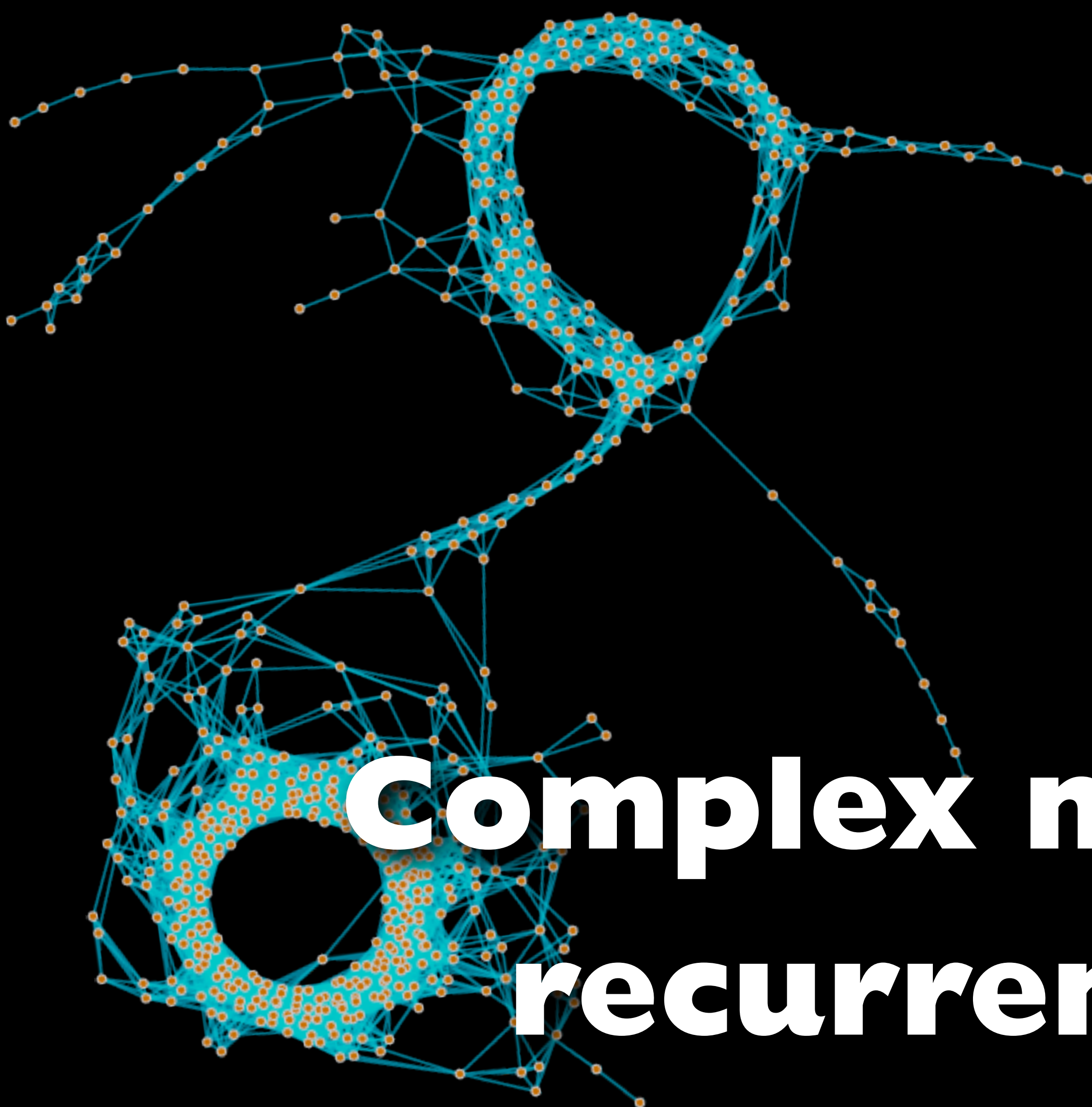

recurrence
analysis
quantification
nonlinear
dynamics
plot
plots
time-series
chaos
chaotic
entropy
complexity
signal
detection
complex
using
data
human
synchronization
heart
systems
rate
variability
networks
protein
based
theory
structure
activity
pressure
signals
cardiovascular
postural
methods
dimension
flow
patterns
conference
nonlinearity
proceedings
control
system
network
fractal
noise
phase
space
test
determinism
dynamical
time
recurrences
prediction
correlation
magnetic
classification
application
coupling

Potsdam Institute for Climate Impact Research

PIK



Norbert Marwan

Jonathan Donges, Reik Donner, Yong Zou, Jürgen Kurths

Complex network analysis of recurrences in phase space

Leibniz Gemeinschaft

Outline

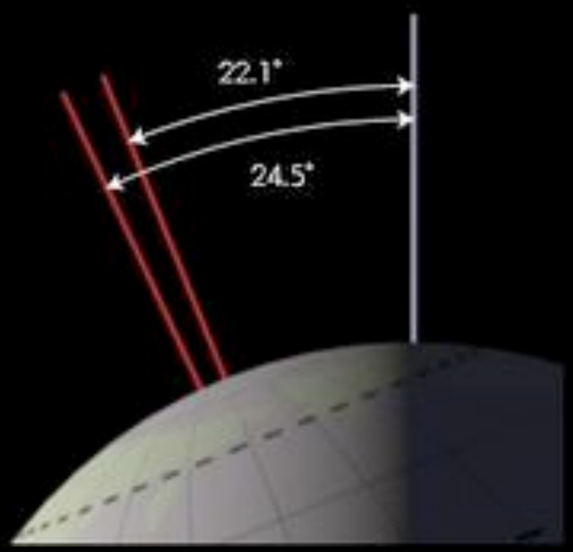
1. Recurrences
2. Recurrence plots
    - definition, structures, quantification, examples
3. Network analysis of recurrences
    - definition, network measures, clustering, examples

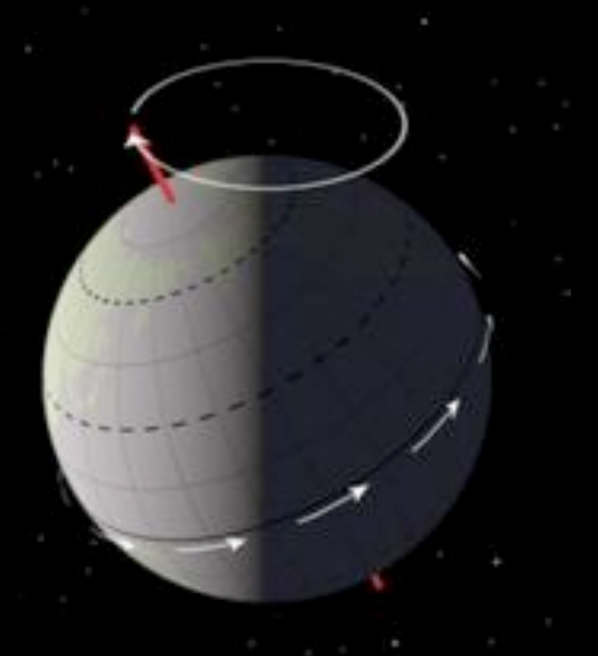
Recurrences

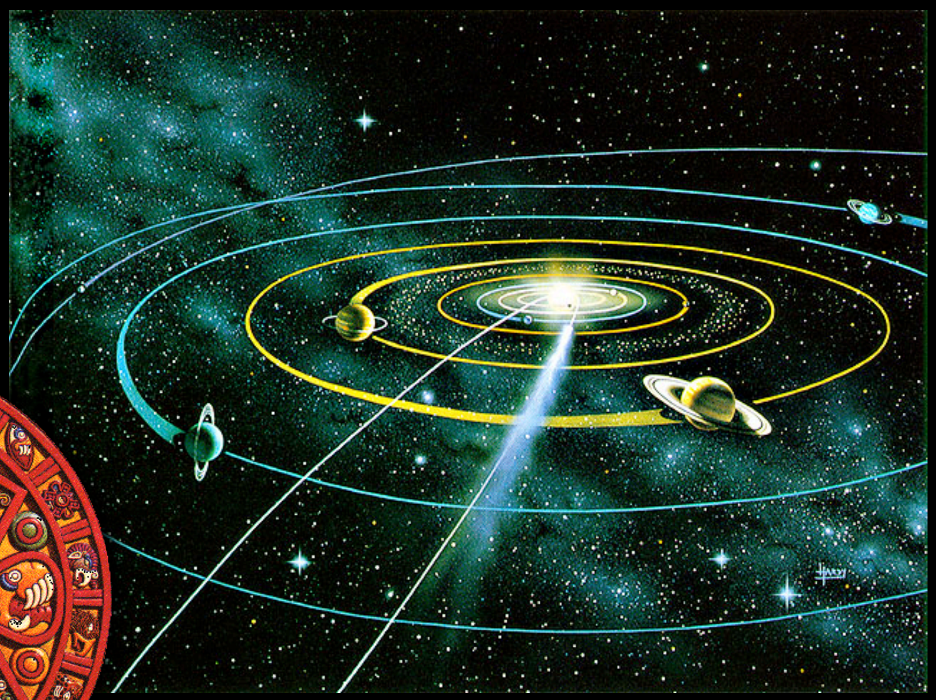
Recurrence

- fundamental characteristic of many dynamical systems
- recurrences in real life:

  Milankovich cycles, weather after storm, El Niño phenomenon, heart beat after exertion, Maya calendar etc.







Recurrence

- Anaxagoras, approx. 450 BC:

  perichoresis: chaotic circular movement

Recurrence

- Poincaré, 1890:

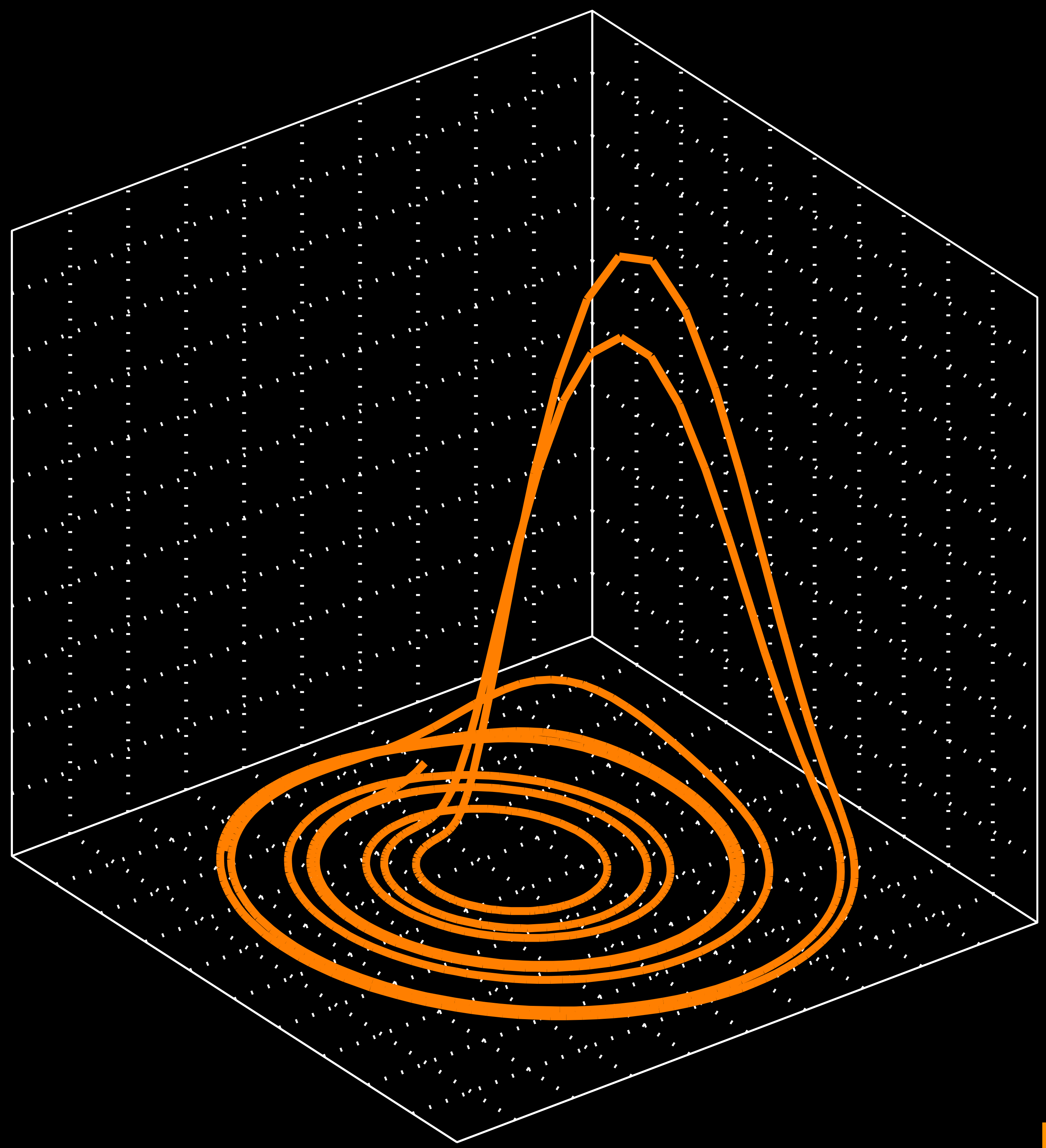
  "a system recurs infinitely many times as close as one wishes to its initial state"

Investigating Recurrence

- Poincaré map
- Recurrence time statistics
- First return map
- Recurrence plot
- Network analysis of recurrences

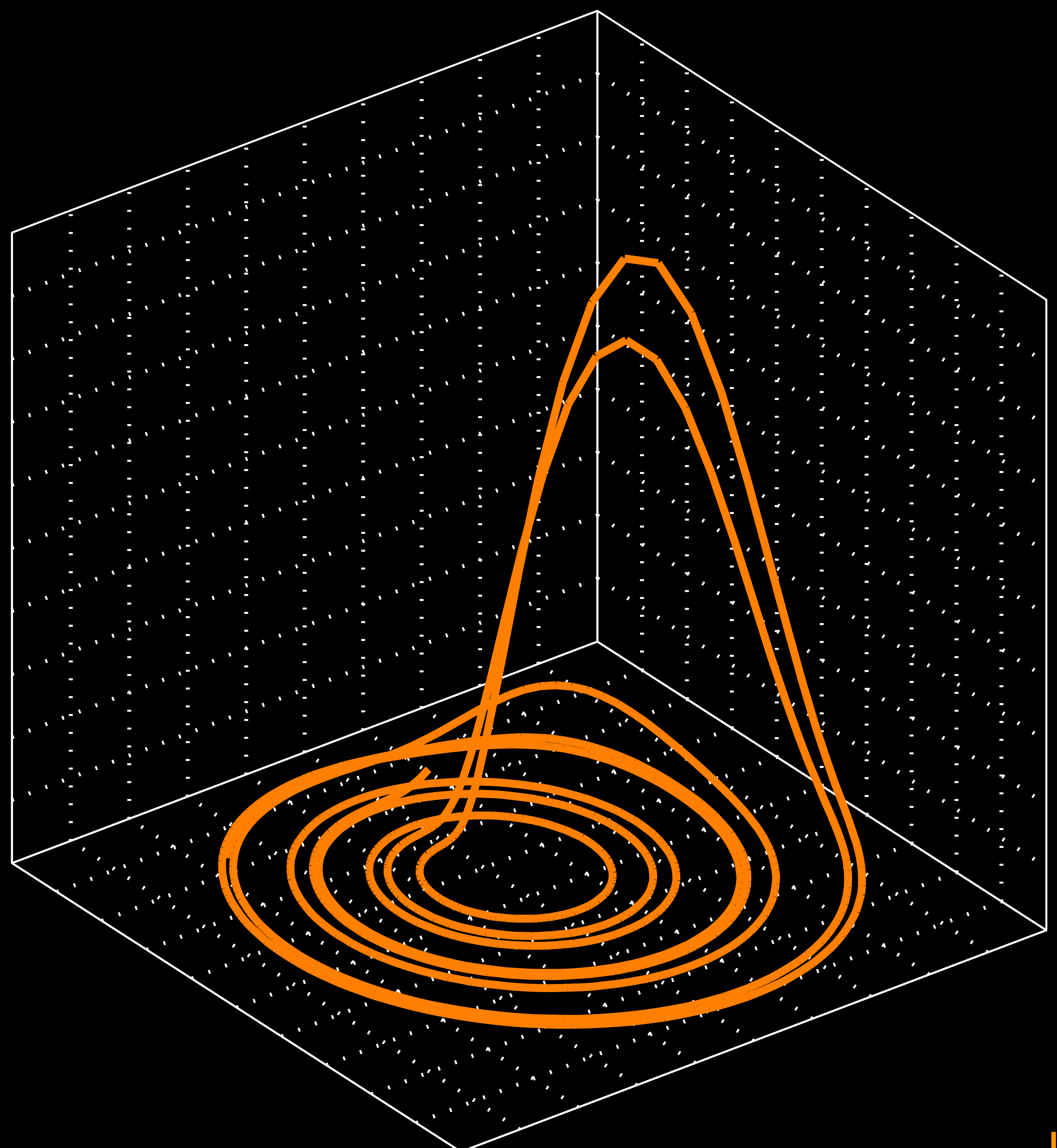
Recurrence Plots

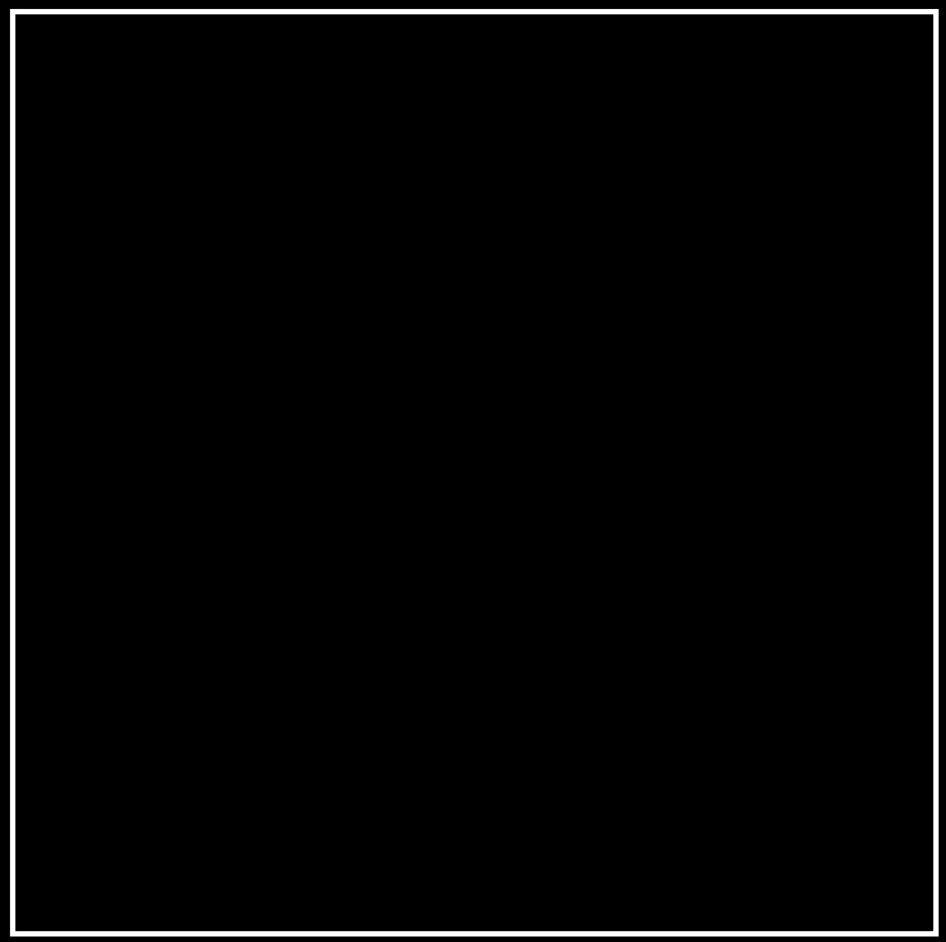
Recurrence Plot



J.-P. Eckmann, S. O. Kamphorst, D. Ruelle, Europhysics Letters, 5, 1987

Recurrence Plot





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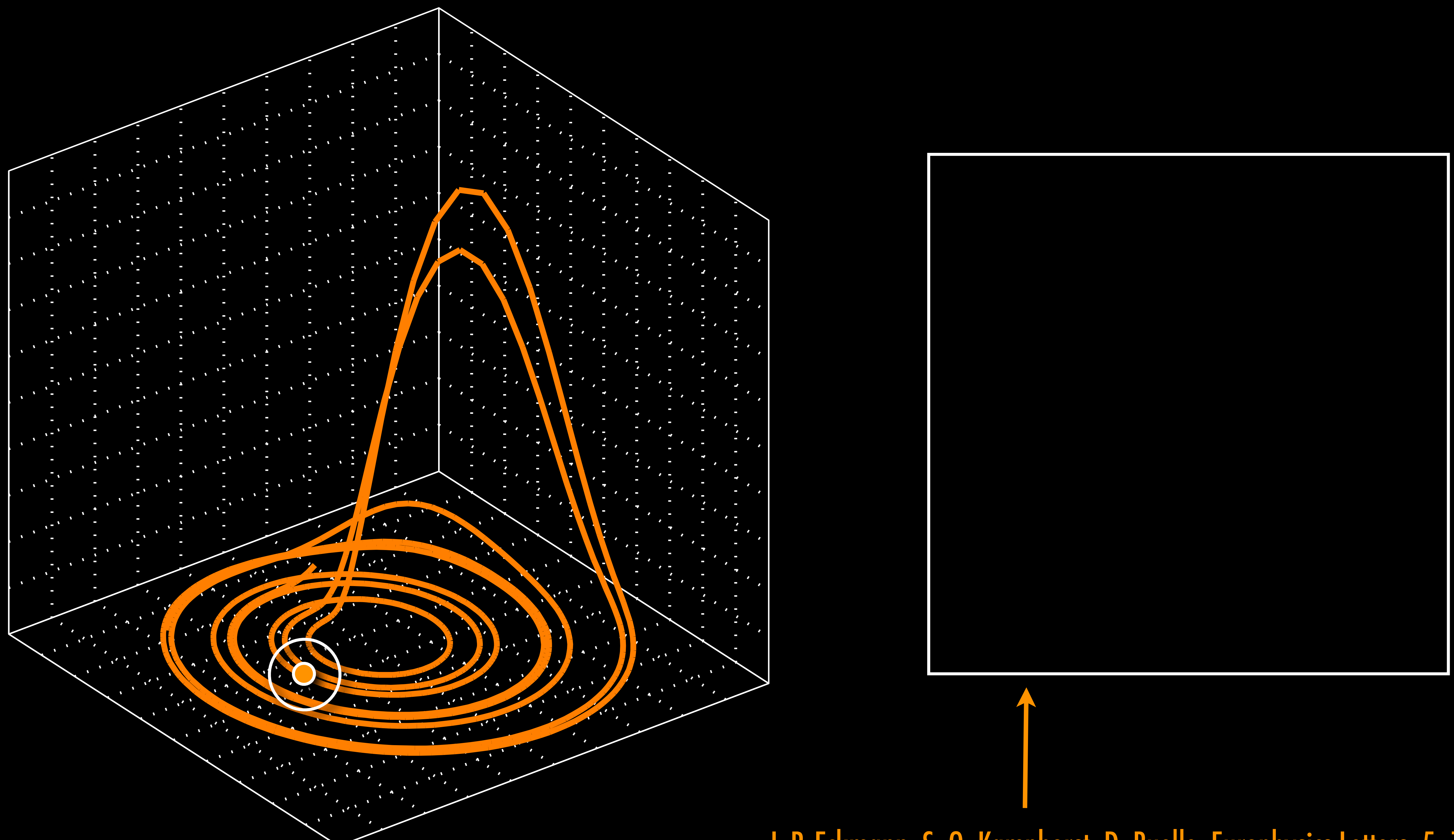

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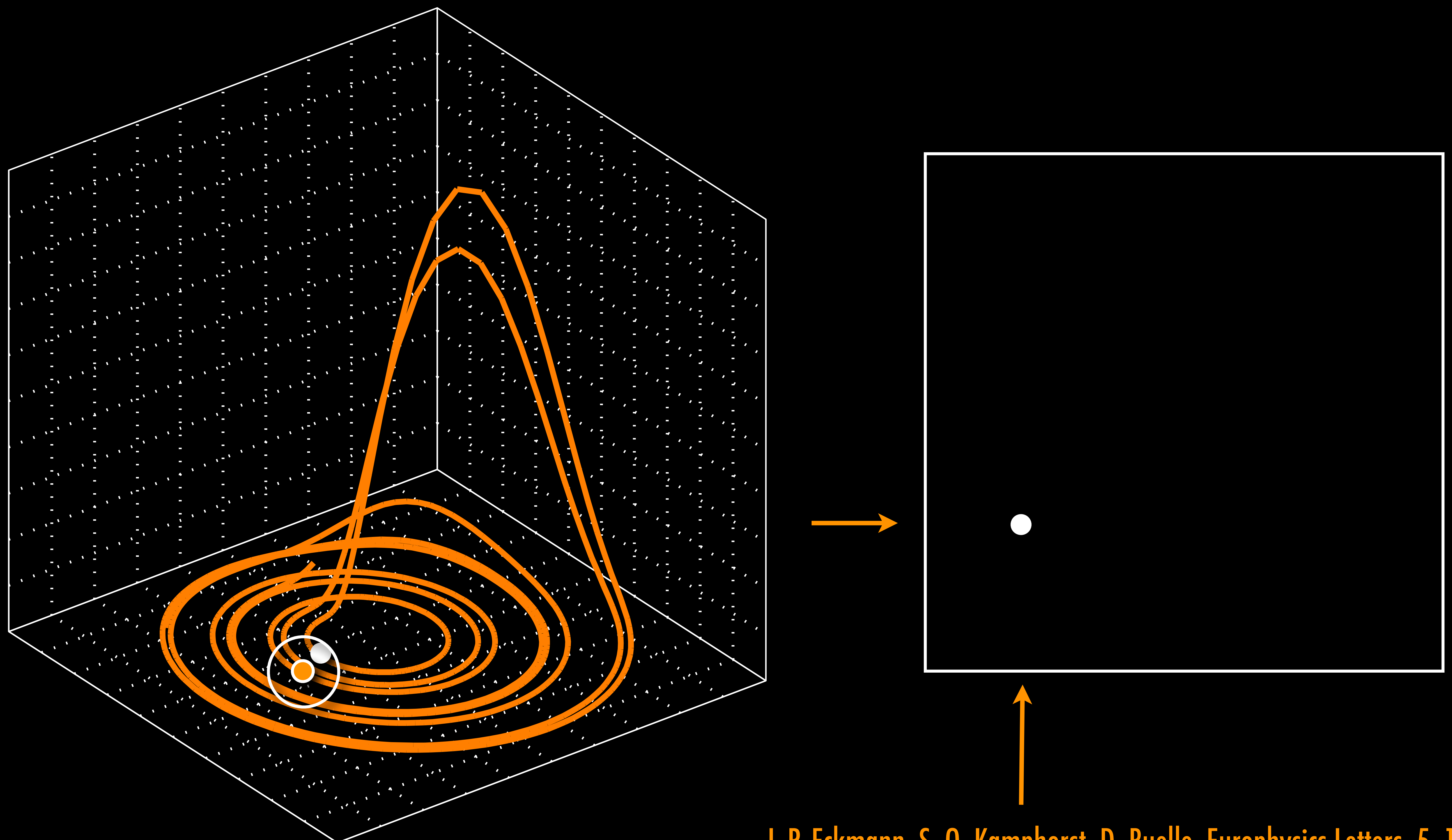

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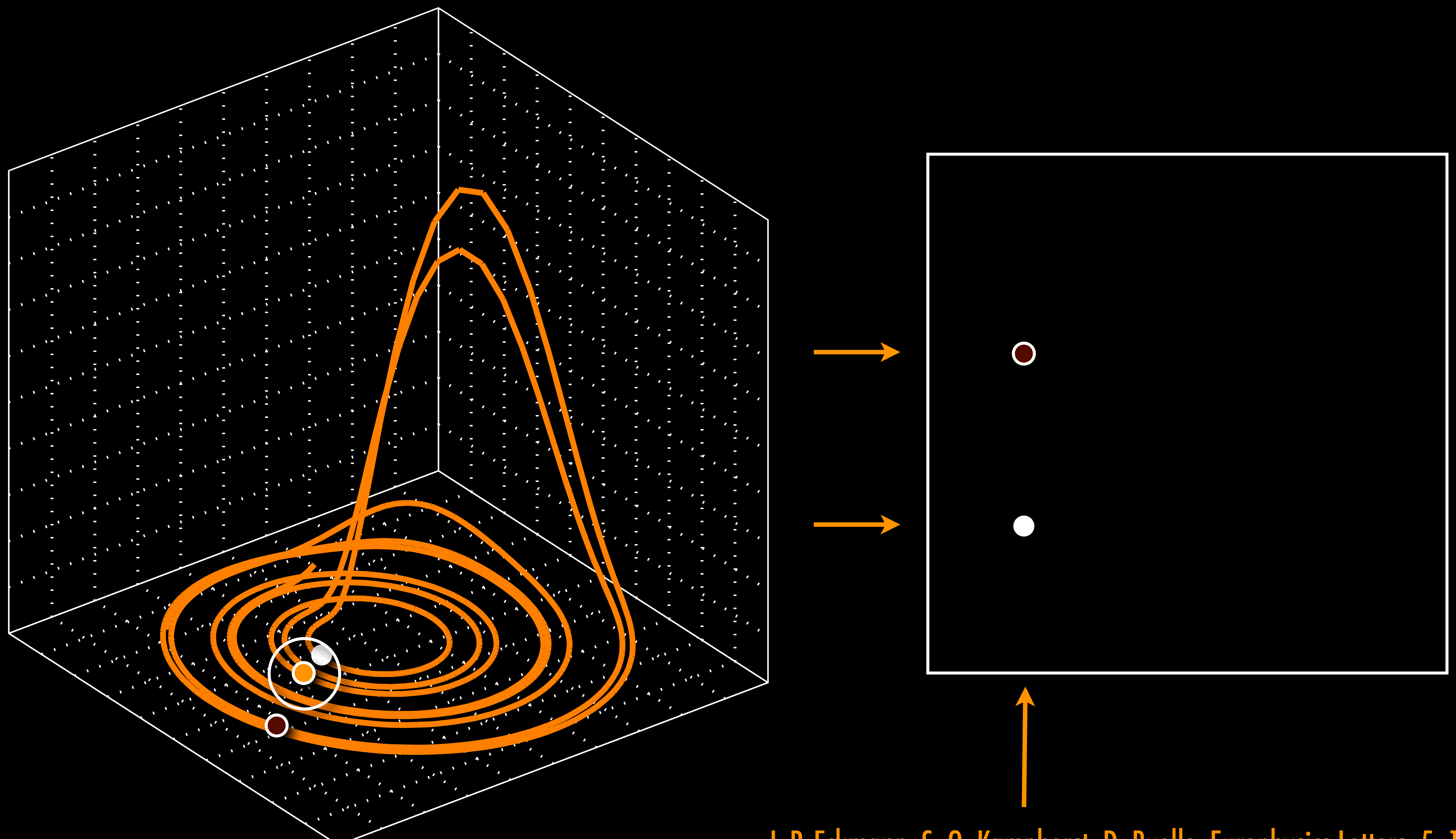

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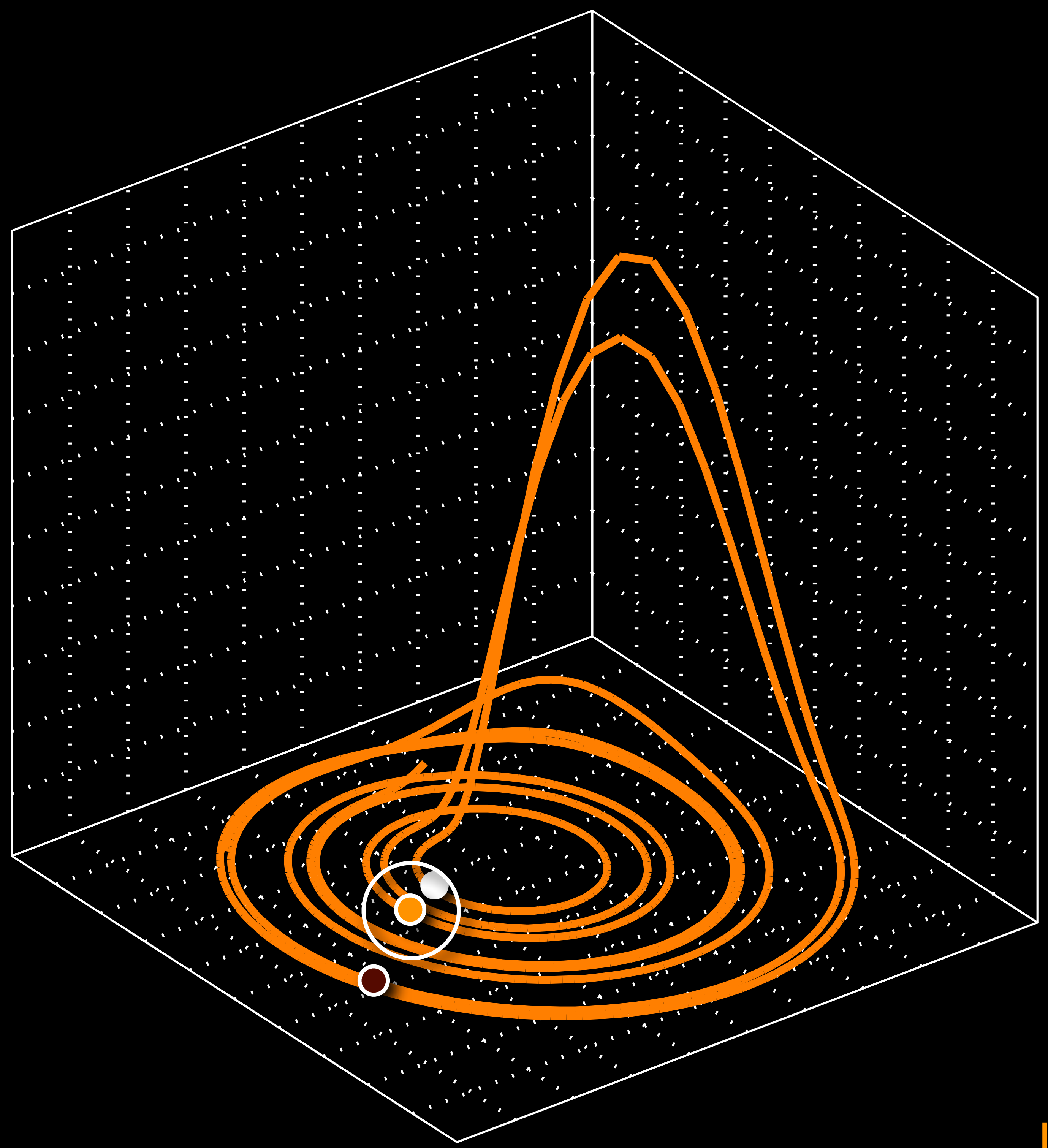
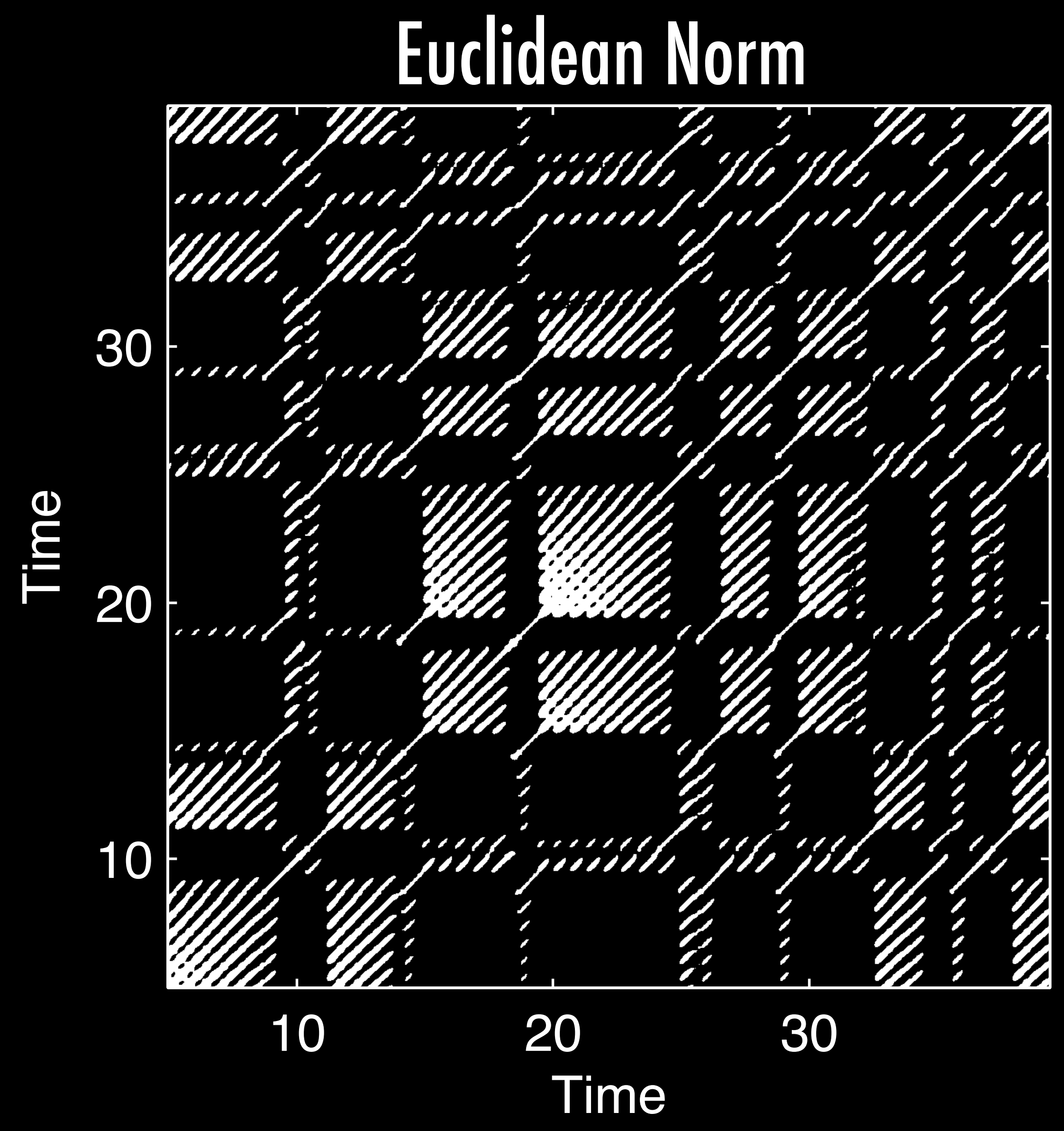
Recurrence Plot

$$\mathbf{R}_{t_1,t_2} = \Theta\left(\varepsilon - \|\vec{x}(t_1) - \vec{x}(t_2)\|\right)$$





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Recurrence Plot

Eckmann et al, EPL, 1987:

> To conclude, we wish to stress that the recurrence plots are rather easily obtained aids for the diagnosis of dynamical systems. They display important and easily interpretable information about time scales which are otherwise rather inaccessible.

Recurrence Plot

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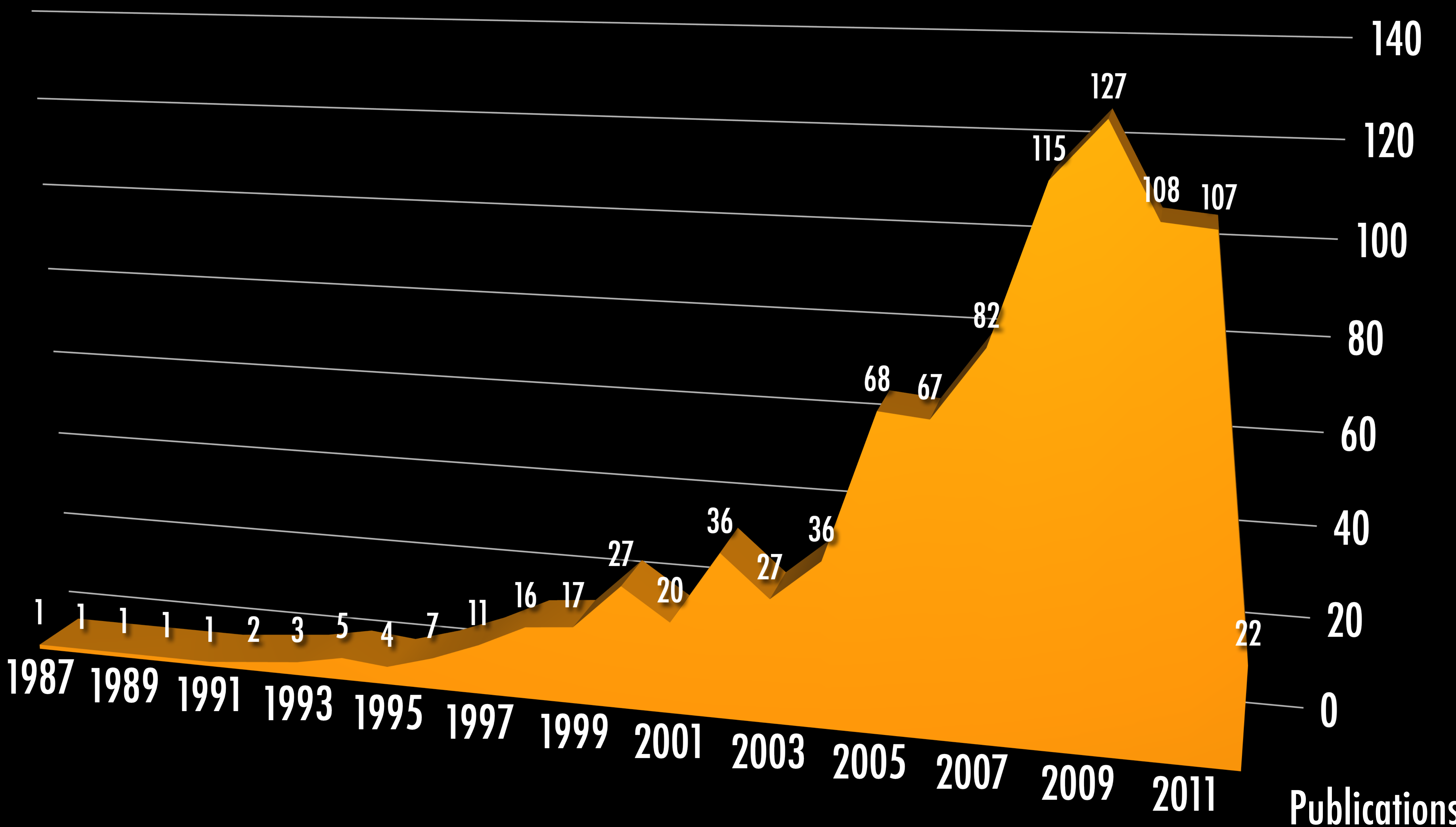
To conclude, we wish to stress that the recurrence plots are rather easily obtained aids for the diagnosis of dynamical systems. They display important and easily interpretable information about time scales which are otherwise rather inaccessible.

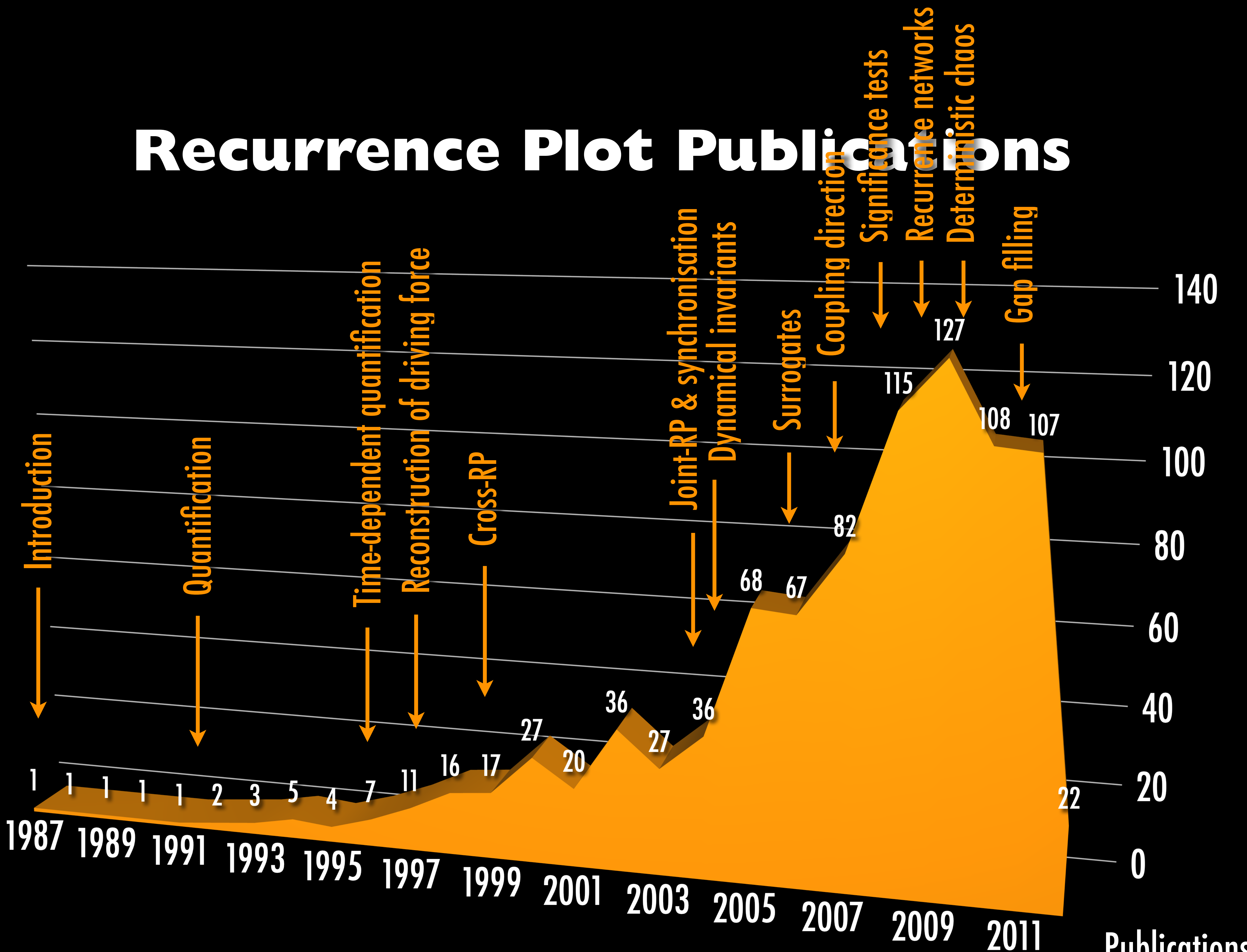
Eckmann et al, EPJST, 2008:

paper. It is of course rewarding to discover that a small paper has, after a dormant period, led to an active field, with many ramifications we certainly had not anticipated. One can wonder what

Recurrence Plot Publications



Recurrence Plot Publications



Recurrence Plot

- to visualise the phase space trajectory by its recurrences

- recurrence matrix:
  - binary
  - symmetric

$R_{i,j} =$

1	1	0	0	1
1	1	1	0	1
0	1	1	0	0
0	0	0	1	1
1	1	0	1	1

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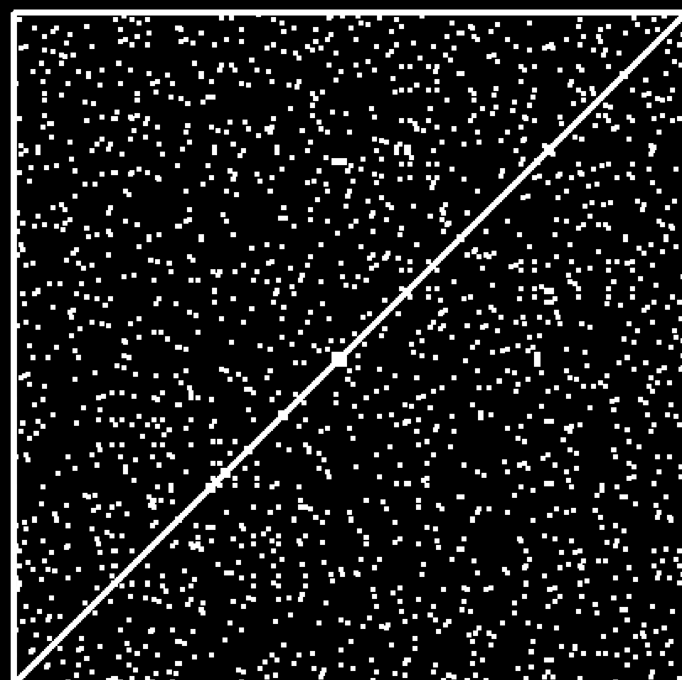
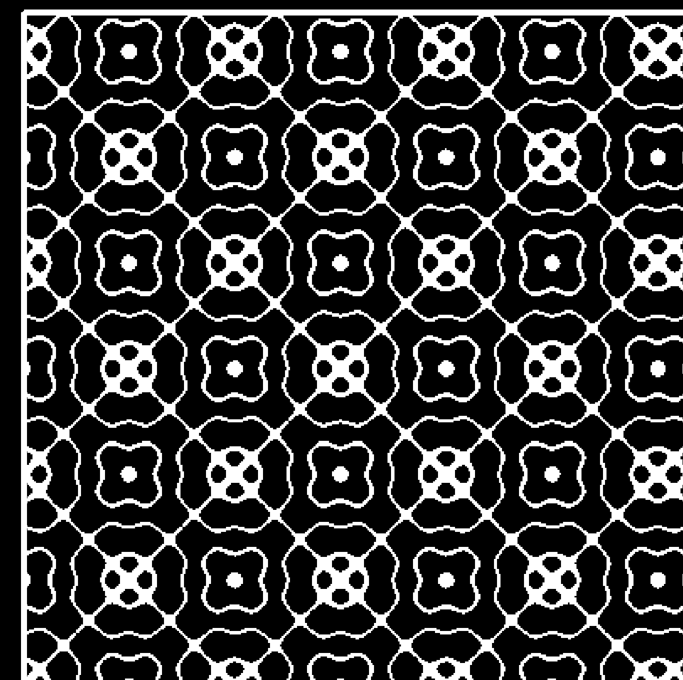
N. Marwan et al., Physics Reports, 438, 2007

Recurrence Plot Typology

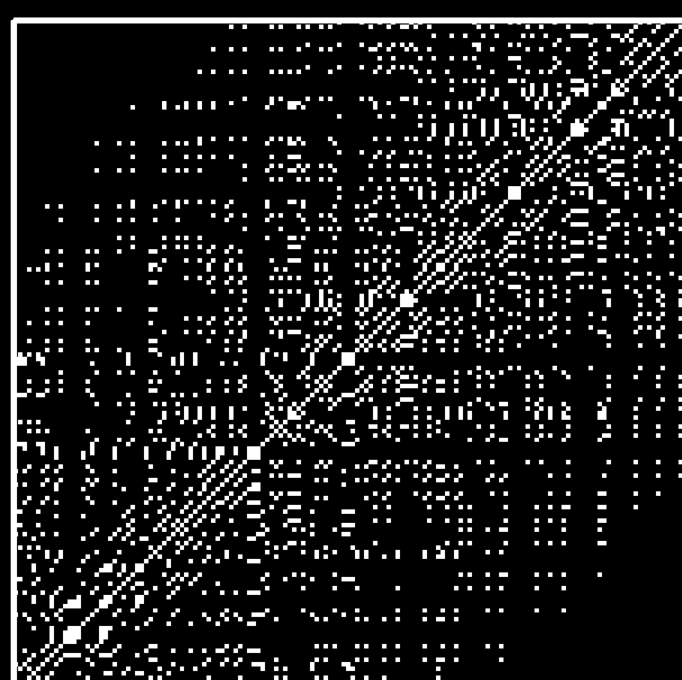
homogeneous

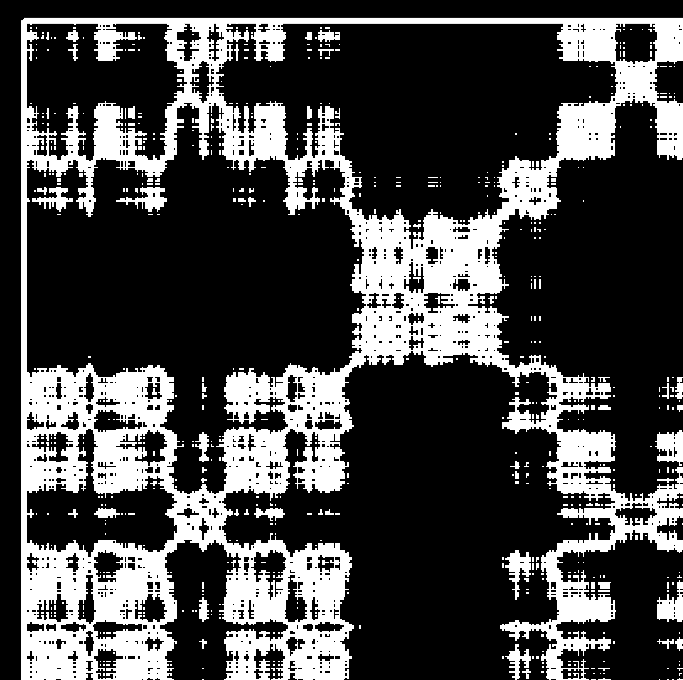




periodic

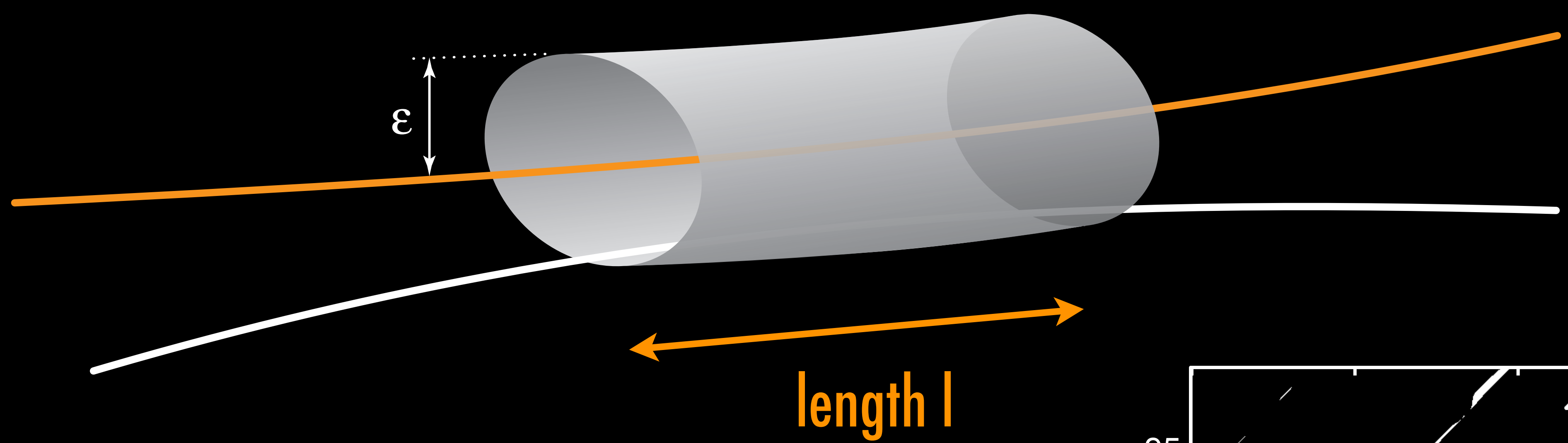
drifty

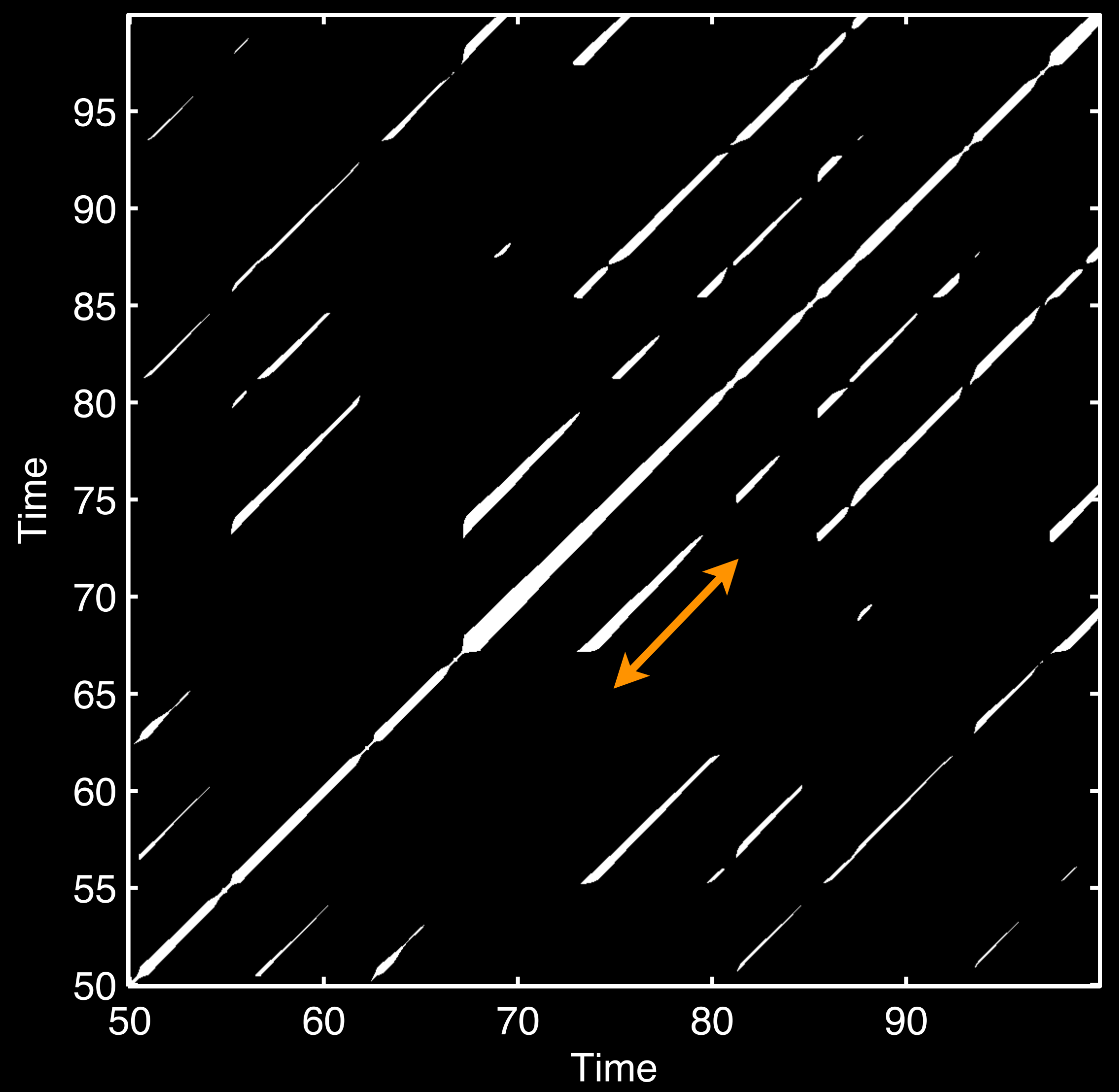




disrupted

Recurrence Quantification





- number of lines of exactly length l
  - histogram P(l)

J. P. Zbilut & C. L. Webber Jr., Phys. Lett. A 171, 1992
N. Marwan et al., Phys. Rev. E 66, 2002

Recurrence Quantification

- Recurrence rate

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}$$

Probability that any state recurs

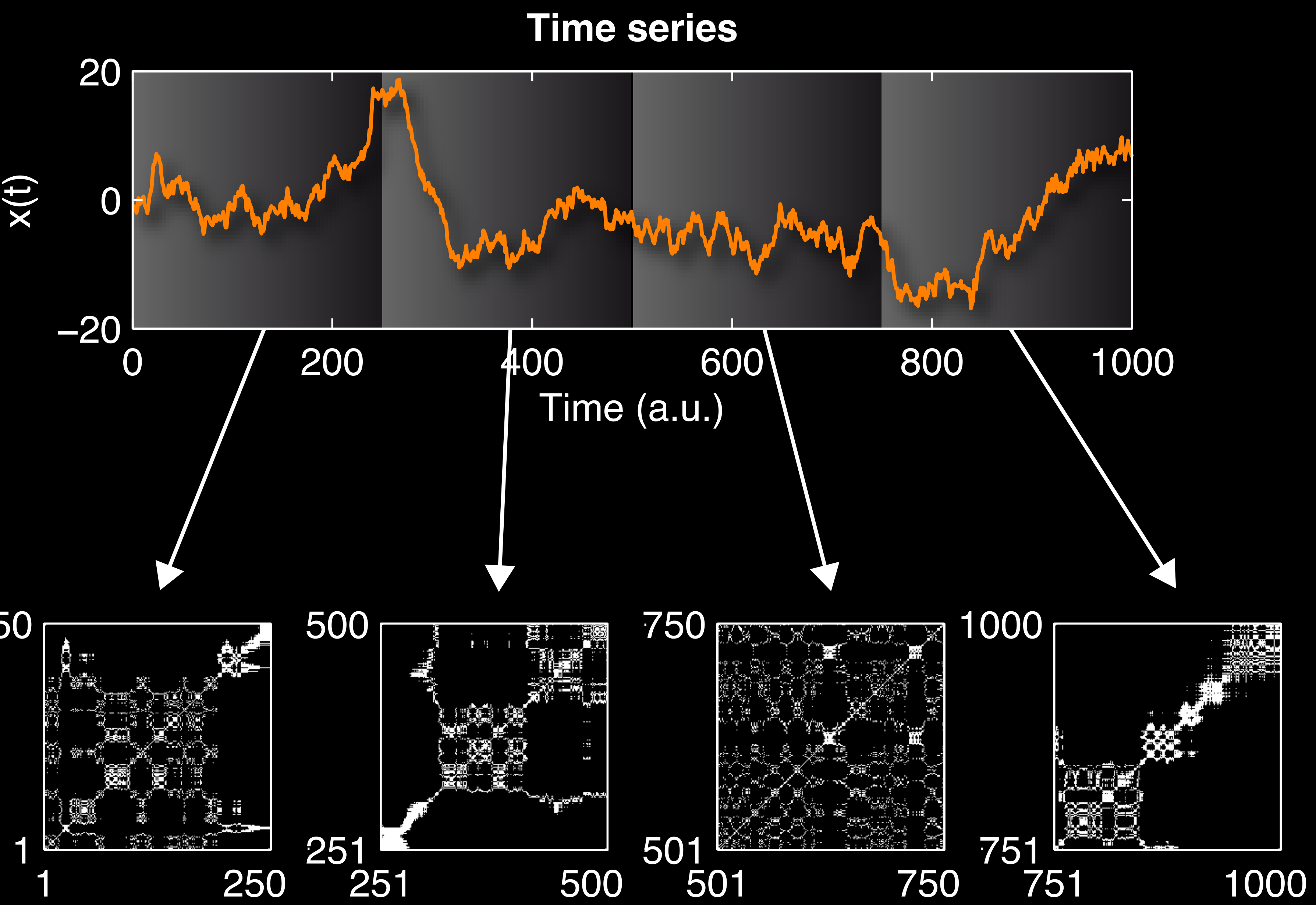
- Determinism

$$DET = \frac{\sum_{l=l_{\min}}^{N} l\, P(l)}{\sum_{l=1}^{N} l\, P(l)}$$

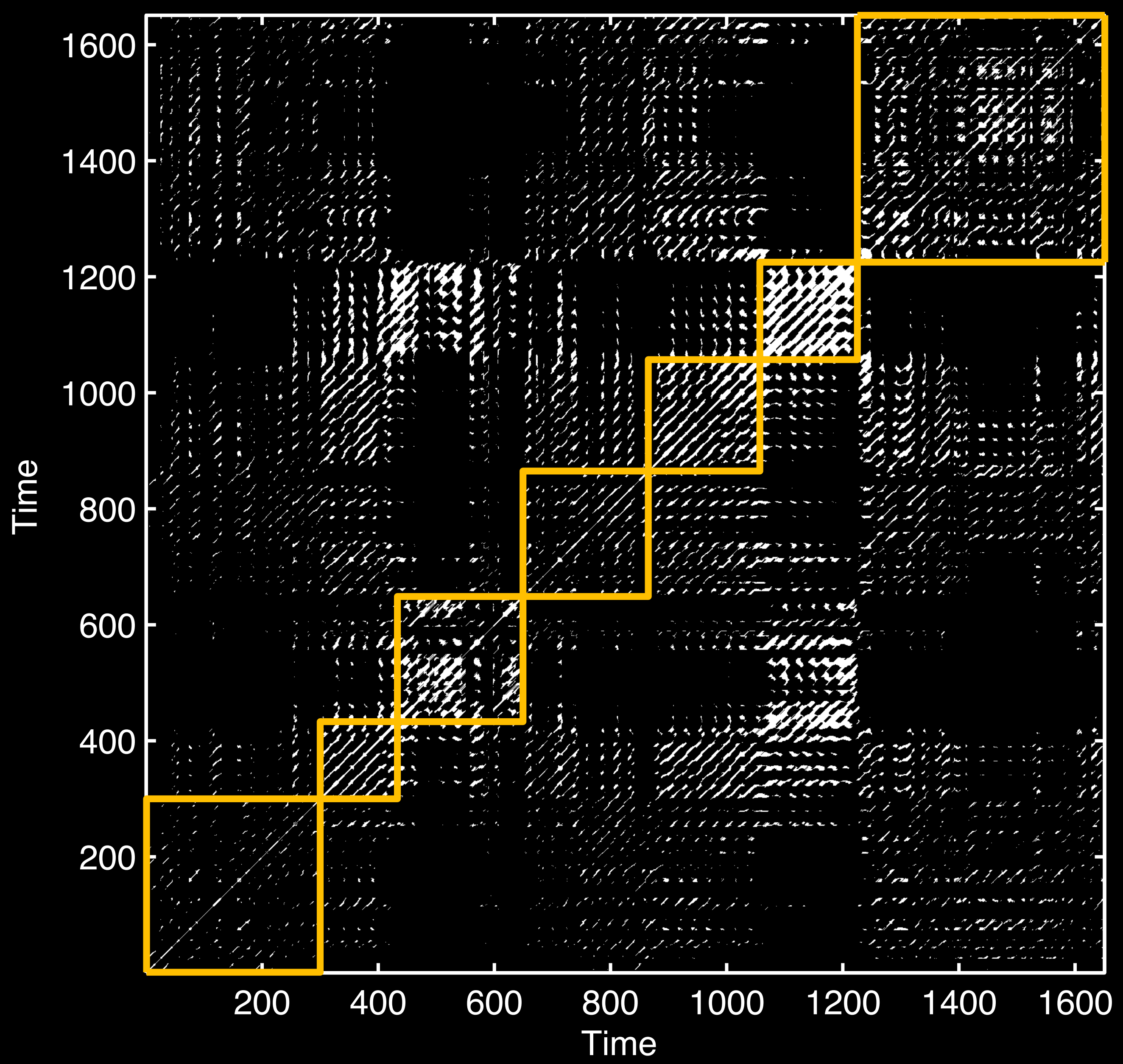
Probability that recurrences further recur

Time Depending Analysis

- sliding window: detection of dynamical transitions



Dynamics of Oxygen Crises in a Lake



Facchini et al., Ecological Modelling, 203, 2007

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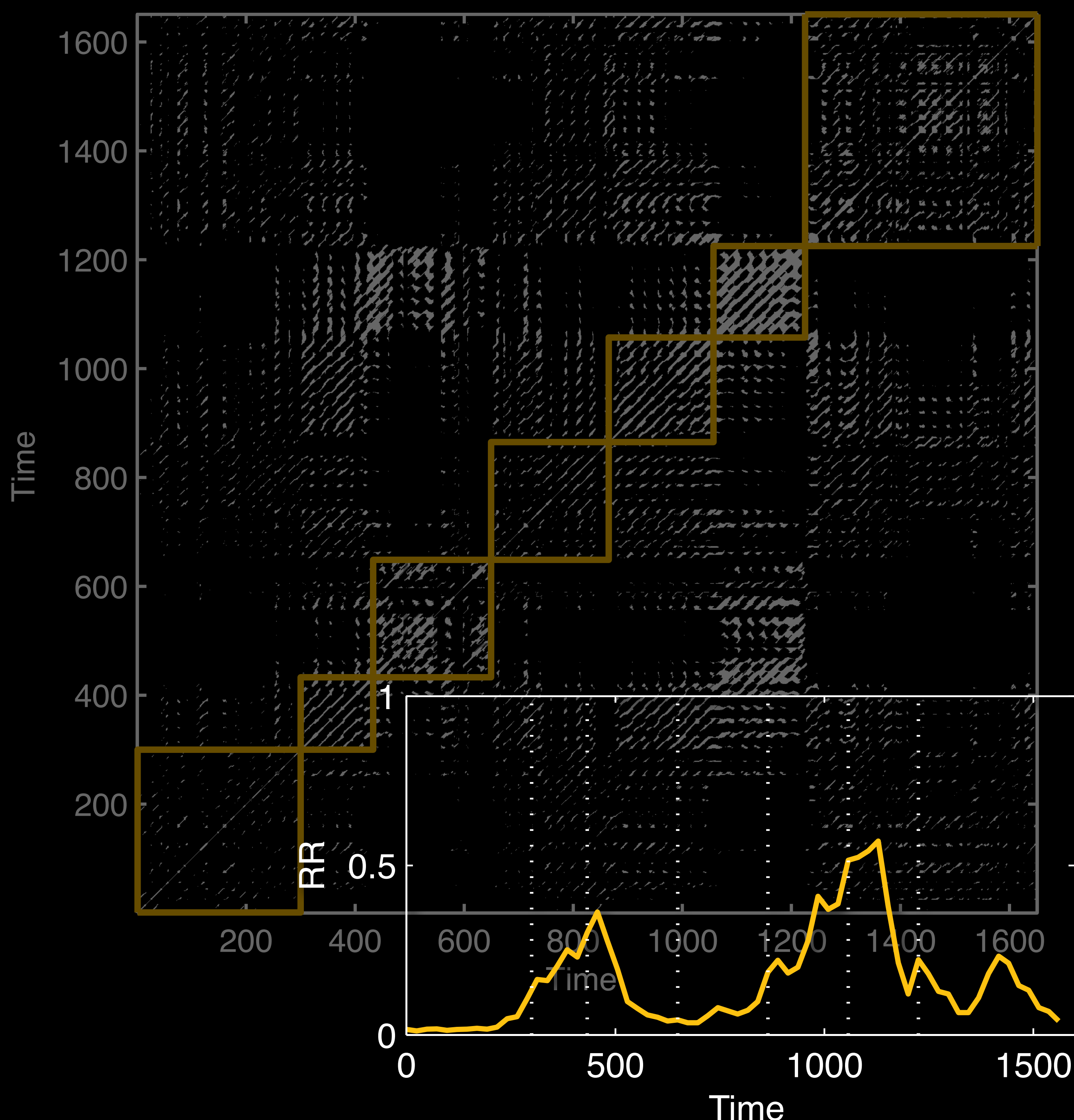
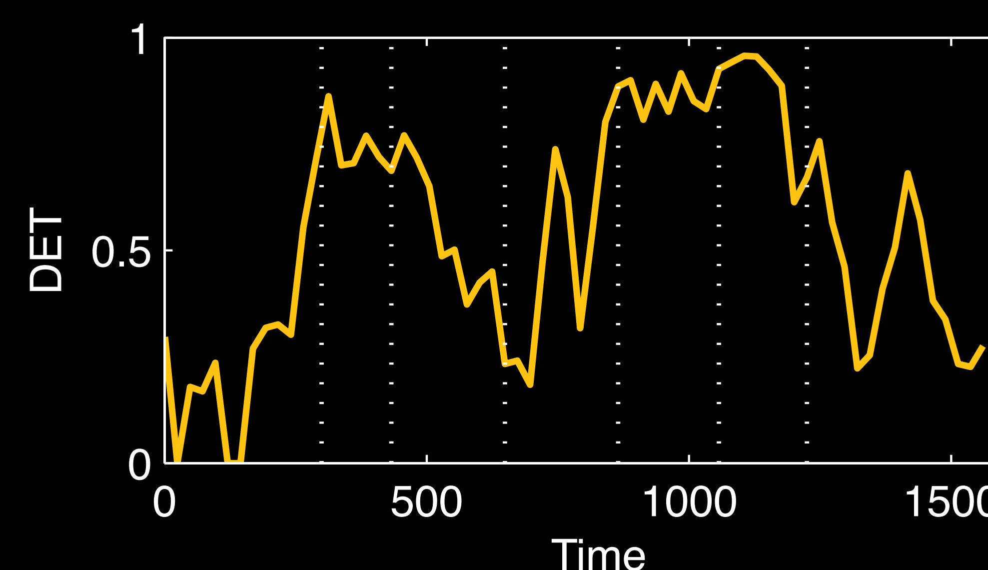
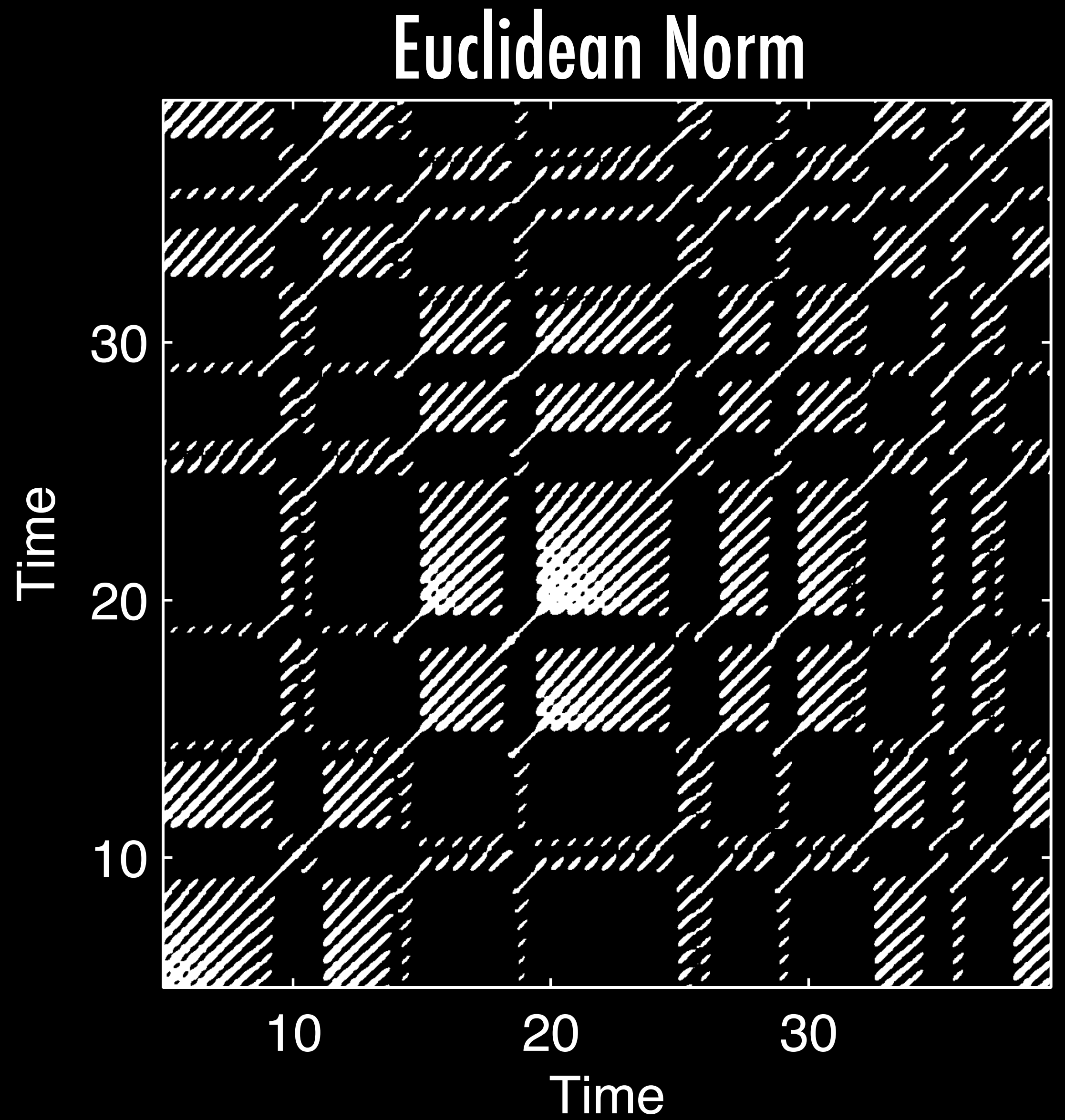




Facchini et al., Ecological Modelling, 203, 2007

Recurrence Plot

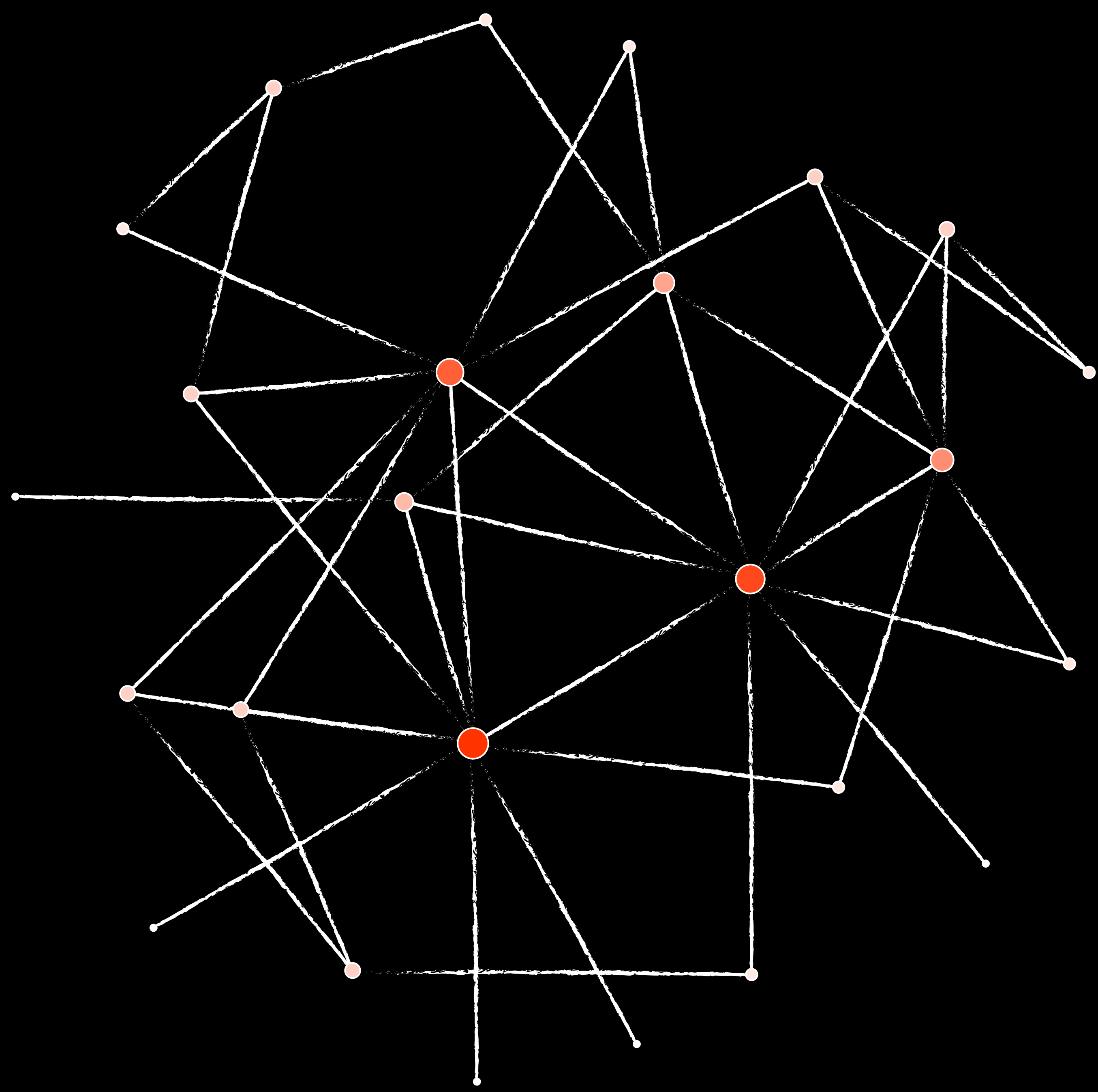
- Transition detection
- Differentiate dynamics
- Finding time scales
- Interrelation detection
- Synchronisation analysis
- Surrogates
- Recurrence time statistics
- etc.



N. Marwan et al., Physics Reports, 438, 2007

Complex Networks

Complex Networks

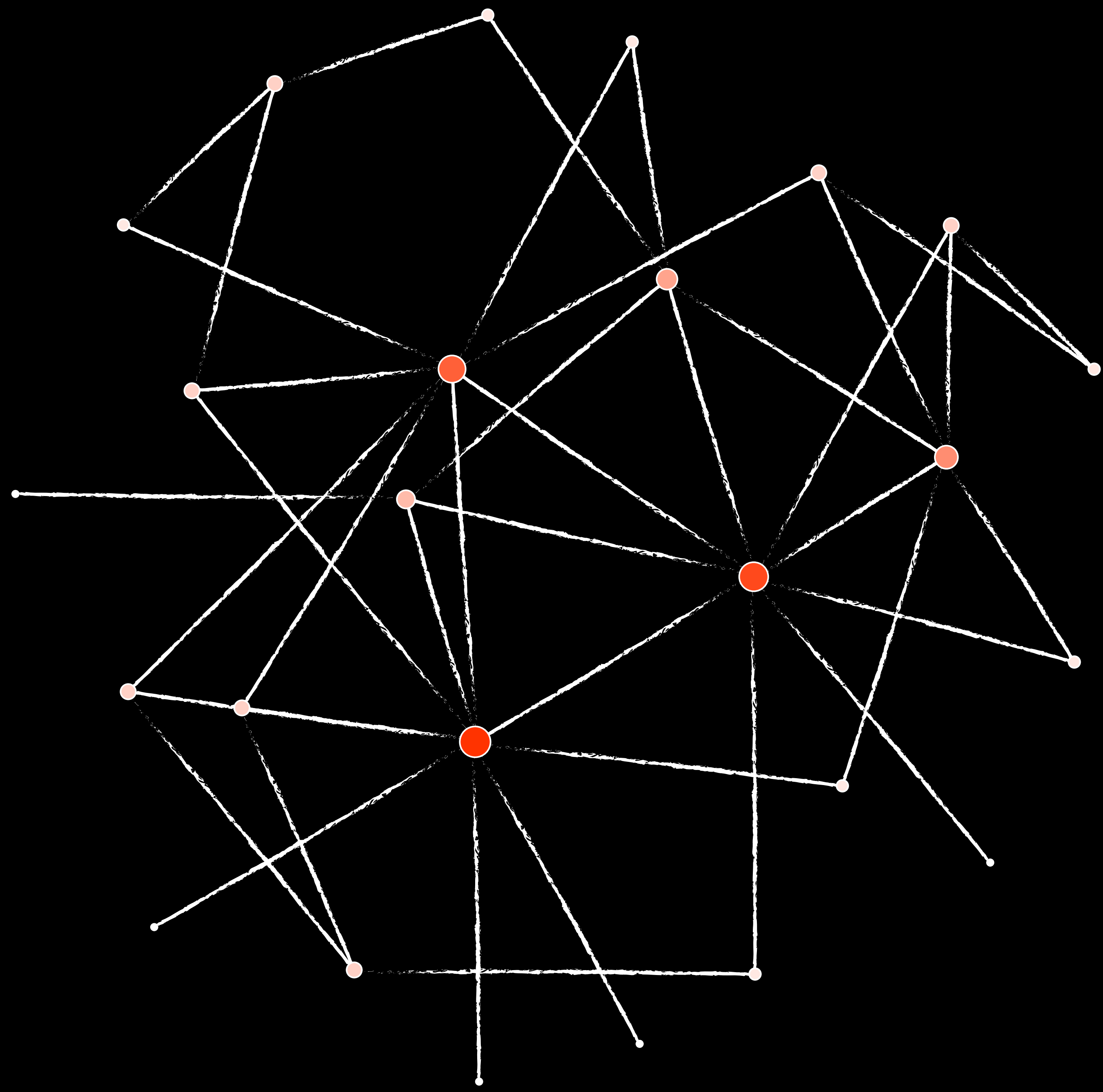


- link matrix (undirected, unweighted network):
  - binary
  - symmetric

$A_{i,j}$ =

0	1	0	0	1
1	0	1	0	1
0	1	0	0	0
0	0	0	0	1
1	1	0	1	0

Complex Networks



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0	1	0	0	1
1	0	1	0	1
0	1	0	0	0
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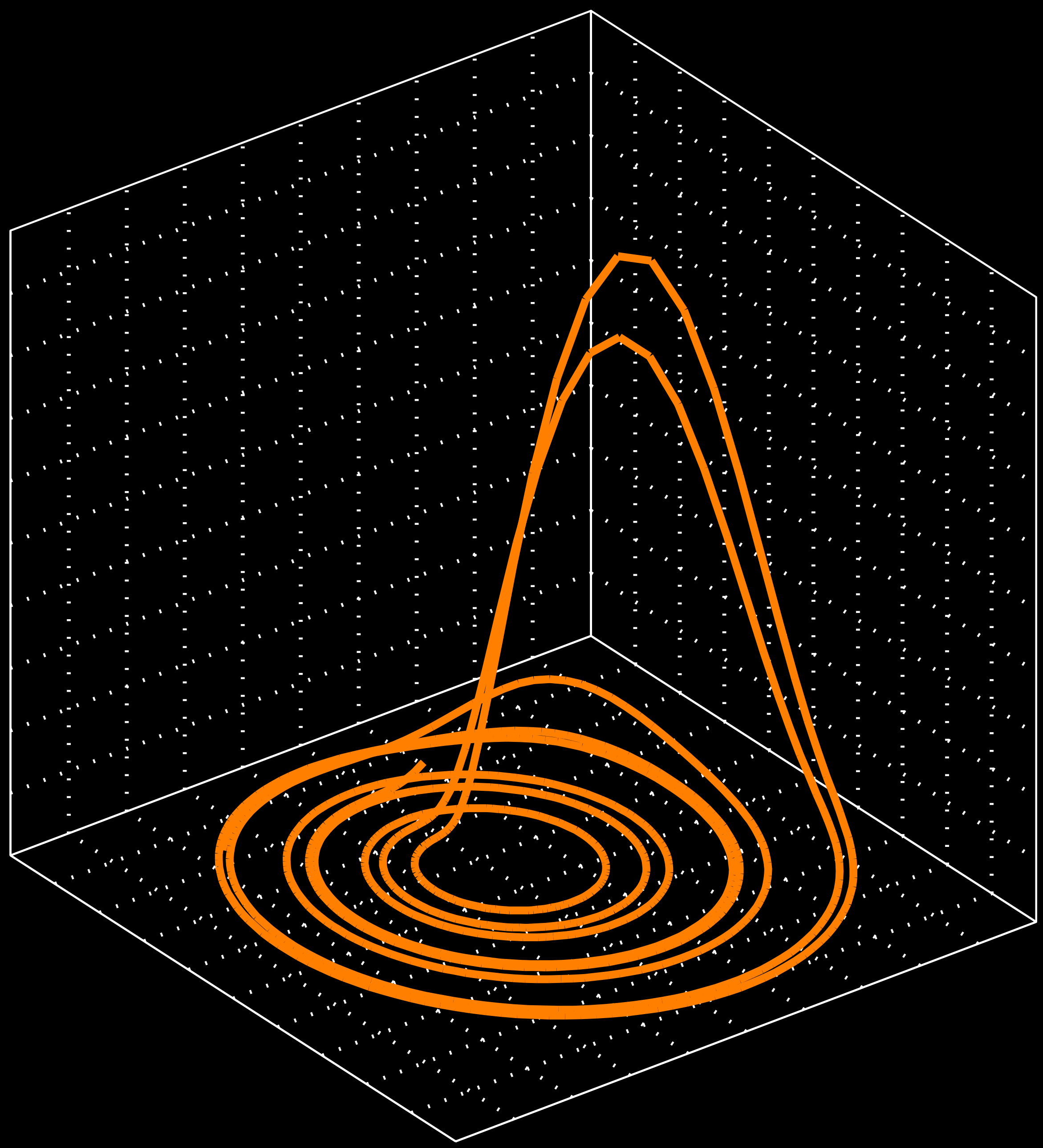
1	1	0	1	0

- link matrix: similar to recurrence plot

Time Series Analysis using Complex Networks

- Link matrix = recurrence matrix of time series
  - Nodes: states in phase space
  - Links: local neighbours of states (i.e. recurrence)

- Path: connected neighbourhoods



Marwan et al., Phys. Lett. A 373, 2009

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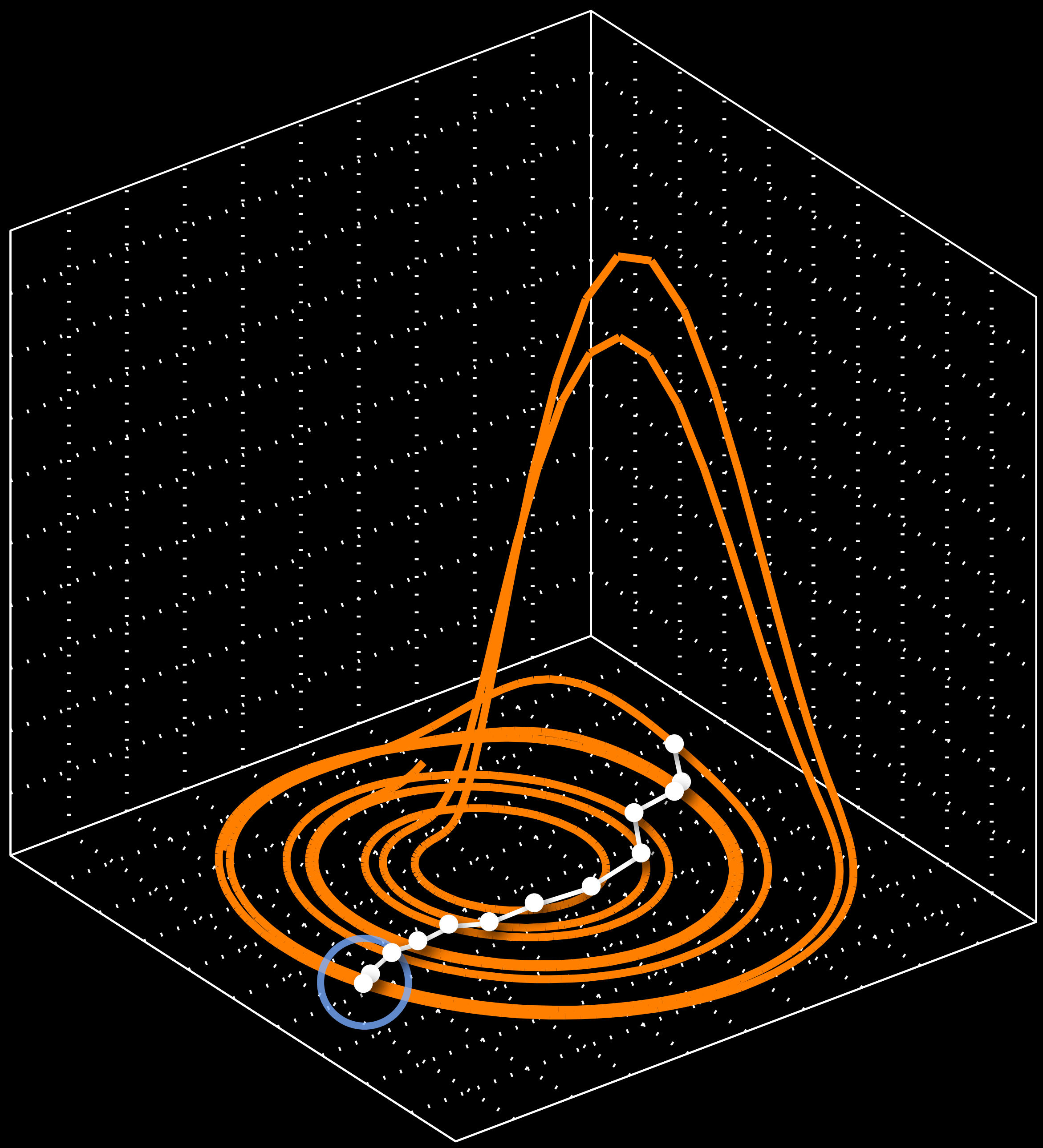

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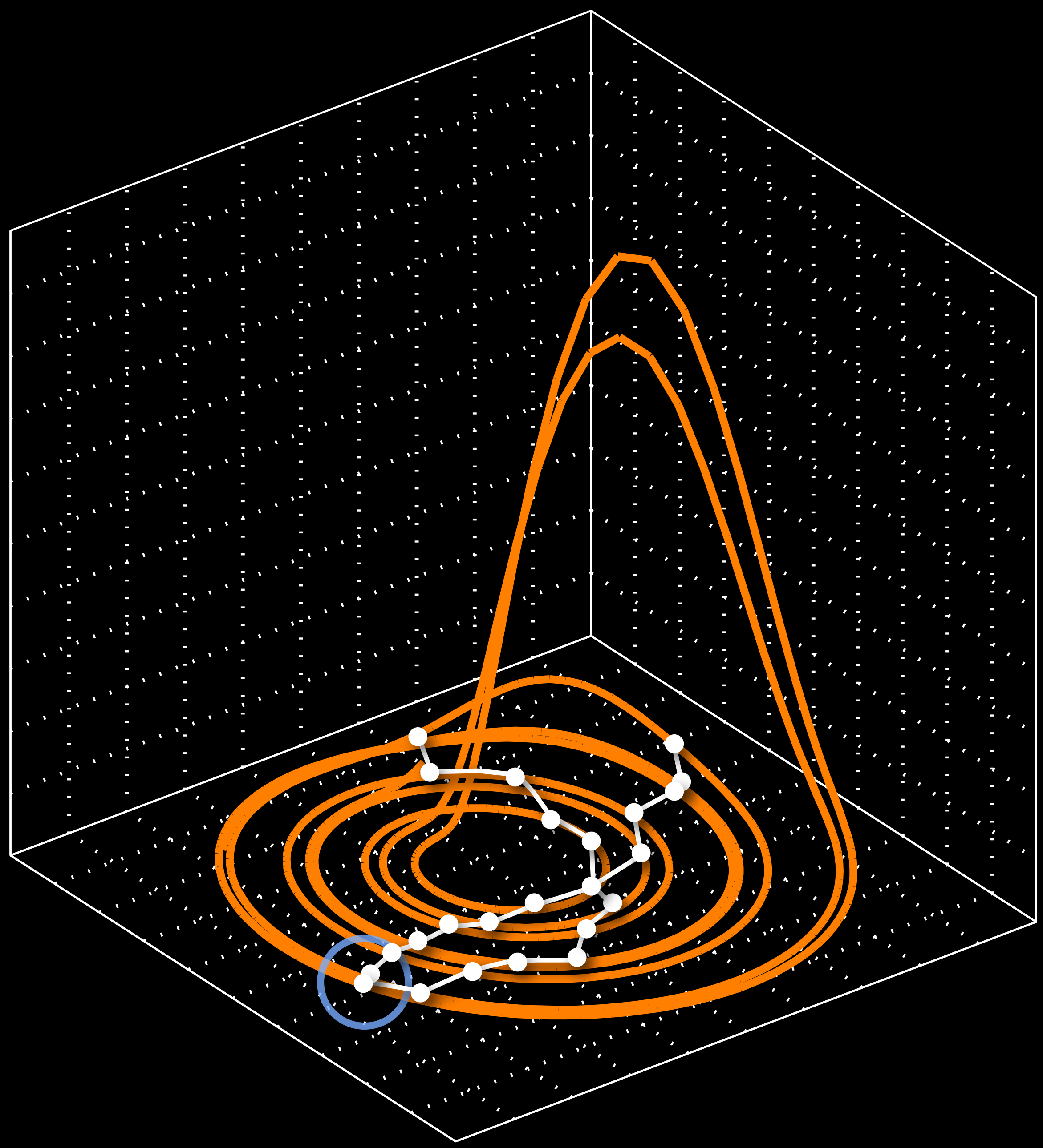
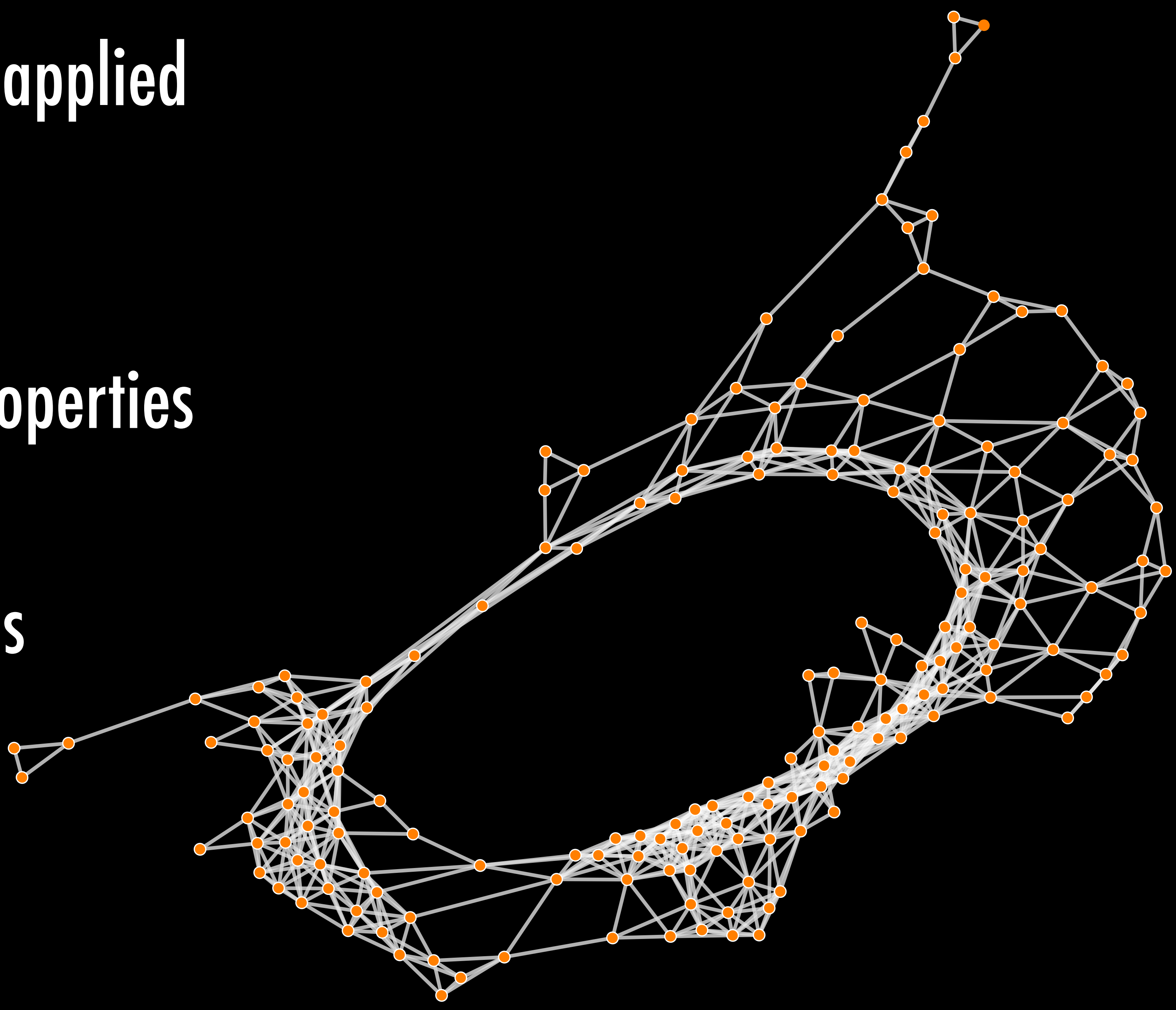


Marwan et al., Phys. Lett. A 373, 2009

Time Series Analysis using Complex Networks

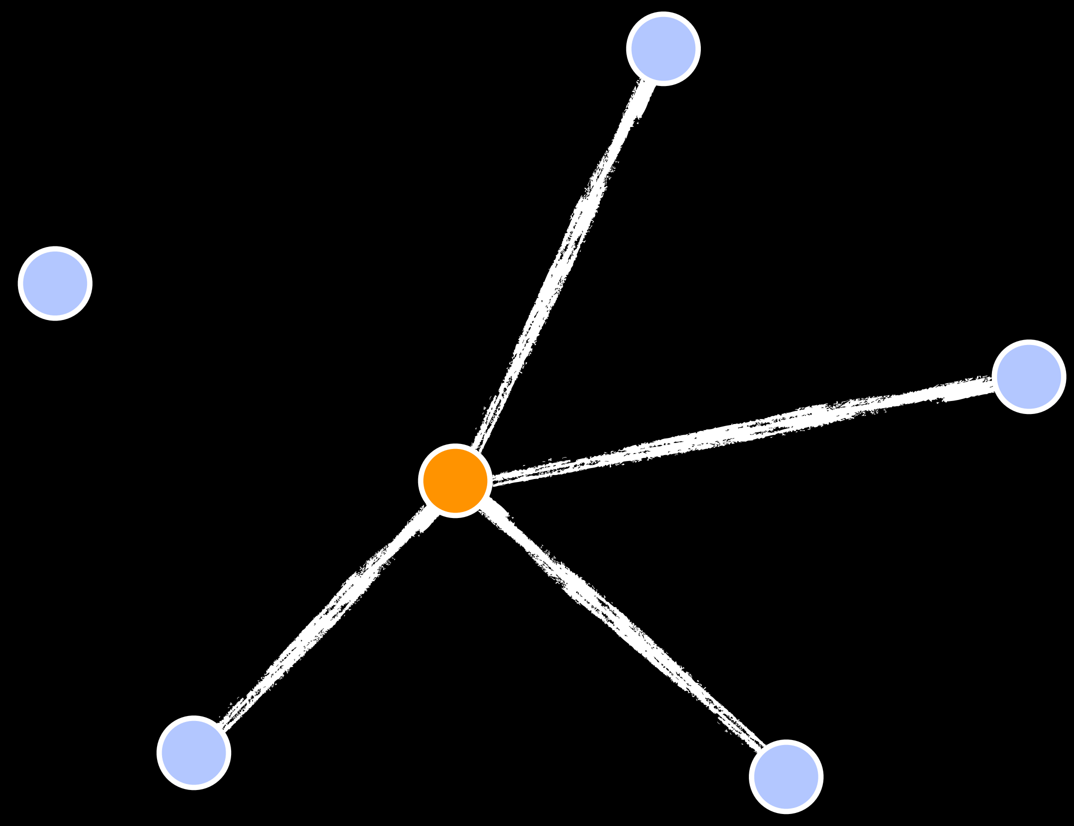
- Complex network measures applied to recurrence plot
  - measures of complexity explaining topological properties of complex systems
  - local and global measures
- „recurrence network“



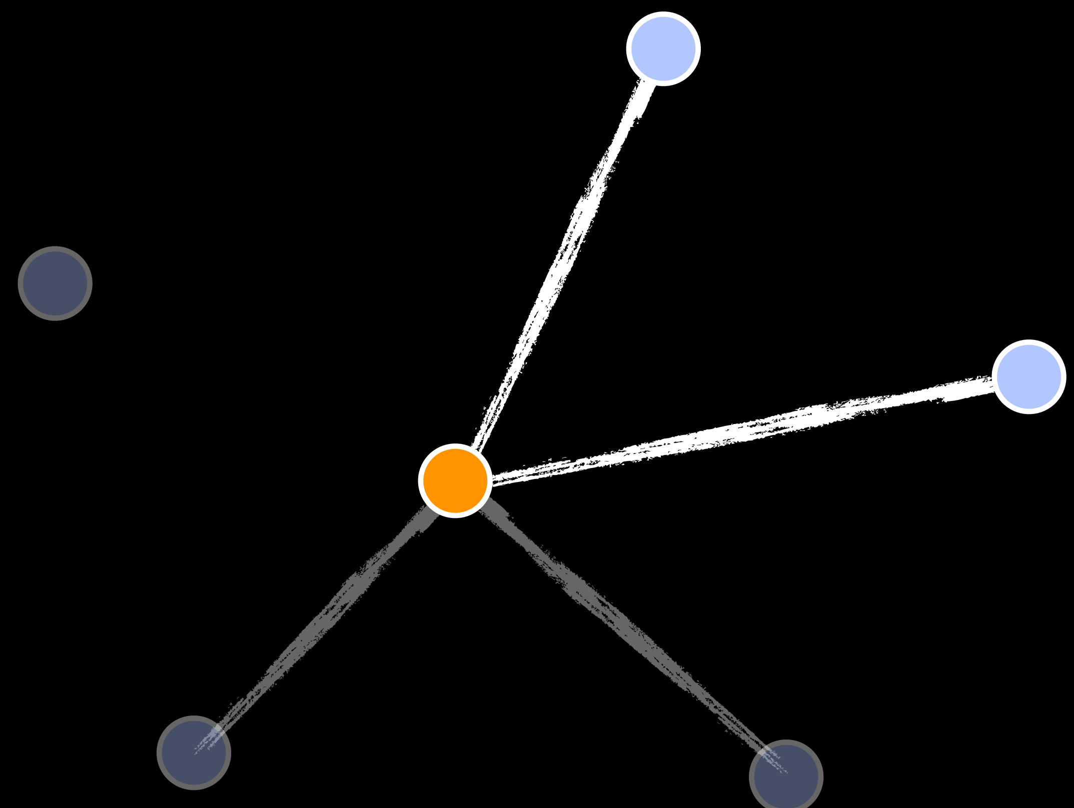
Marwan et al., Phys. Lett. A 373, 2009
Donner et al., IJBC 21, 2011

Scale	Network measure	Phase space
Local	link density	global recurrence rate
	degree centrality	local recurrence rate
Intermediate	clustering coefficient	invariant objects, local dimension
	local degree anomaly	local heterogeneity of phase space density
	assortativity	continuity of phase space density
	matching index	twinness
Global	average path length	mean phase space separation
	network diameter	phase space diameter
	closeness centrality	local centeredness in phase space
	betweenness centrality	local attractor fractionation
	global transitivity/ clustering	regular dynamics
	motif distribution	dynamical classification

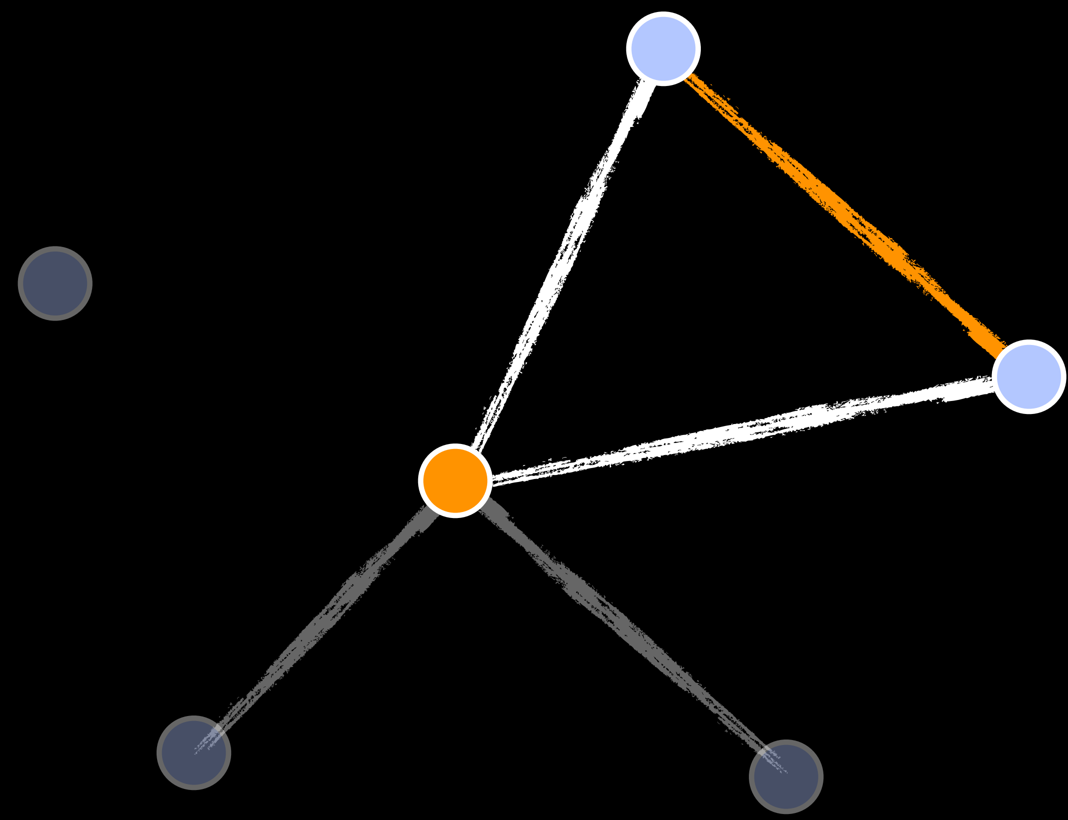
Clustering Coefficient



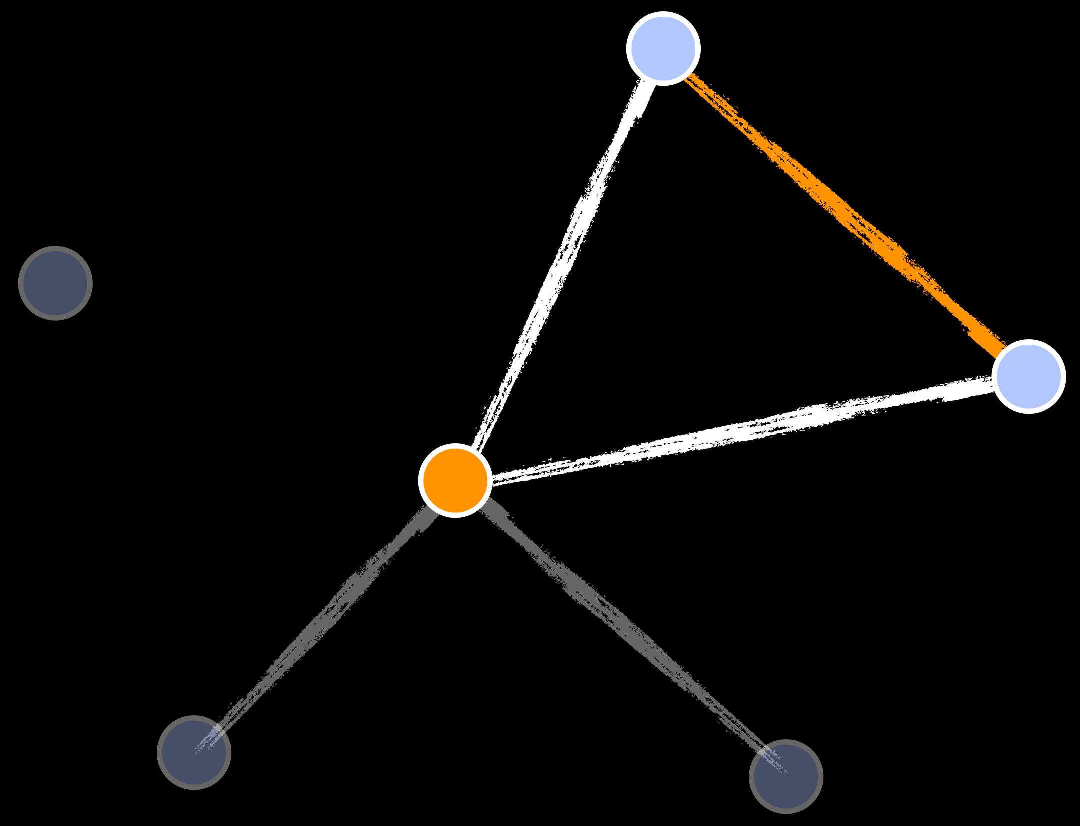
Clustering Coefficient



Clustering Coefficient

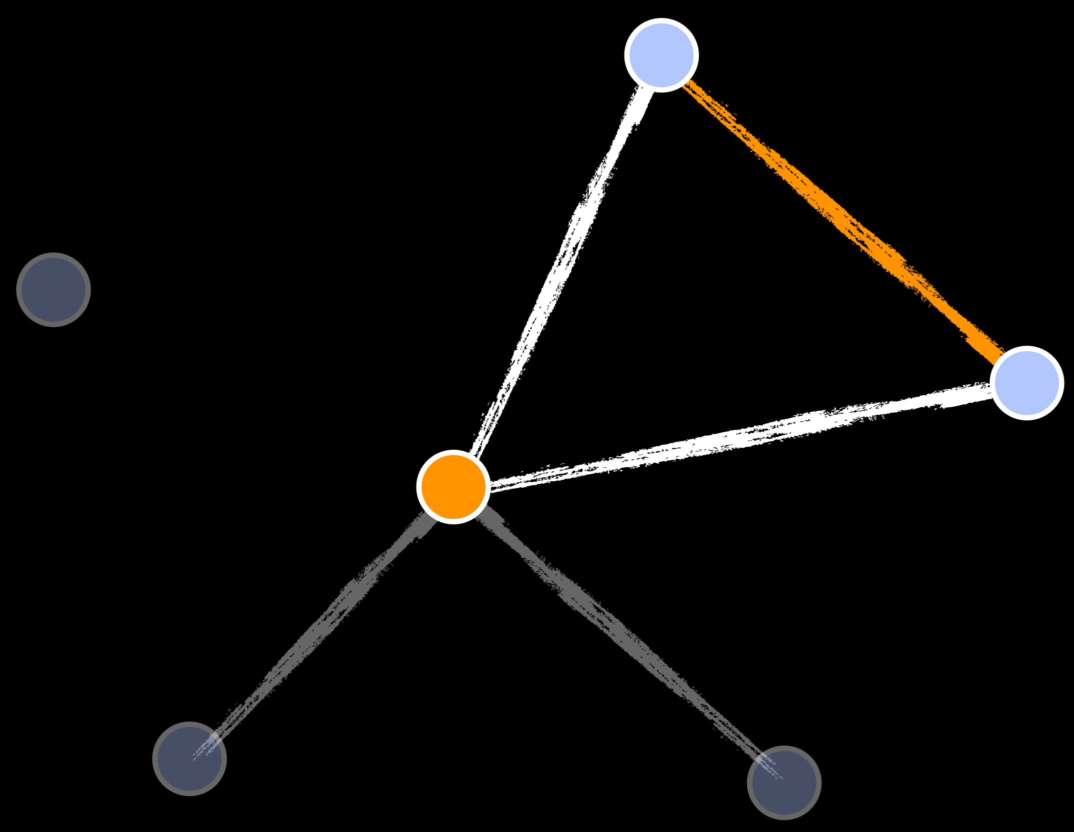


Clustering Coefficient



- probability that neighbours of a node are also connected

Clustering Coefficient

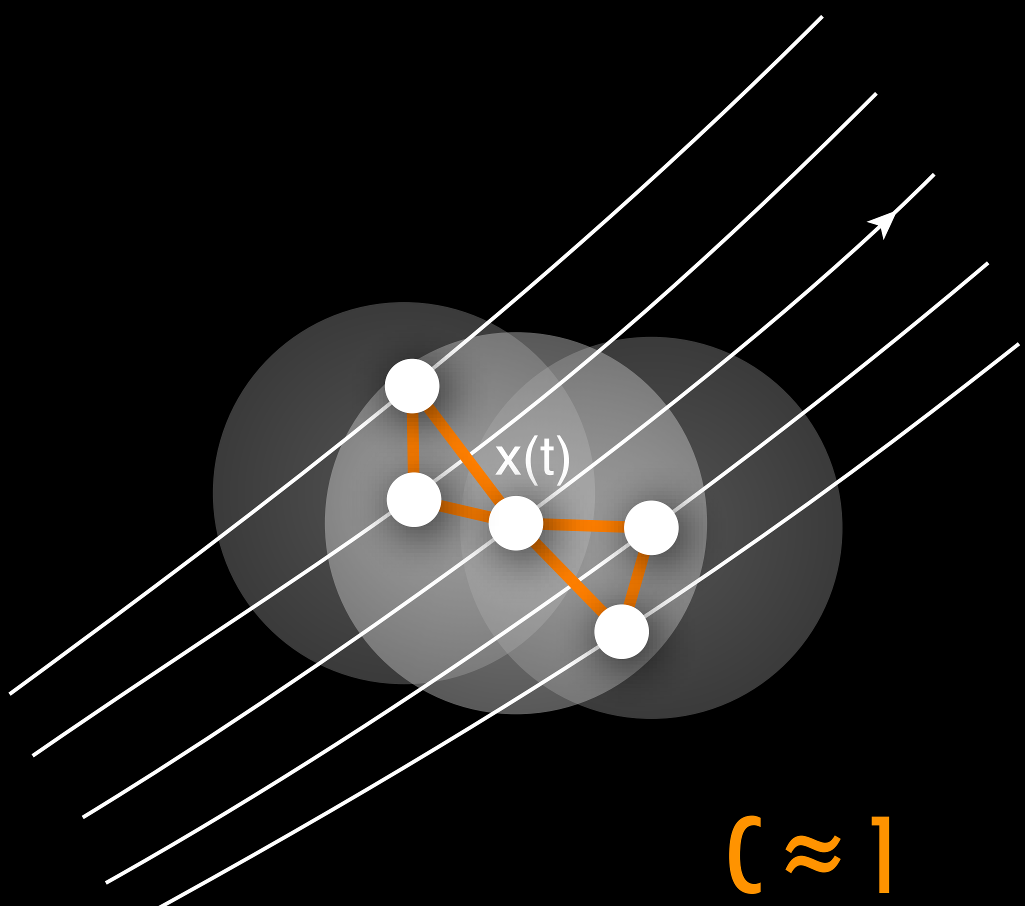


$$C_v = \frac{\sum_{i,j} A_{v,i} A_{i,j} A_{j,v}}{k_v(k_v - 1)}$$

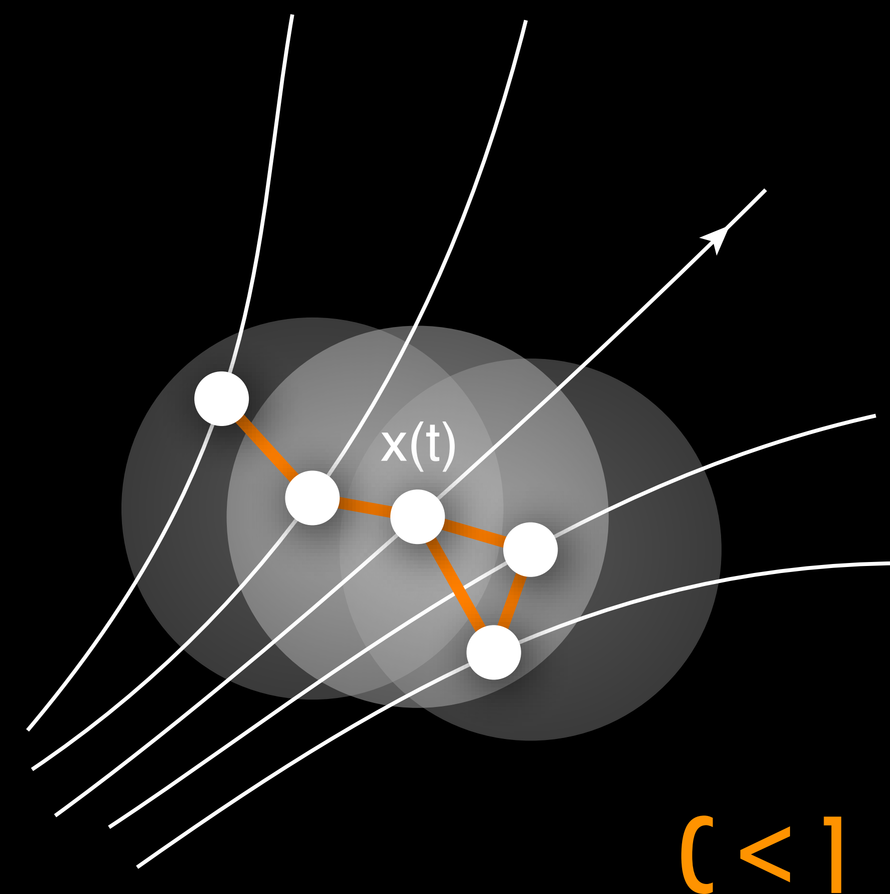
- probability that neighbours of a node are also connected

Clustering Coefficient in Phase Space

Regular/ periodic

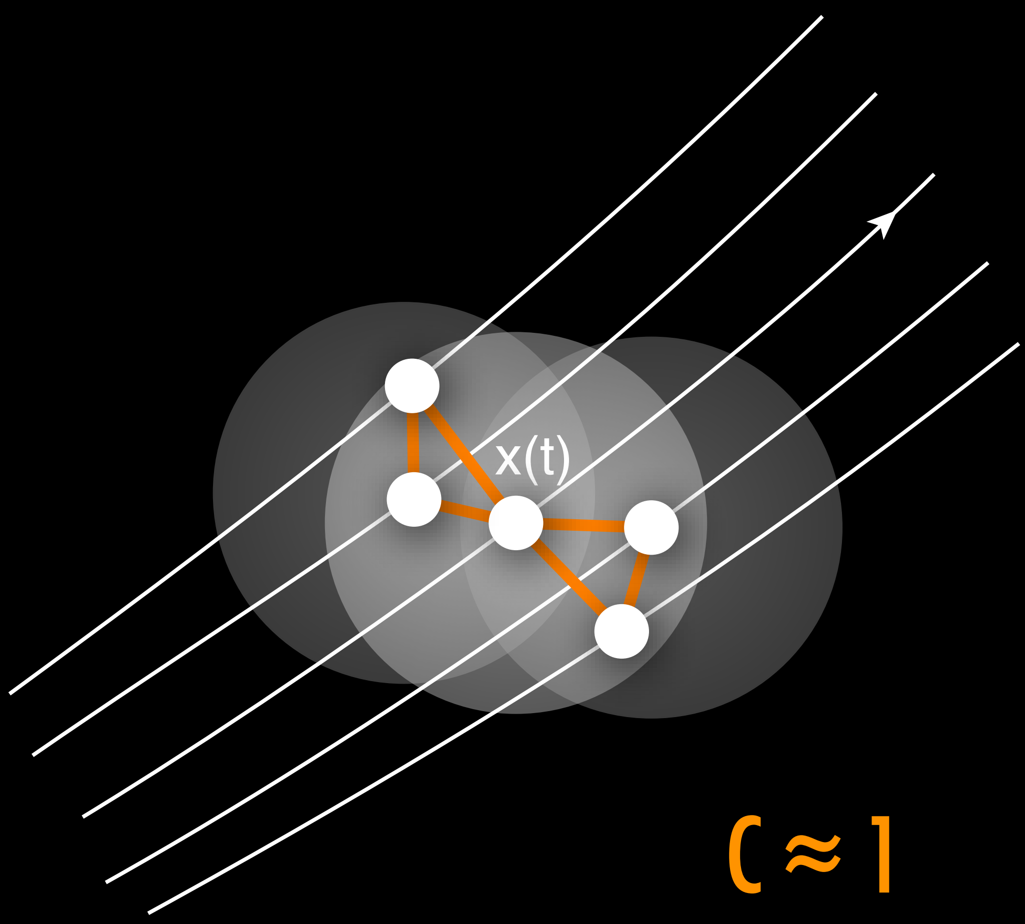


Diverging/ chaotic



Clustering Coefficient in Phase Space

Regular/ periodic



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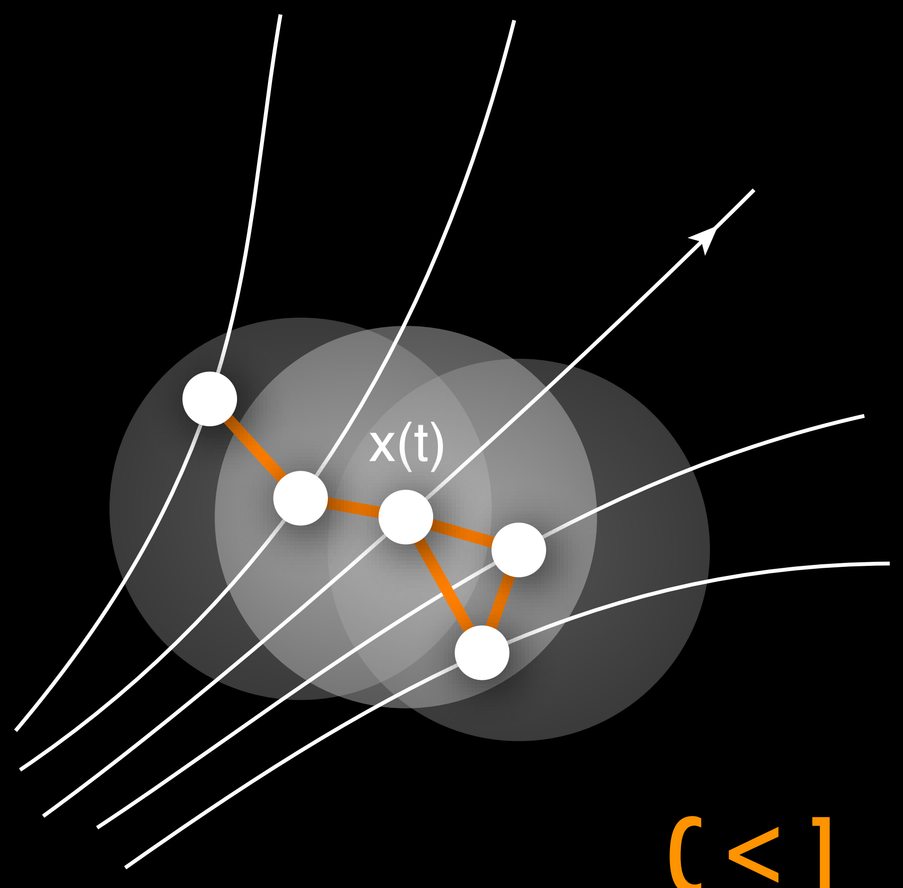
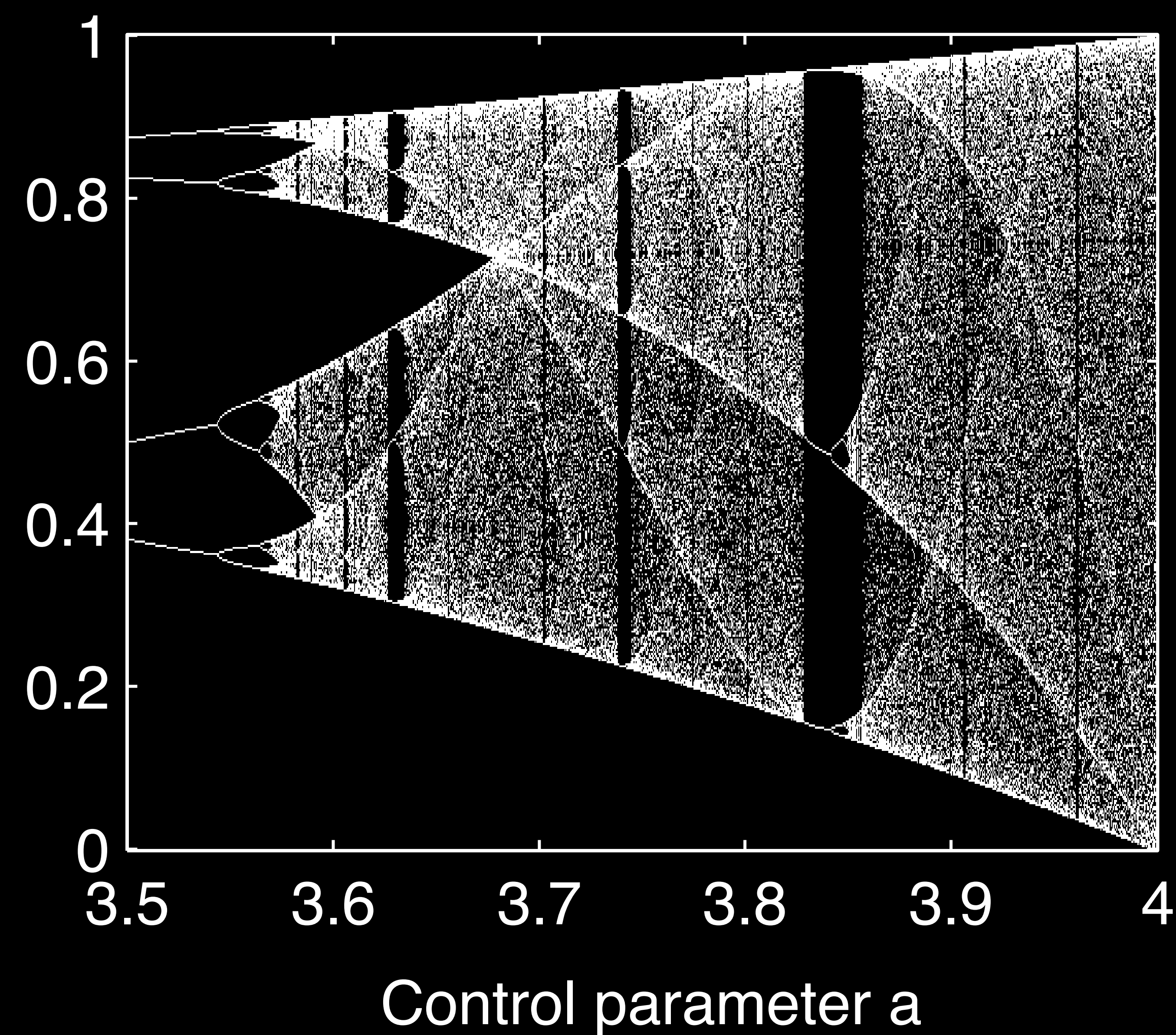


- clustering coefficient: regularity of dynamics, system dimension

Example: Logistic Map

- Logistic map:

$$x_{i+1} = a\, x_i(1 - x_i)$$



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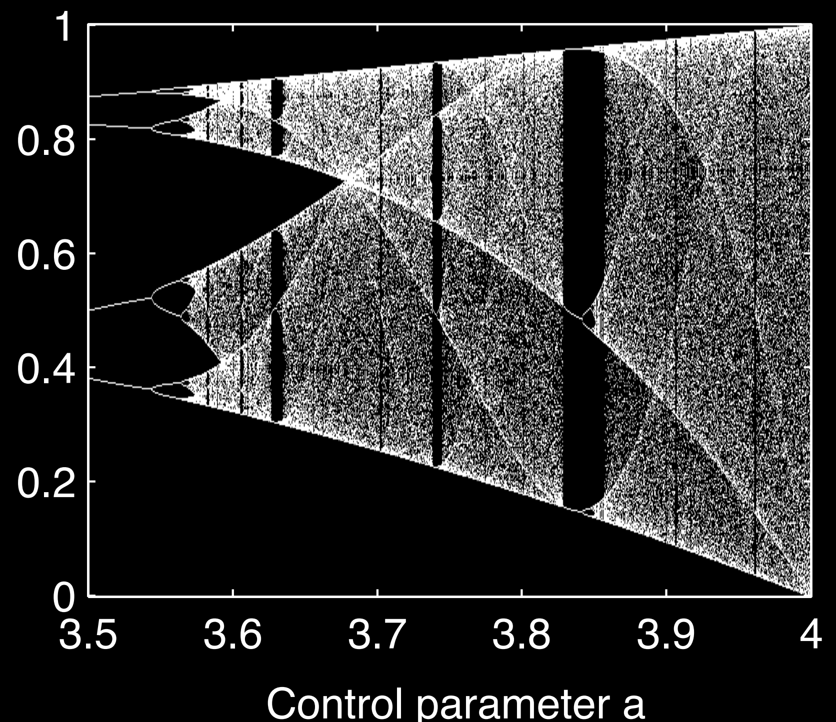

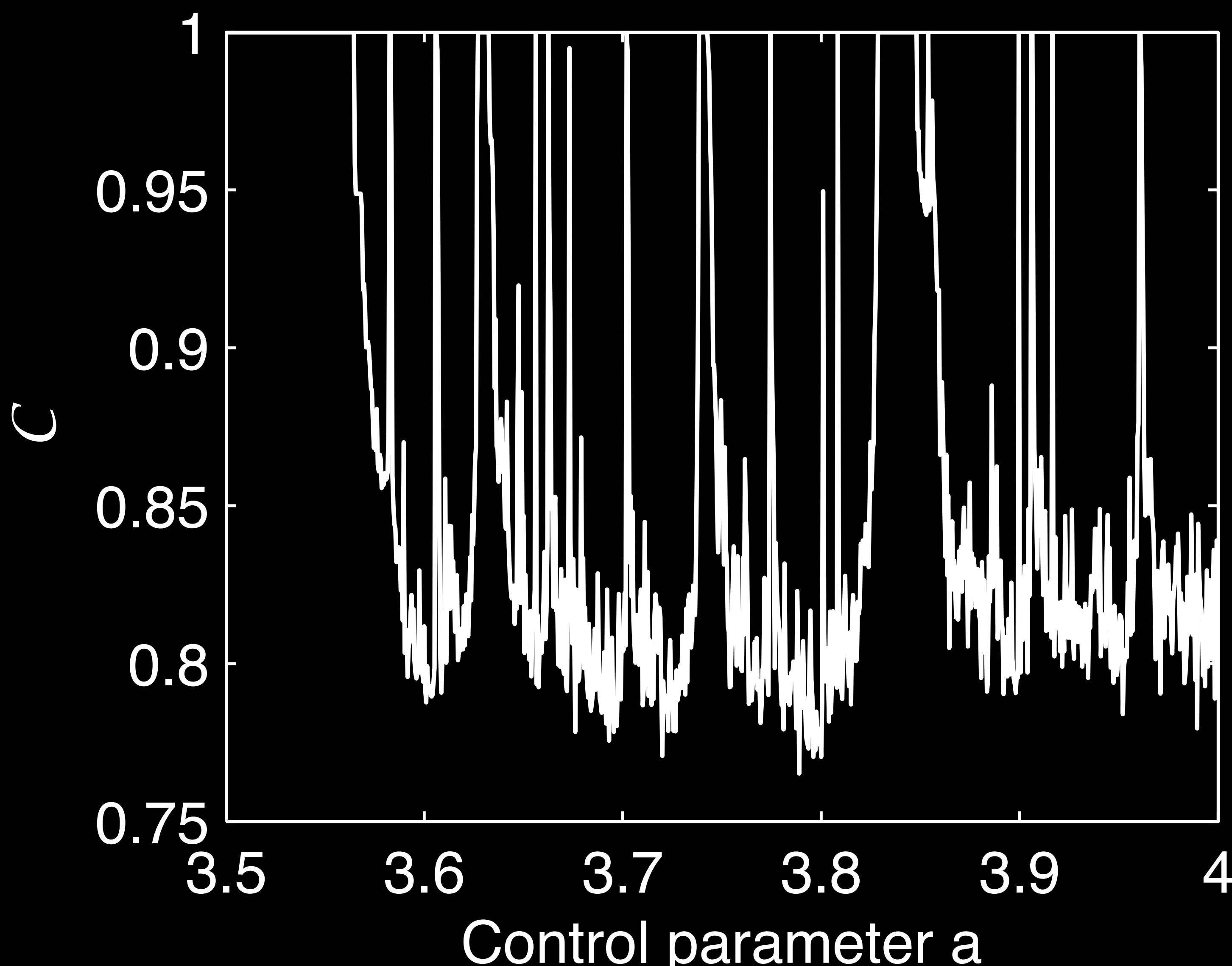

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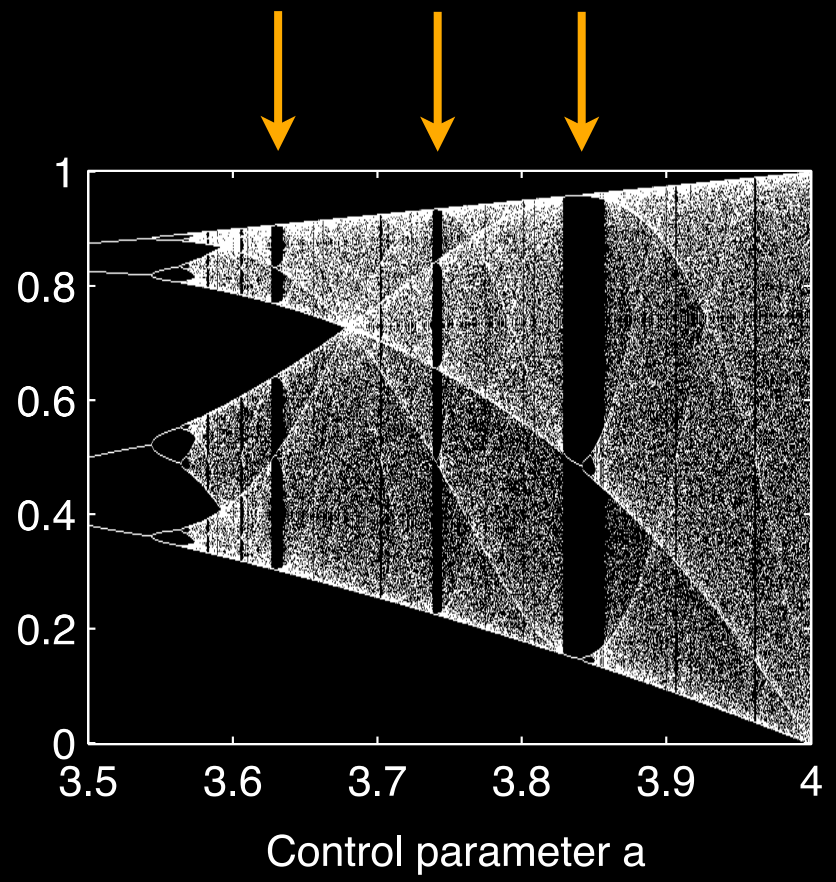
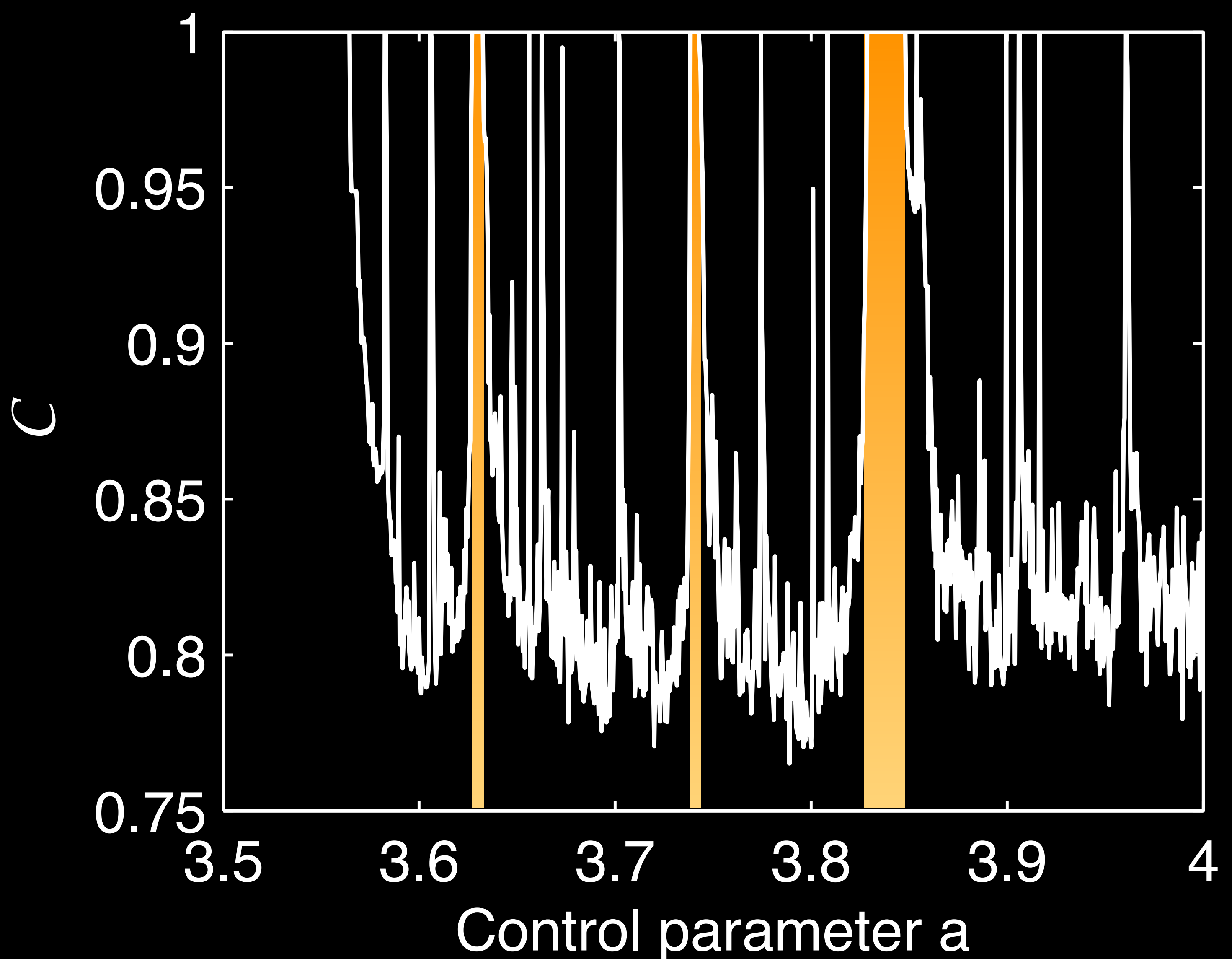
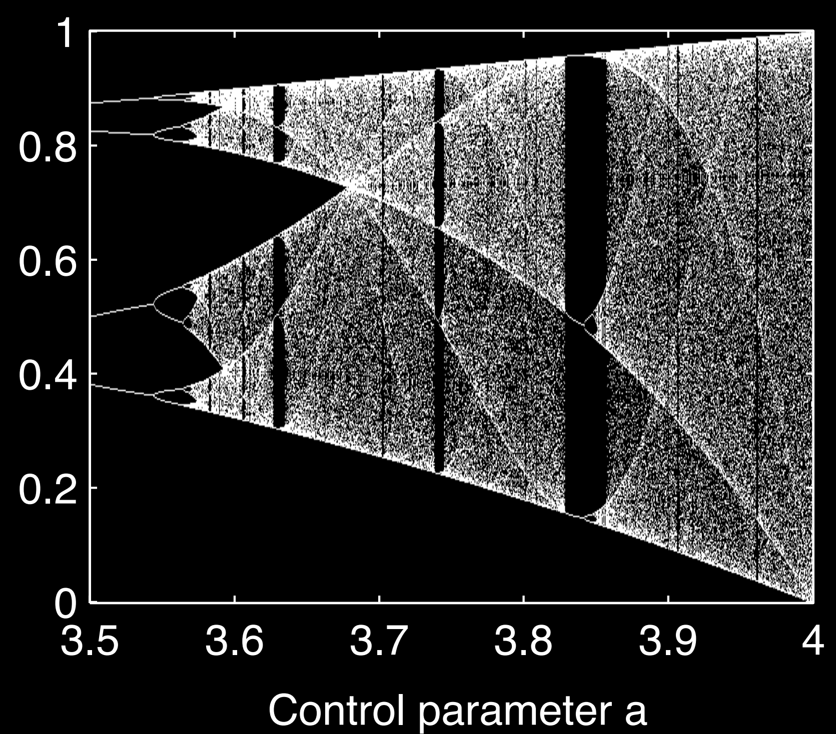
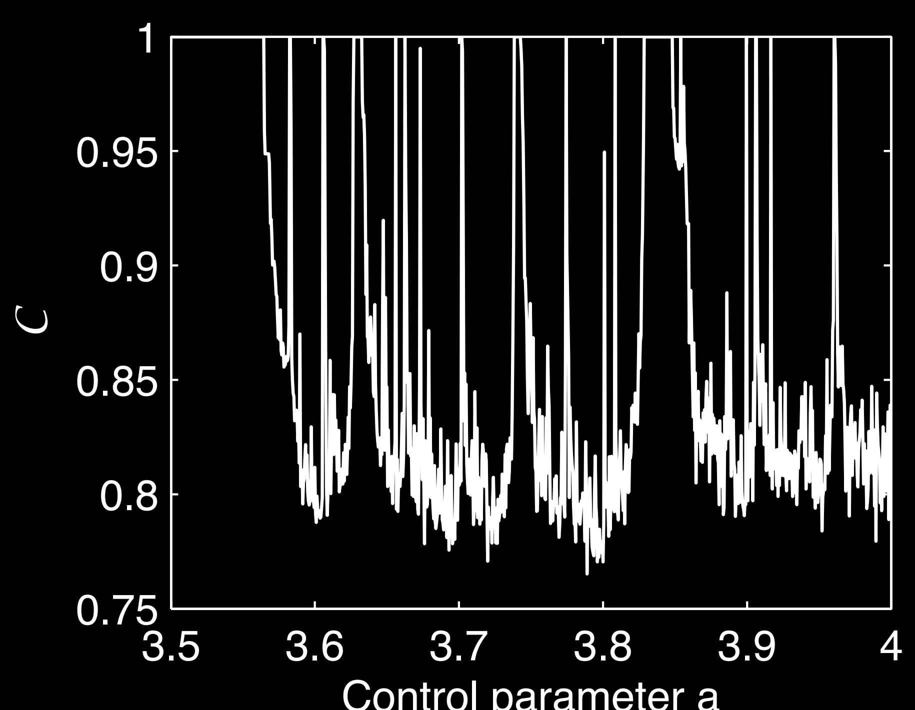




- periodic windows

Example: Logistic Map





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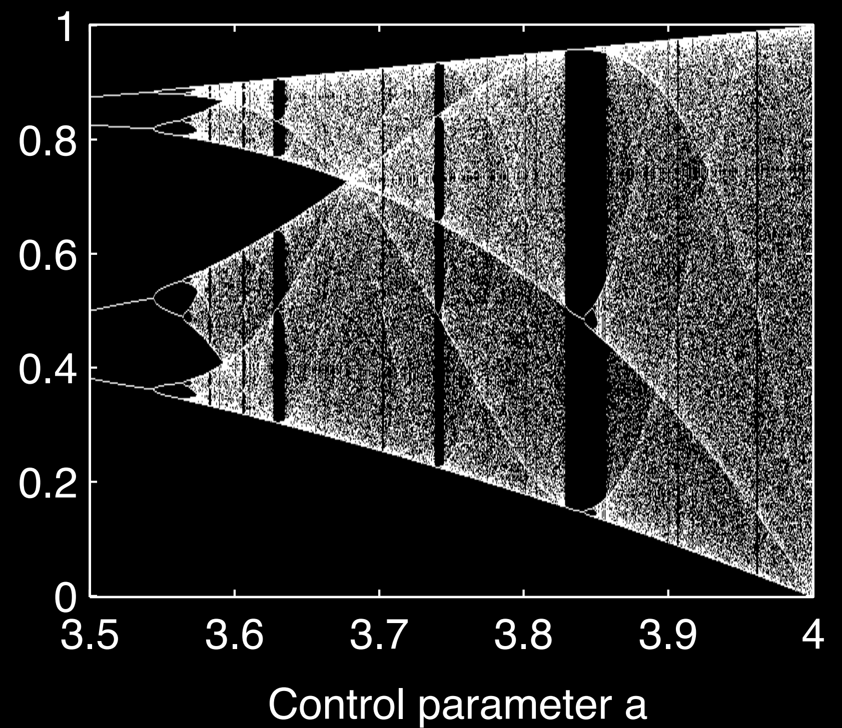
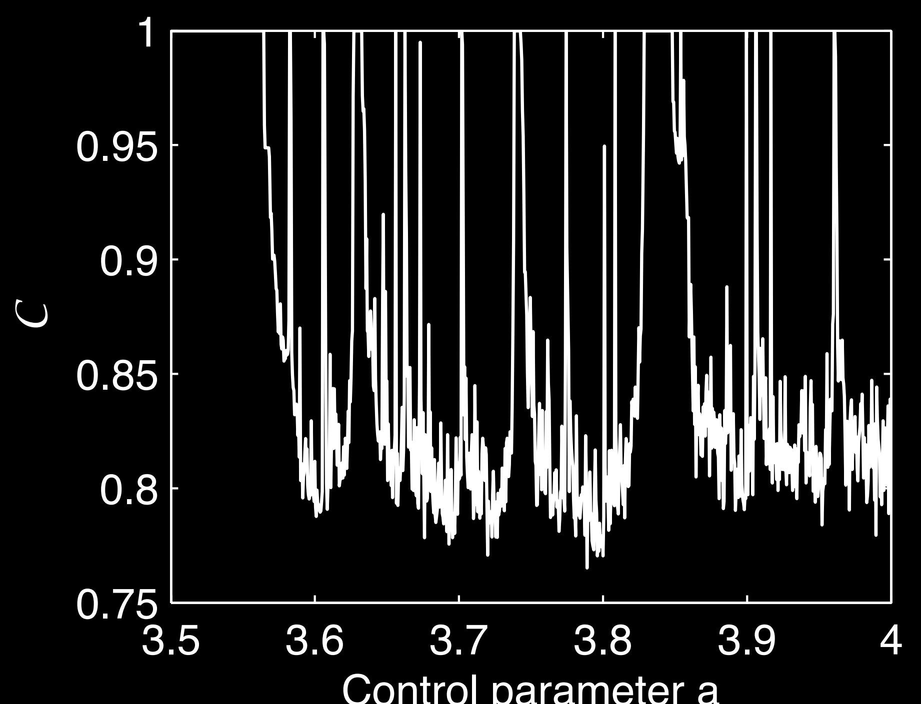
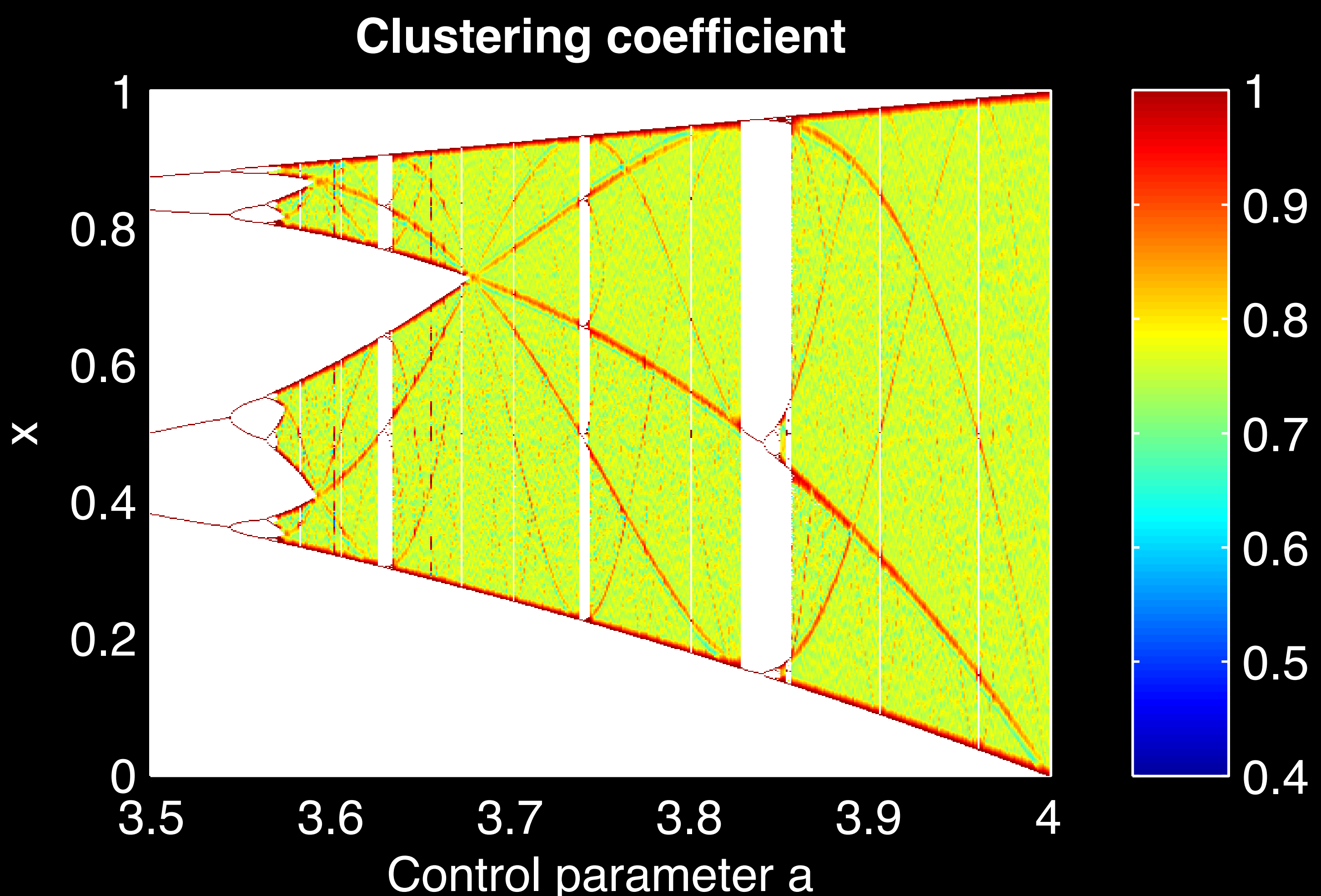
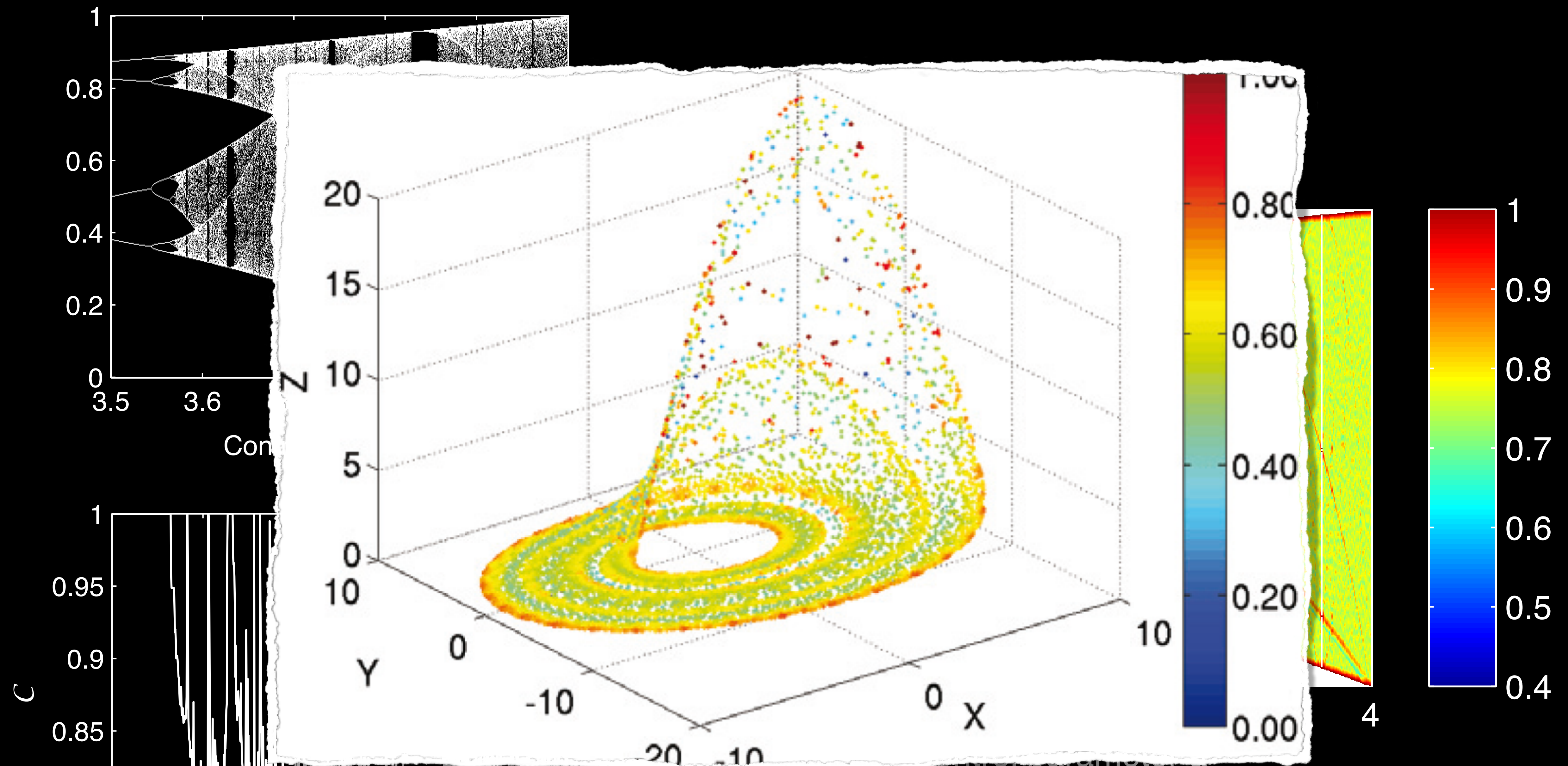


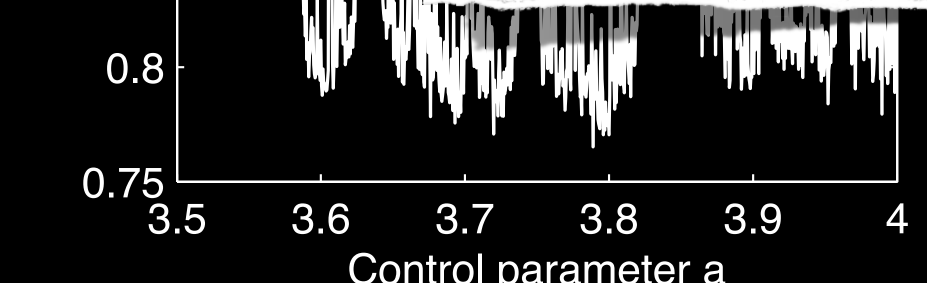




- intermittent phases

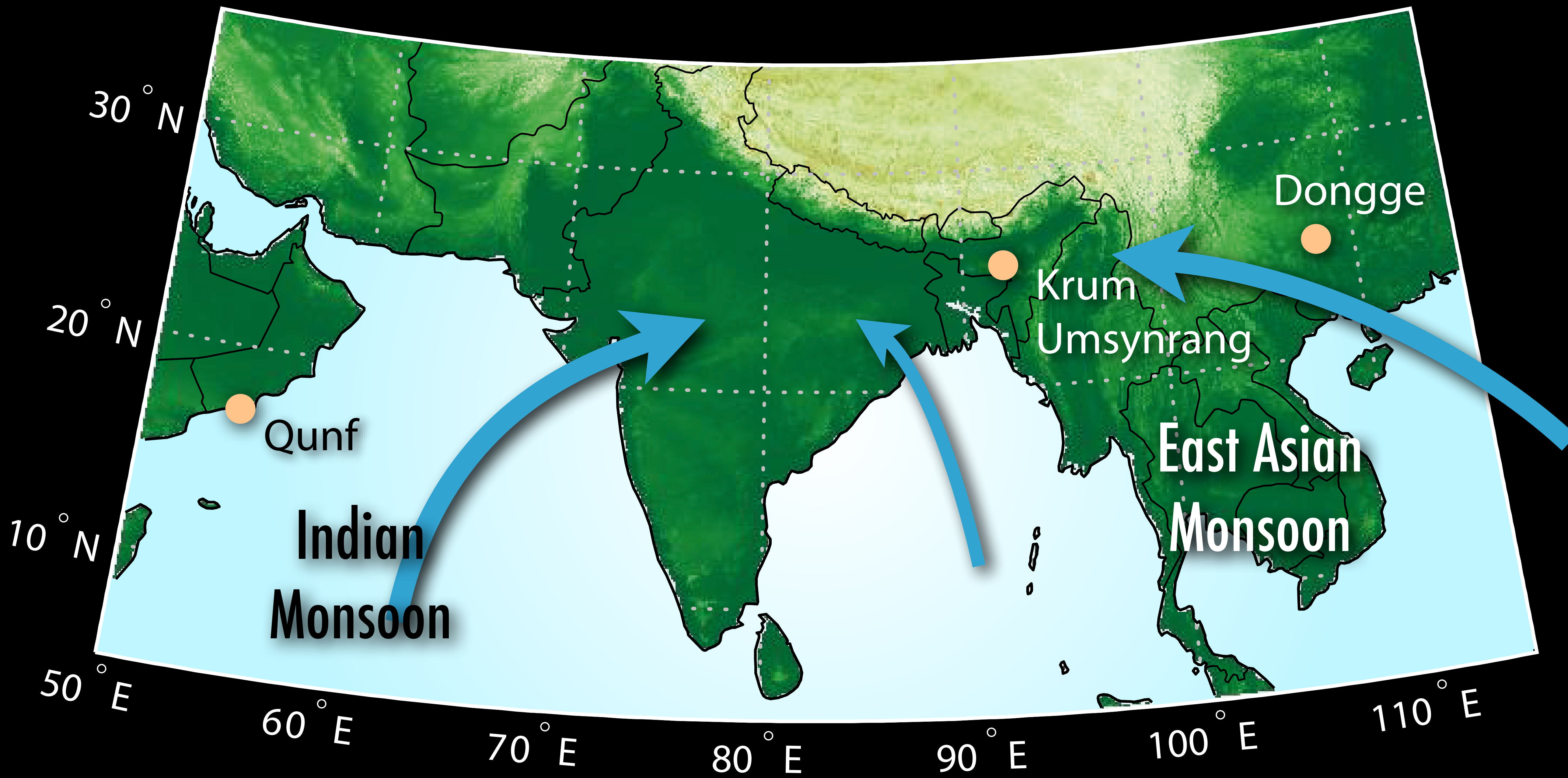
Example: Logistic Map

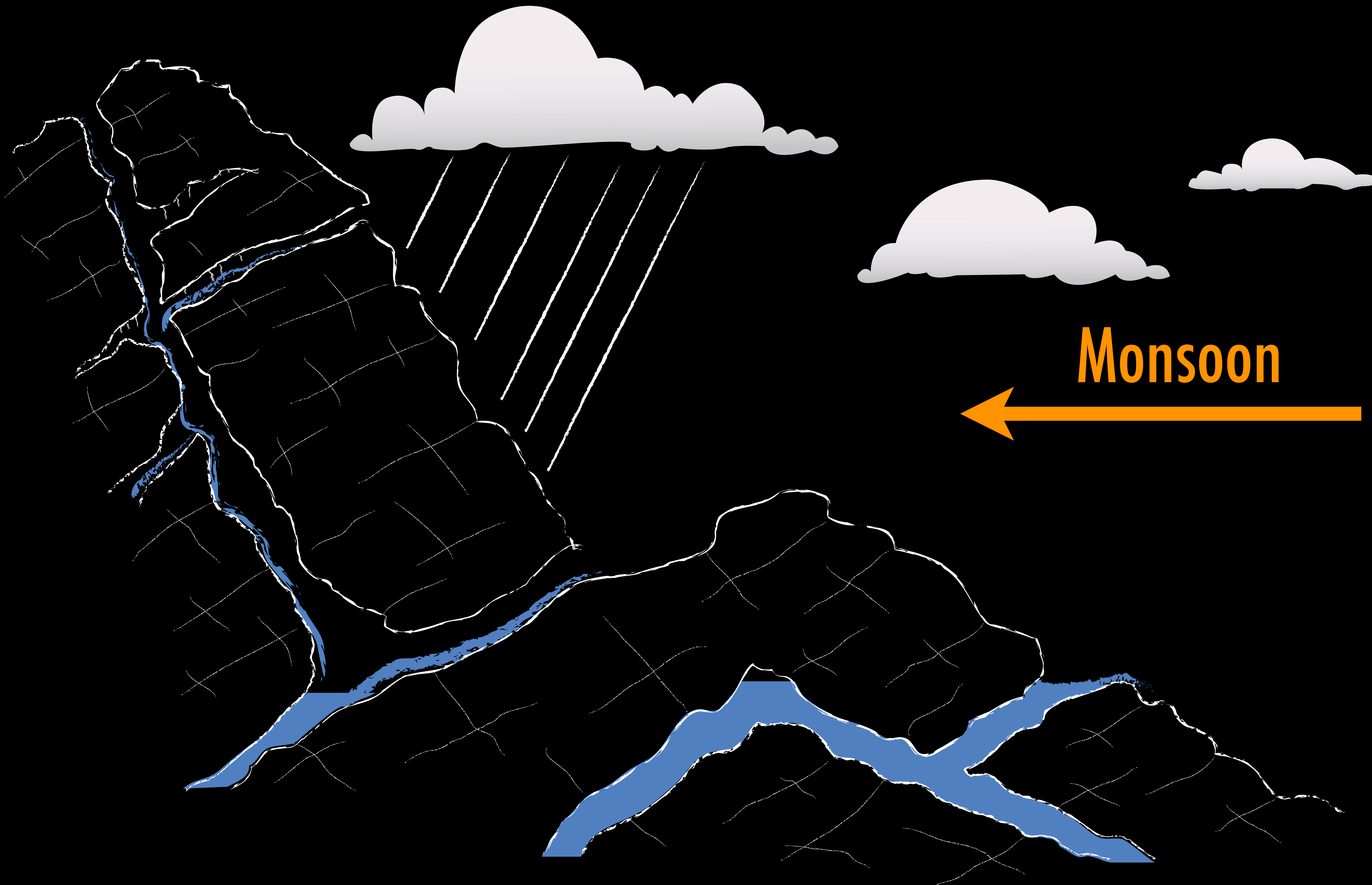


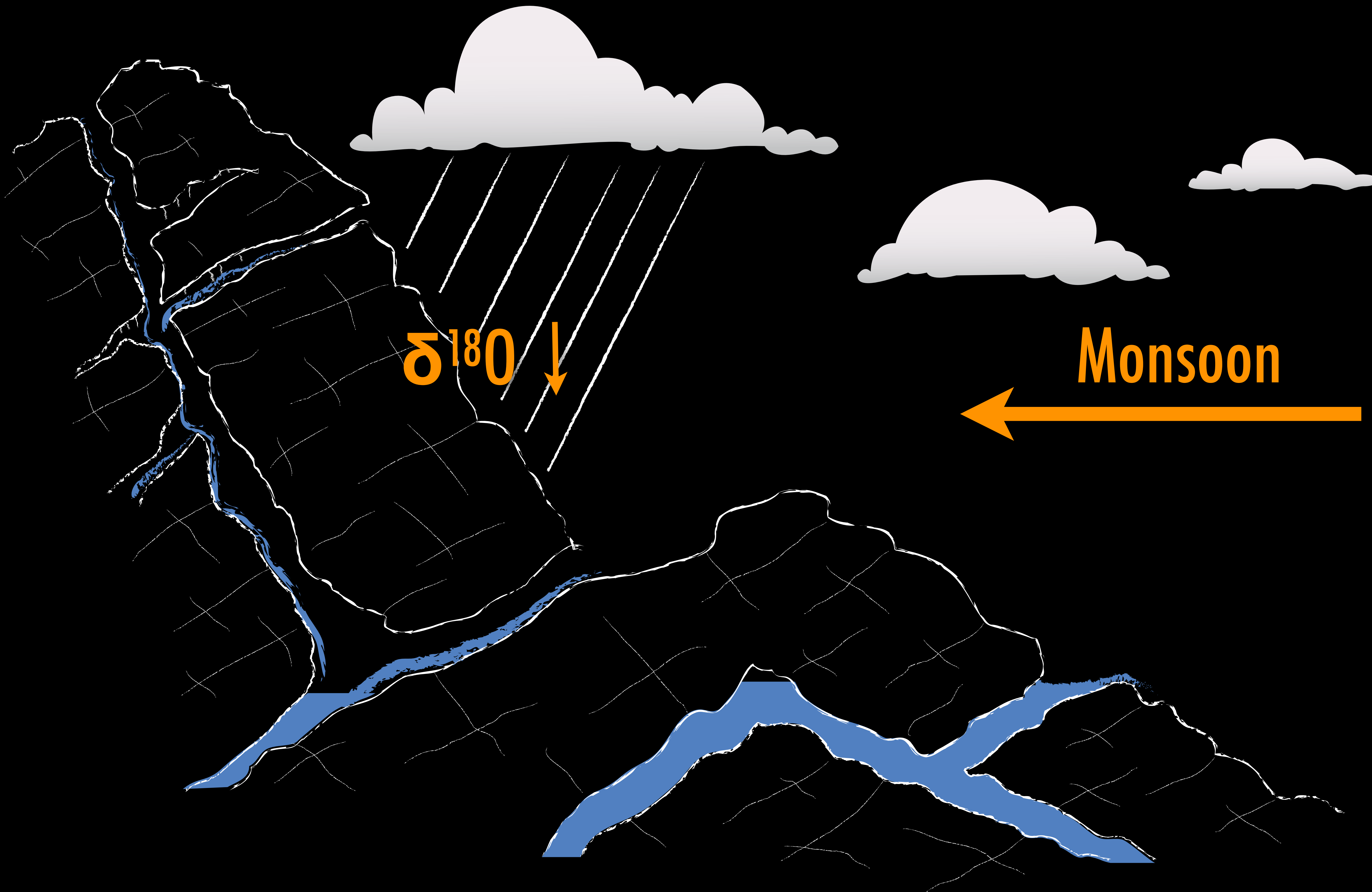


- intermittent phases

Asian Monsoon




Monsoon

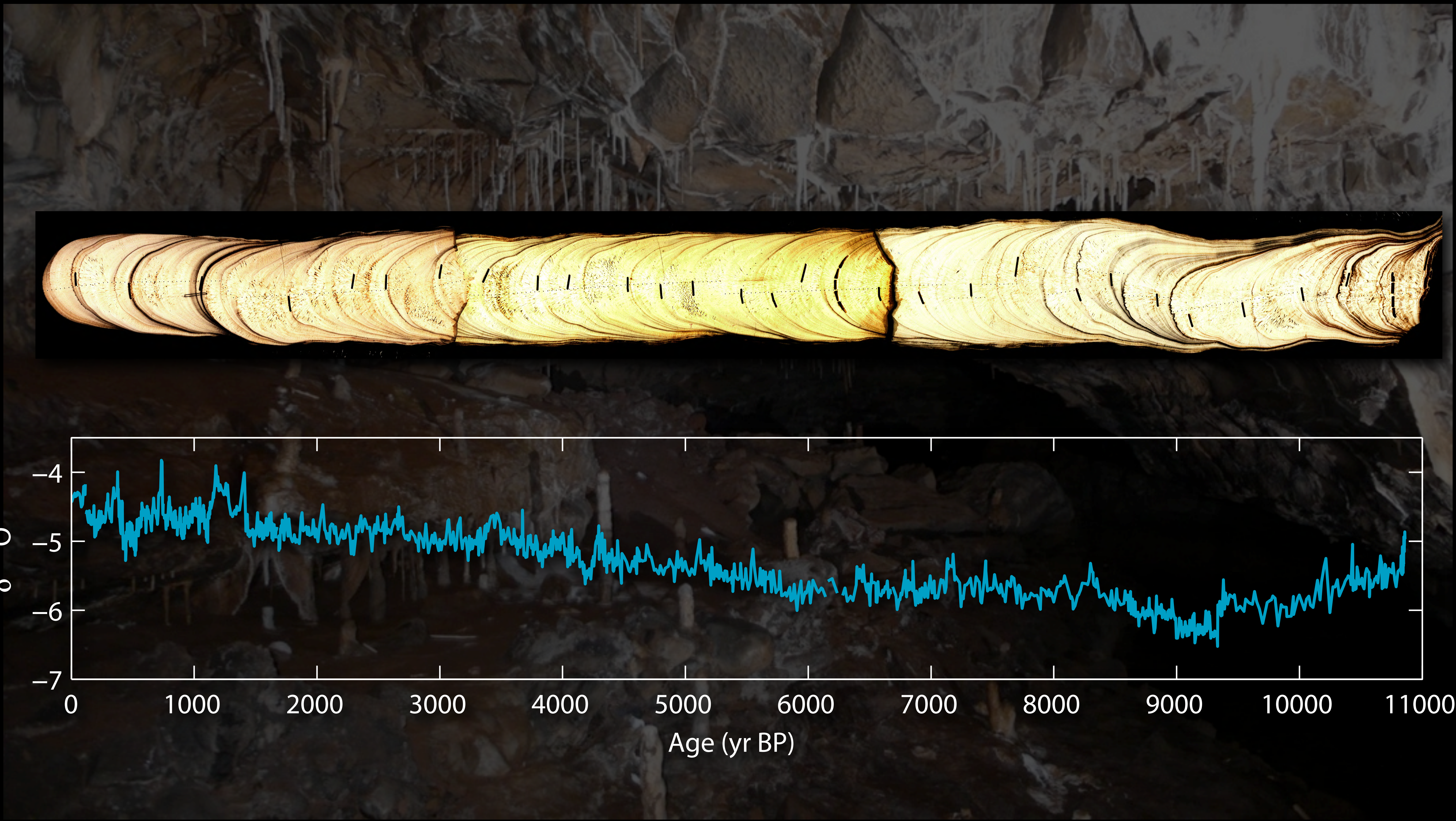


$\delta^{18}O$ ↓
Monsoon

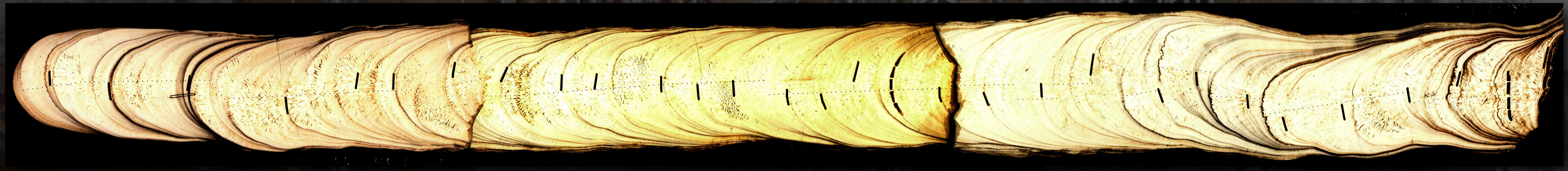
Asian Monsoon

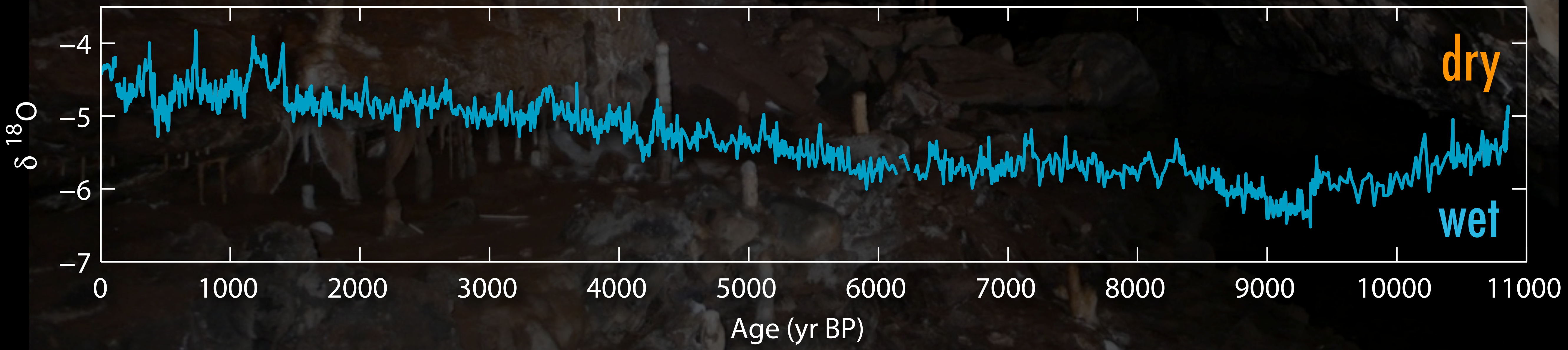
Asian Monsoon

Asian Monsoon



Asian Monsoon





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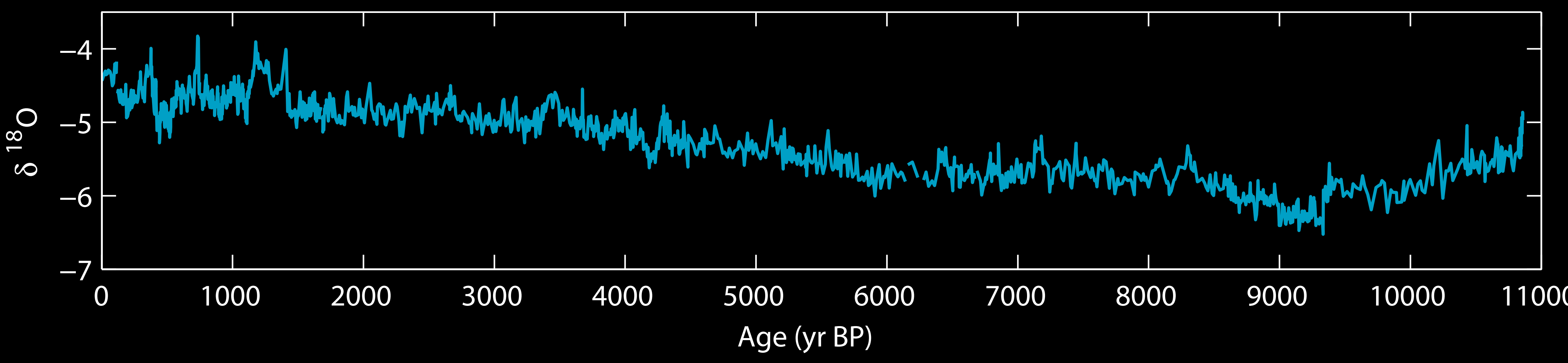

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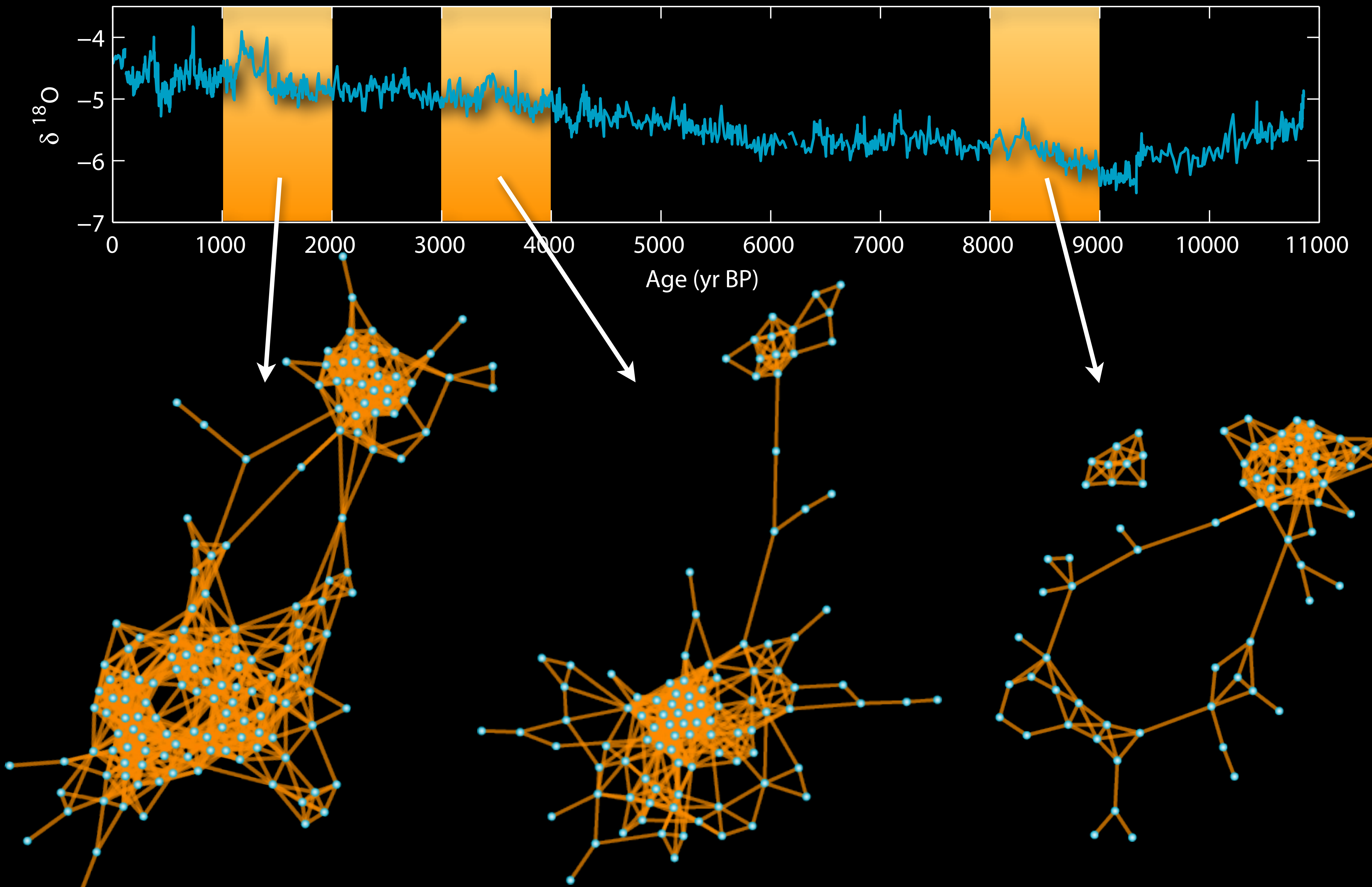

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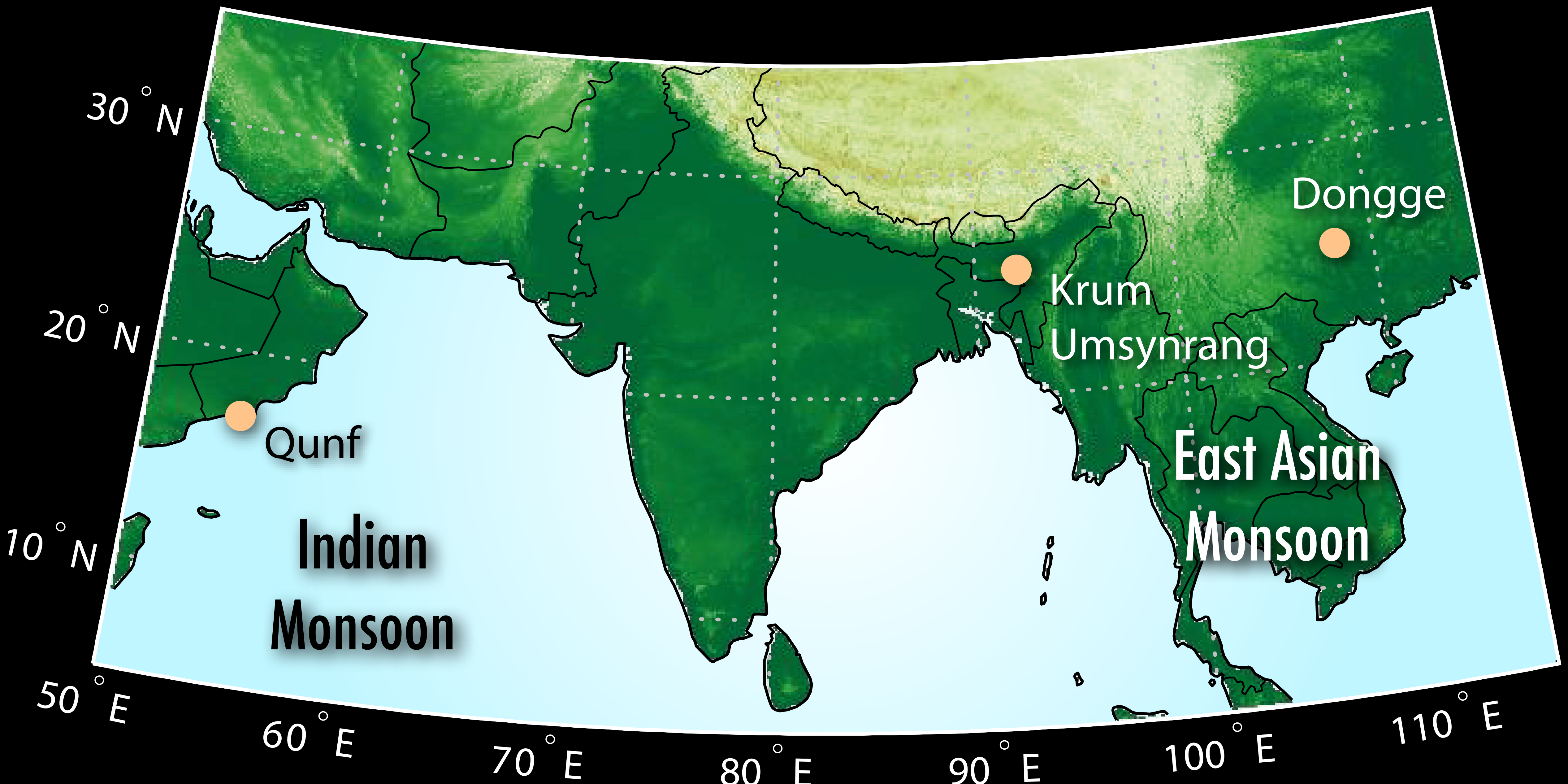
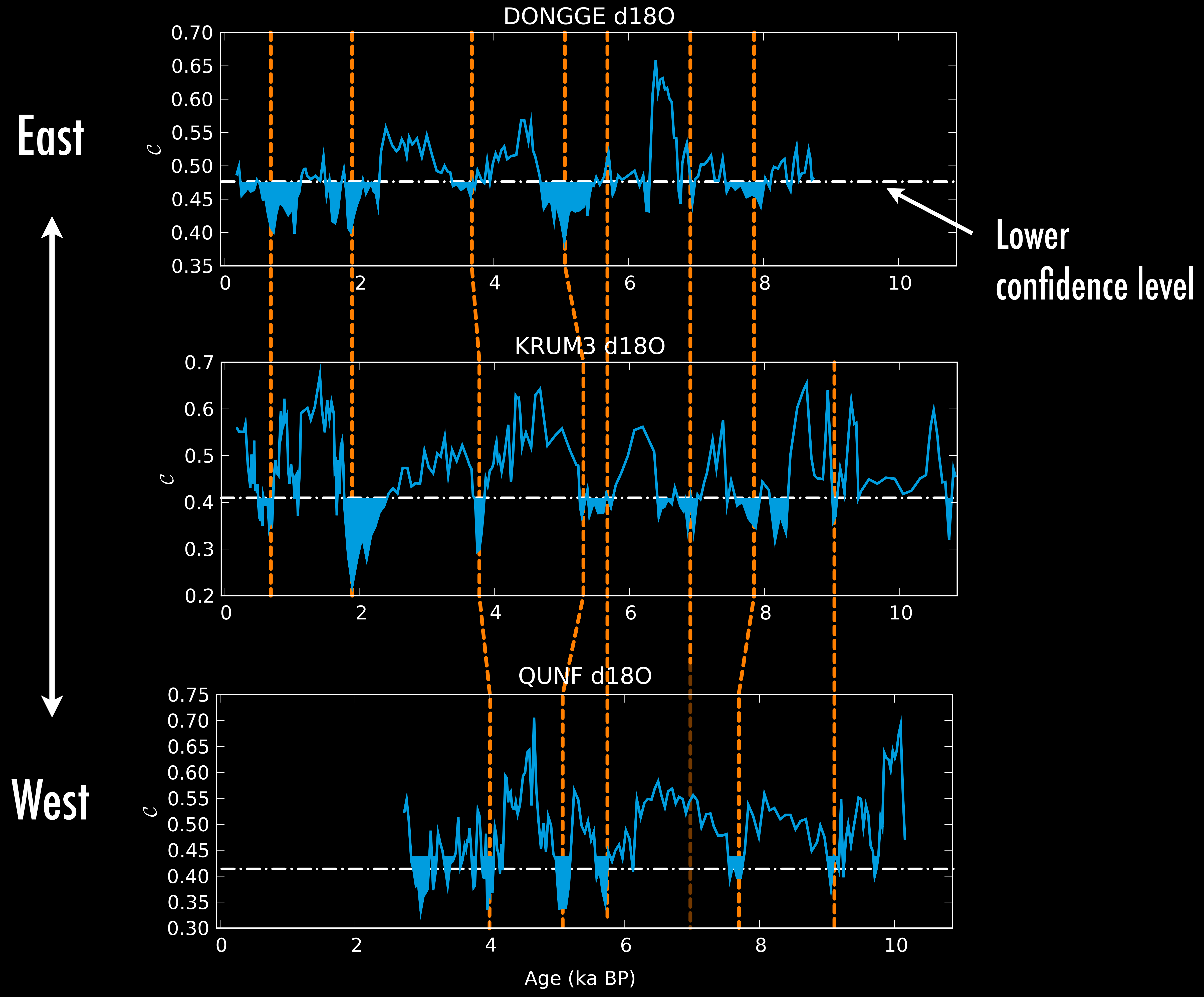
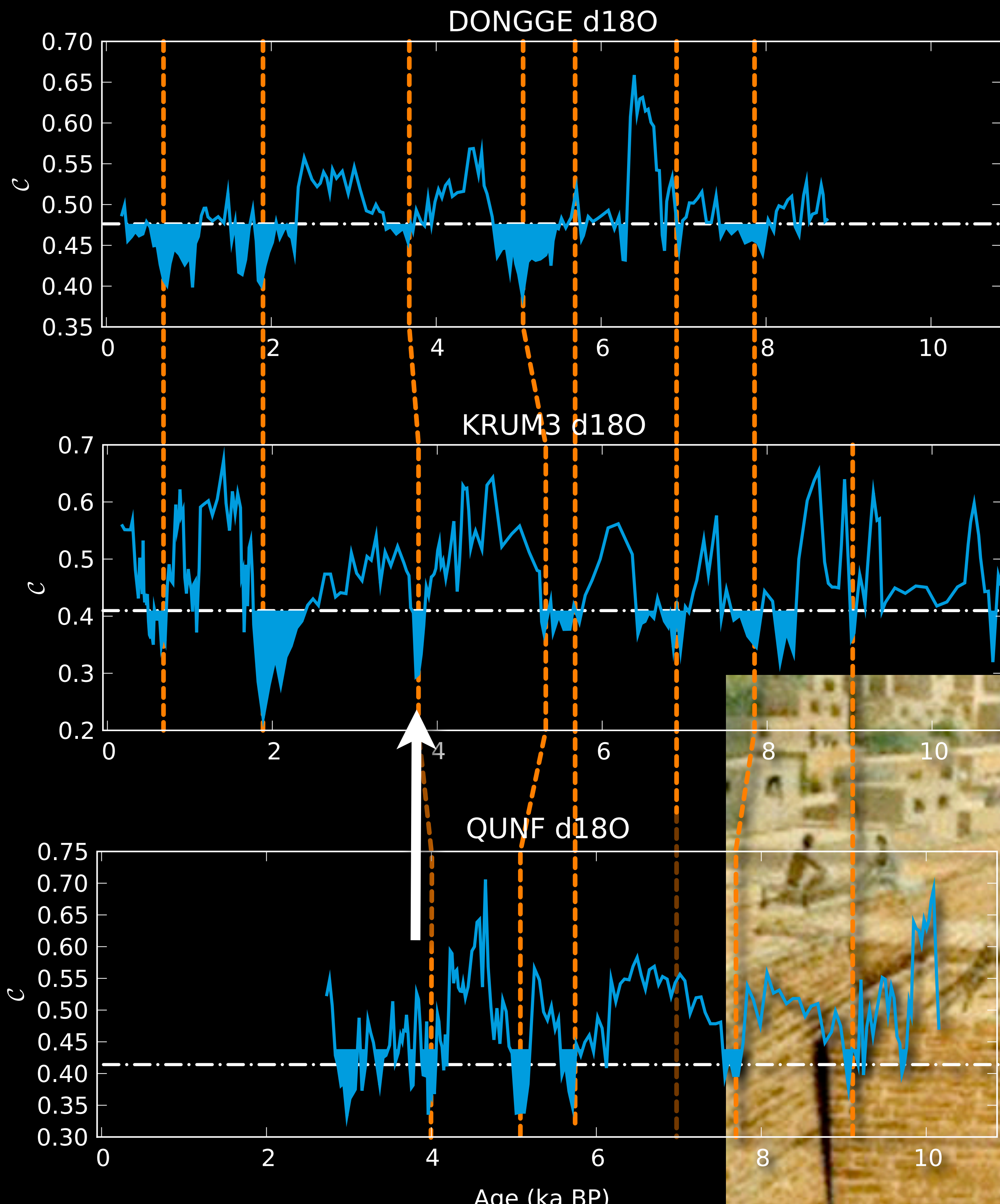
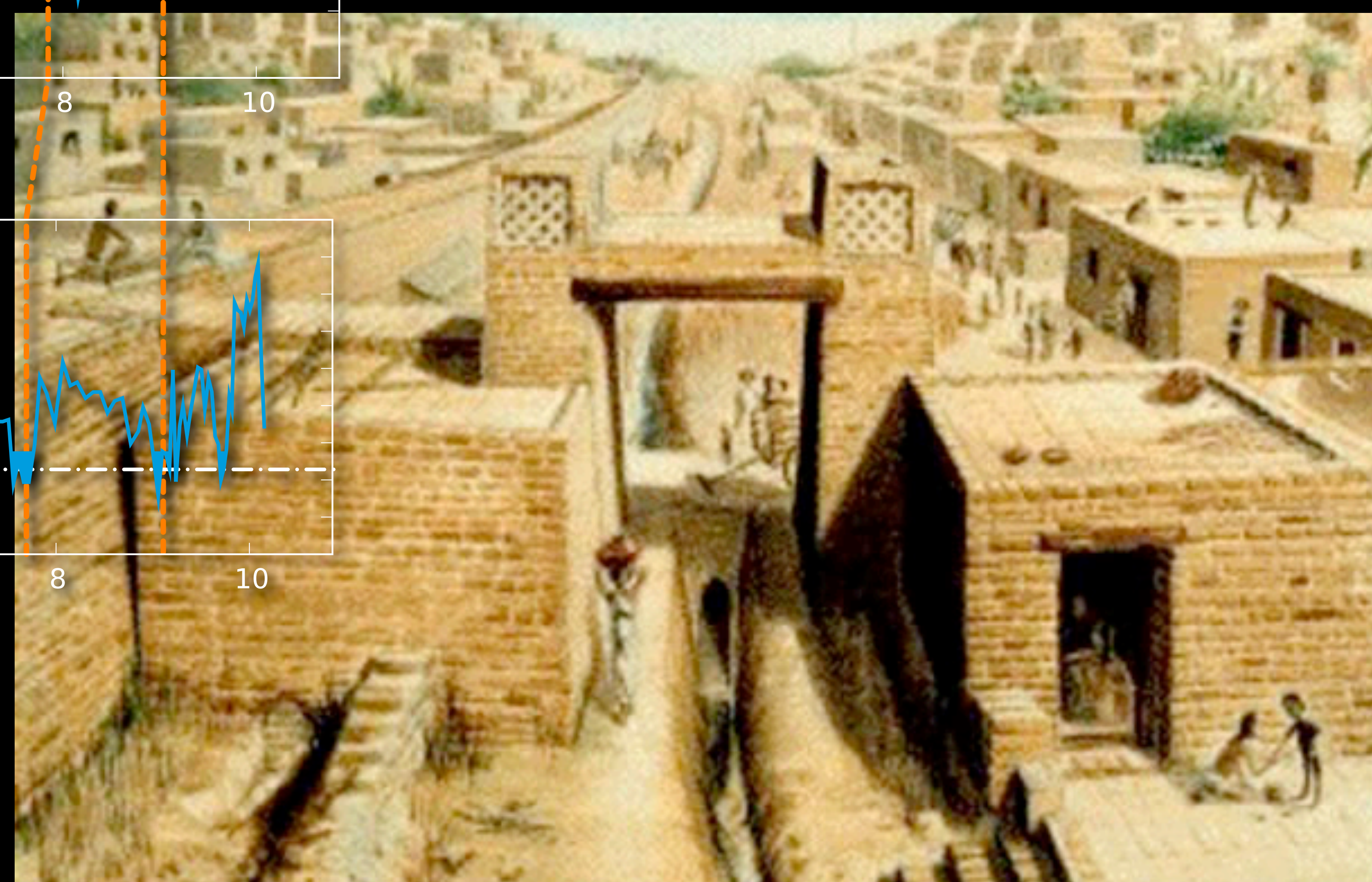



East
West
DONGGE d18O
KRUM3 d18O
QUNF d18O
c
Age (ka BP)
Lower
confidence level



3900-3700 yr BP
Harappan culture vanished

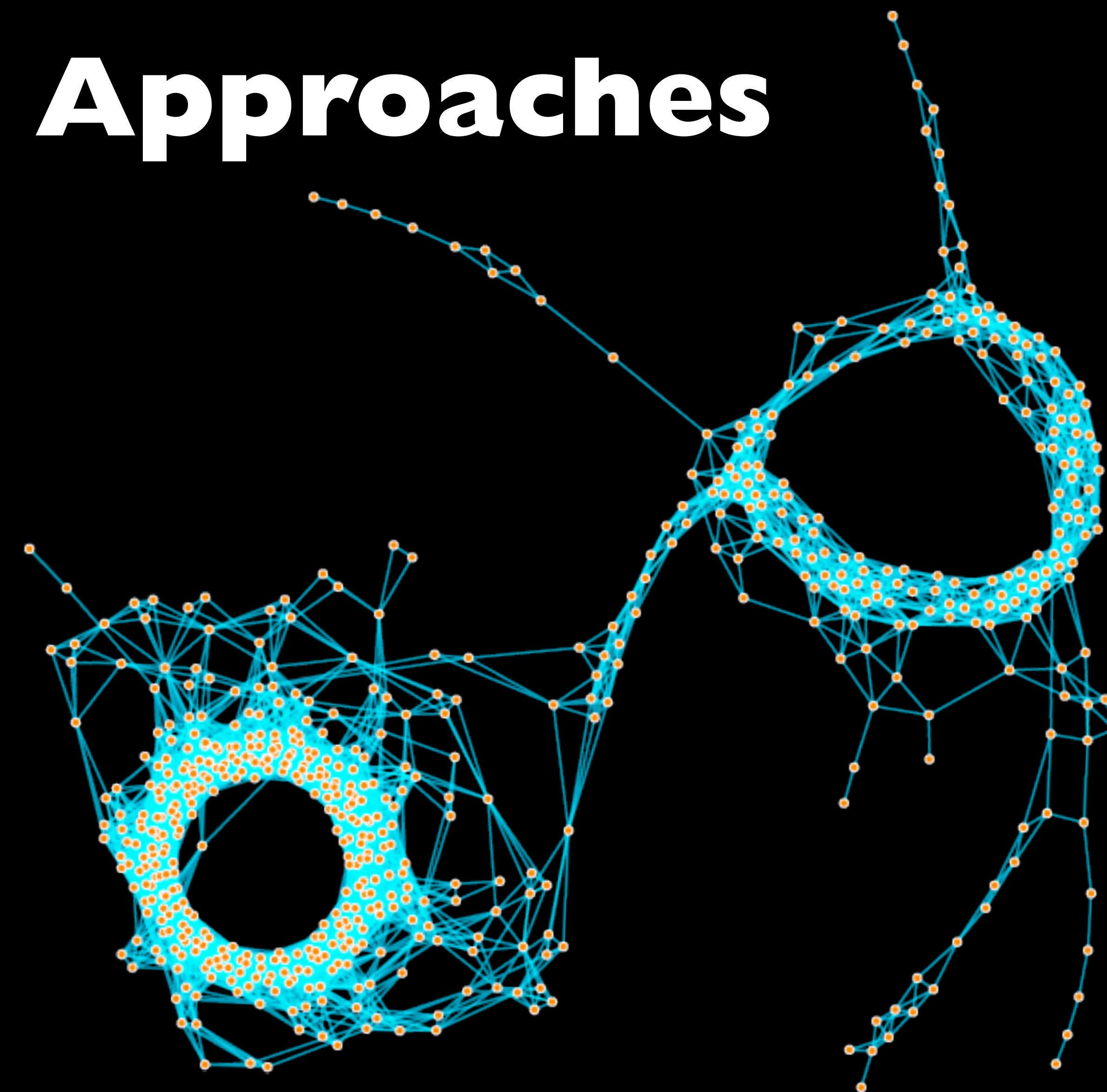


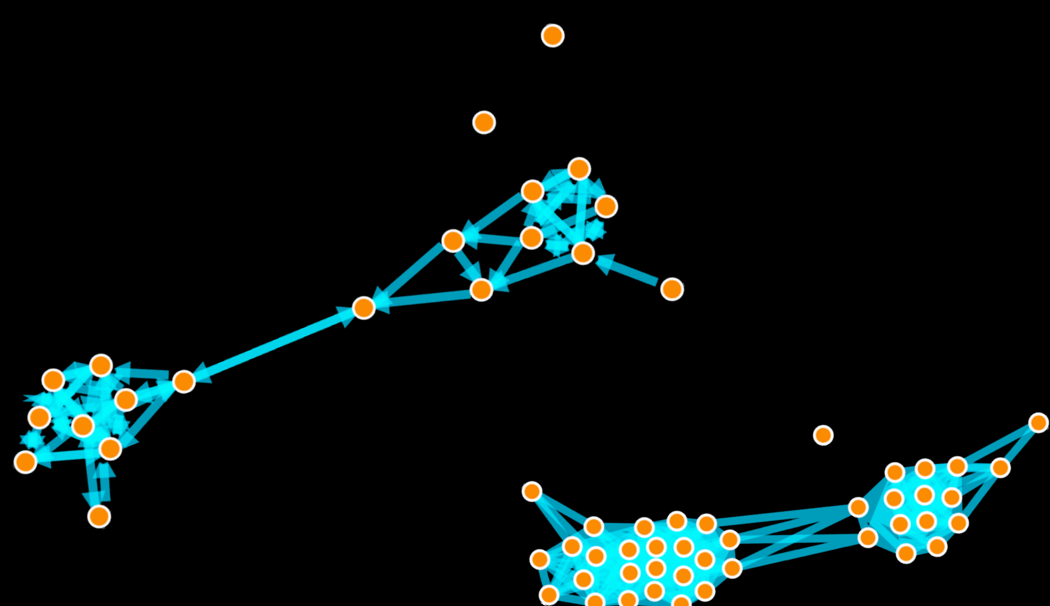
Summary

- Complex networks from time series
- Identification and classification of dynamics (regular – chaotic)
- Detection of transitions in dynamics (bifurcations, structural discontinuities)
- Complementary analysis to traditional recurrence analysis

Alternative Approaches

- Visibility graph
- Cycle network
- Correlation network
- Transition network





Key Publications

- N. Marwan, J. F. Donges, Y. Zou, R. V. Donner, J. Kurths: Complex network approach for recurrence analysis of time series, Physics Letters A, 373(46), 4246–4254 (2009).
- J. F. Donges, R. V. Donner, M. H. Trauth, N. Marwan, H. J. Schellnhuber, J. Kurths: Nonlinear detection of paleoclimate-variability transitions possibly related to human evolution, Proceedings of the National Academy of Sciences, 108(51), 20422–20427 (2011).
- R. V. Donner, M. Small, J. F. Donges, N. Marwan, Y. Zou, R. Xiang, J. Kurths: Recurrence-based time series analysis by means of complex network methods, International Journal of Bifurcation and Chaos, 21(4), 1019–1046 (2011).