Geometric Structures in Recurrence Plots

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Recurrence Plots

The method of recurrence plots (RP) was first introduced to visualize time dependent behavior of the dynamics of systems $\boldsymbol{x}(t)$, which can be pictured as a trajectory in the phase space (Eckmann 1987). It represents the recurrence of the phase space trajectory to a certain state. The main step of this visualization is the calculation of the $N \times N$-matrix

$$\boldsymbol{R}(t,\tilde{t}) = \Theta(\varepsilon - \|\boldsymbol{x}(t) - \boldsymbol{x}(\tilde{t})\|)$$

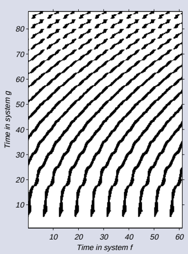
where ε is a cut-off distance and $\Theta(\cdot)$ is the Heaviside function.

The recurrence plot exhibits characteristic large-scale and small-scale patterns which are caused by typical dynamical behavior (Marwan 2003), e. g. diagonal lines (similar local evolution of different parts of the trajectory) or horizontal and vertical black lines (state does not change for some time).

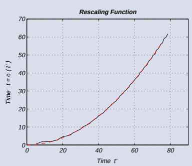
A recent developement introduces cross recurrence plots (CRP), which can be used in order to investigate potential time transformations in two time series (Marwan 2002). Such an analysis can be applied, e. g. for the adjustment of time scales of borehole data or in dendrochronology.

CRPs as a tool for time scale adjustment:

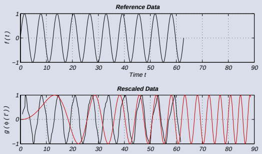
$f(t)=\sin(\varphi\, t)$ and $g(t)=\sin(\psi\, t^2)$.



The CRP for two sine functions, which is the base for the determination of the rescaling function between both data series ($m = 2$, $\tau = 1$, $\varepsilon = 0.2$). The differences in the time domain cause a distorted main diagonal in the form of the parabolic function $\phi \sim t^2$.



The rescaling function (black) determined from the CRP shows the expected parabolic shape. In red the square function.



Reference data series f (upper panel) and second data series g before (red) and after (black) time-adjustment achieved by using the rescaling function (lower panel).

Line Structures in Recurrence Plots

In a more general sense the line structures in an RP exhibit locally the time relationship between the current trajectory segments. A line structure in an RP of length l corresponds to the closeness of the segment $\boldsymbol{x}(T_1(t))$ to another segment $\boldsymbol{x}(T_2(t))$, where $T_1(t)$ and $T_2(t)$ are the local time scales (or transformations of an imaginary absolute time scale t) which preserve that $\boldsymbol{x}(T_1(t)) \approx \boldsymbol{x}(T_2(t))$ for some time $t = 1 \ldots l$.

Under some assumptions (e. g. piecewise existence of an inverse of the transformation $T(t)$) the local slope $b(t)$ of a line in an RP represents the local time derivative of the inverse second time scale $T_2^{-1}(t)$ applied to the first time scale $T_1(t)$

$$b = \partial_t T_2^{-1}(T_1(t))$$

This relationship applies also to cross recurrence plots and enables them to adjust time scales of two different measurements or time series under different time transformations (different sampling times, varying compressions etc).

References

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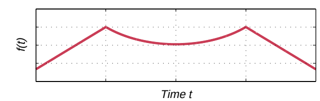
A



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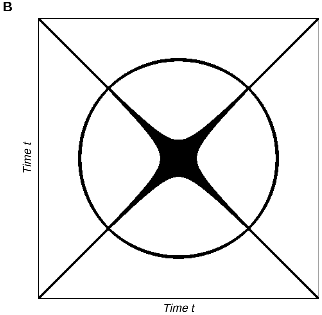
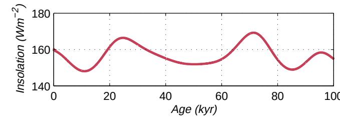


A one-dimensional system (A) represented by a function $f(T)=T(t)$, with a section of a monotonical, linear increase $T_{lin}=t$ and another (hyperbolic) section which follows $T_{hyp} = -(r^2 - t^2)^{1/2}$. We assume that both sections visit the same area in the phase space. Using the inverse of the hyperbolic section, we find the slope

$$b = \frac{t}{\sqrt{r^2 - t^2}}$$

which corresponds with the derivative of a circle line with a radius r. Therefore, a bowed line structure with the form of a circle occurs in the RP (B).

A



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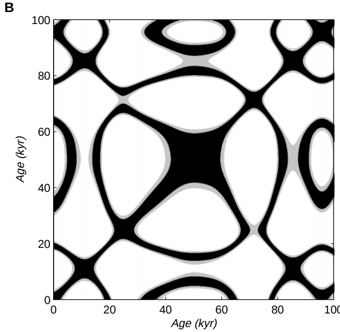

Examplary data from nature (A) reveals that such structures are not only restricted to artificial models. Here the January insolation for the last 100 kyr on the latitude 44° N (Berger 1991) is presented. The corresponding RP (B) shows an emphasized circle. From this geometric structure we can infer that in this epoch the insolation data contains a more-or-less symmetric sequence of a linear increasing, and a hyperbolic decreasing, followed by a reverse of this sequence, a hyperbolic increasing and a linear decreasing part.

Make your own recurrence plots!
http://tocsy.agnld.uni-potsdam.de (*CRP Toolbox*)
http://www.recurrence-plot.tk (*programmes*)