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Network approach for unveiling subtle transitions in dynamical systems

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Abstract : An analogy between the recurrence matrix and the adjacency matrix of an undirected and unweighted complex network is exploited. Some measures of complexity usually used for the characterization of complex networks are then computed. The potential of these recurrence-based network measures in unveiling subtle transitions, such as quasiperiodic - strange nonchaotic - chaotic dynamics, is illustrated.

Approach

- The recurrence matrix R of a dynamical system contains the values 1 or 0 depending whether the phase space vectors $\vec{x}_i$ (i=1,...,N) are close in the phase space (in this case there are recurrences) or not.
- An approach to a network is built assuming the phase space vectors as nodes, the recurrences as links and the path as connected neighborhoods.
- The binary adjacency matrix A, in which a connection between node i and j is marked as $A_{i,j}$=1, is equal to the recurrence matrix from which the identity matrix is subtracted.

Network Measures

- **Link Density (ρ)**

$$\rho = \frac{1}{N(N-1)} \sum_{i,j=1}^{N} A_{i,j}$$

- **Average Path Length (L)**

$$L = \frac{1}{N(N-1)} \sum_{i,j=1}^{N} d_{i,j}$$

$$d_{i,j} = \|\vec{x}_i - \vec{x}_j\|$$

- **Clustering Coefficient (Cl)**

$$Cl = \sum_{v} C_v / N$$

$$C_v = \frac{\sum_{i,j=1}^{N} A_{v,i} A_{i,j} A_{j,v}}{k_v (k_v - 1)}$$

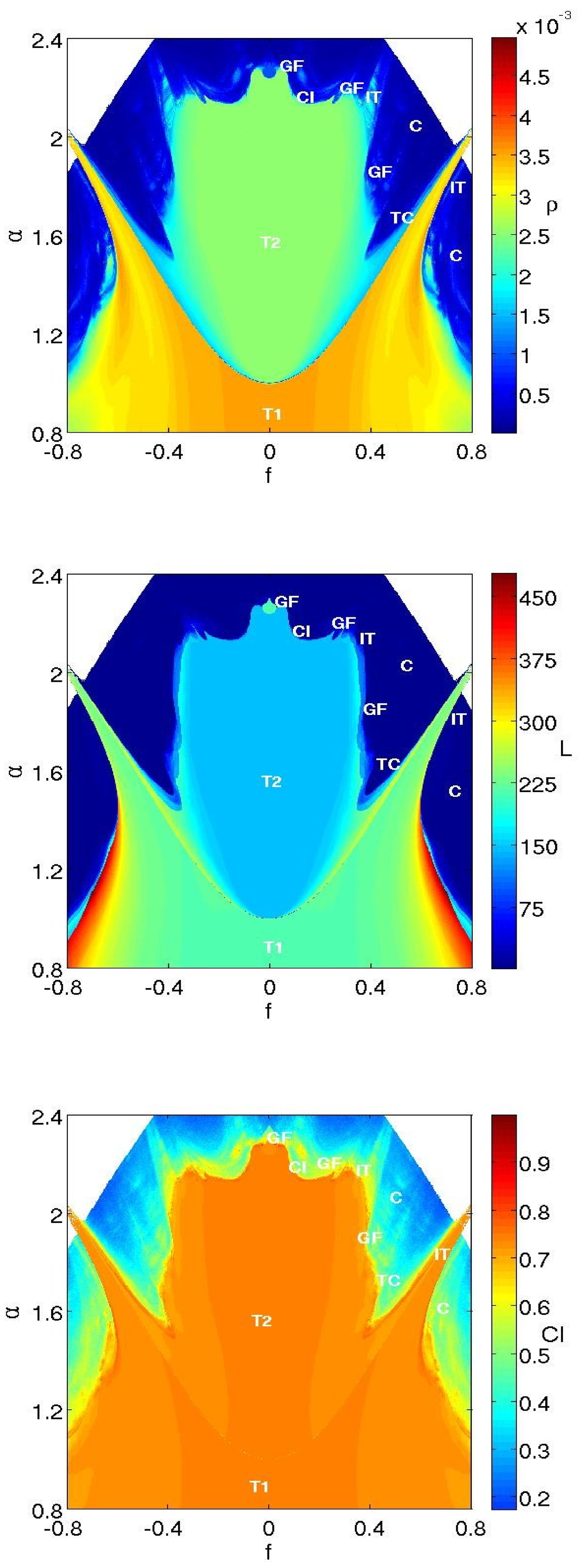
$$k_v = \sum_{i=1}^{N} A_{v,i}$$

Application and Results

System: Cubic Map

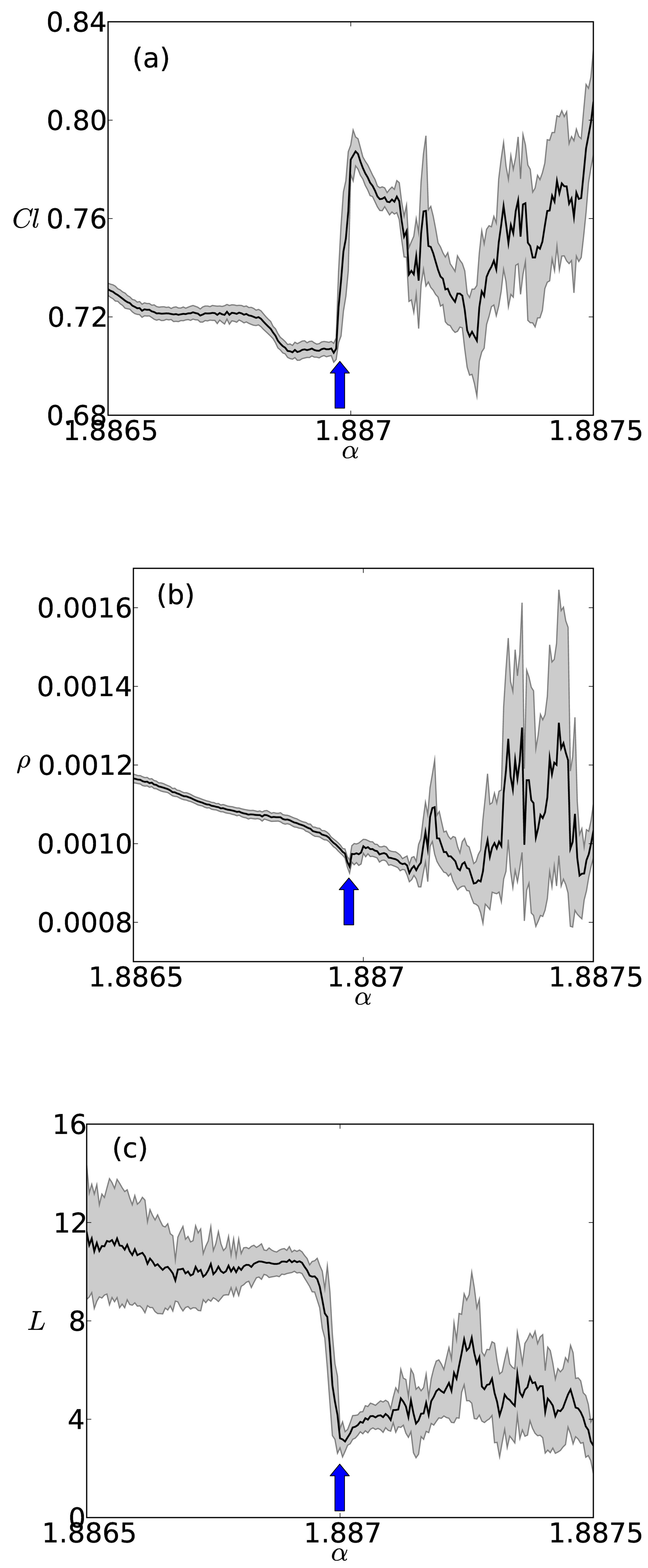
$$\begin{cases} x_{i+1} = f - \alpha x_i + x_i^3 \\ f = 0.7\cos(2\pi\theta_i) \\ \theta_{i+1} = \theta_i + \omega \\ \omega = (\sqrt{5}-1)/2 \end{cases}$$

Bifurcation Diagram



T1 and T2: Tori with period 1 and 2. **GF**: Gradual fractalization of tori leading to strange nonchaotic attractors (SNAs). **TC**: Collision of period-doubled tori with their unstable parents. **IT and CI**: Regions where SNAs appear through intermittency and crisis induced intermittency. **C**: Chaotic regions. White bands in upper-left and upper-right coners: Trajectories escape to infinity.

- **Illustration of TC mechanism** leading to SNAs for a fixed value of the bifurcation parameter *f = 0.7* and α is varied. The critical value at which the collision happens is $\alpha_c \sim 1.88697$.
- The complex network measures are computed for 100 realizations of trajectory of 5000 samples.
- Main result: the network measures are able to detect the critical value α_c at which the SNA is created.



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