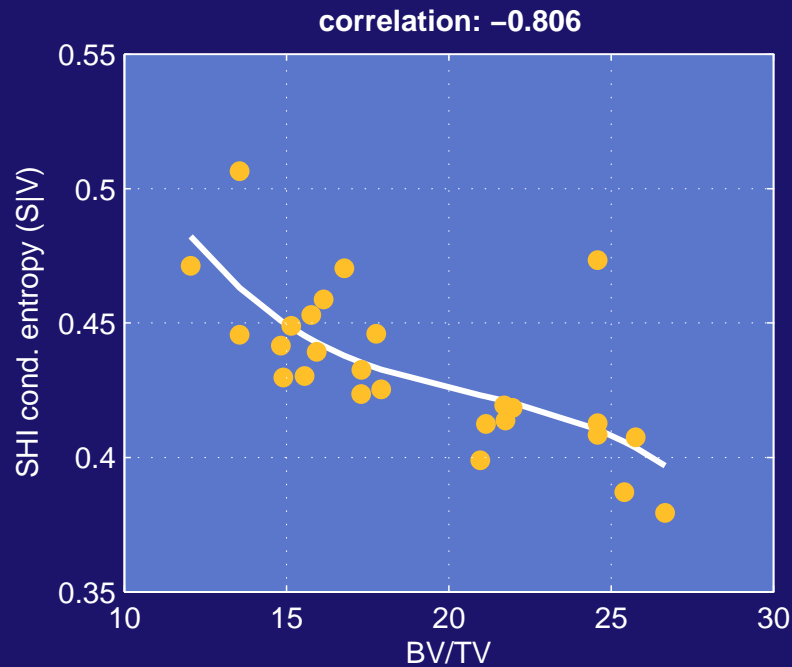
# Mean Shape Index of Proximal Tibia



Averaged SHI over BV/TV and histomorphometric structure measure.

→ SHI average correlates with **connectivity**

# Entropy of Shape Index of Proximal Tibia



Conditional entropy over BV/TV and histomorphometric structure measure.

→ Cond. entropy related with **spacial distribution** of parallel structures

## Conclusions

- 2D-measures successful adopted to 3D (Lacunarity, Moran's and Geary's Index)
  - Lacunarity related with connectivity density
  - Moran's and Geary's Index related with network connectivity

## Conclusions

- 2D-measures successful adopted to 3D (Lacunarity, Moran's and Geary's Index)
  - Lacunarity related with connectivity density
  - Moran's and Geary's Index related with network connectivity
- new 3D measure
  - Shape Index related with several geometrical properties (connectivity, parallel plate model, star volume)