Recurrence Plots for Spatial Data

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Outline

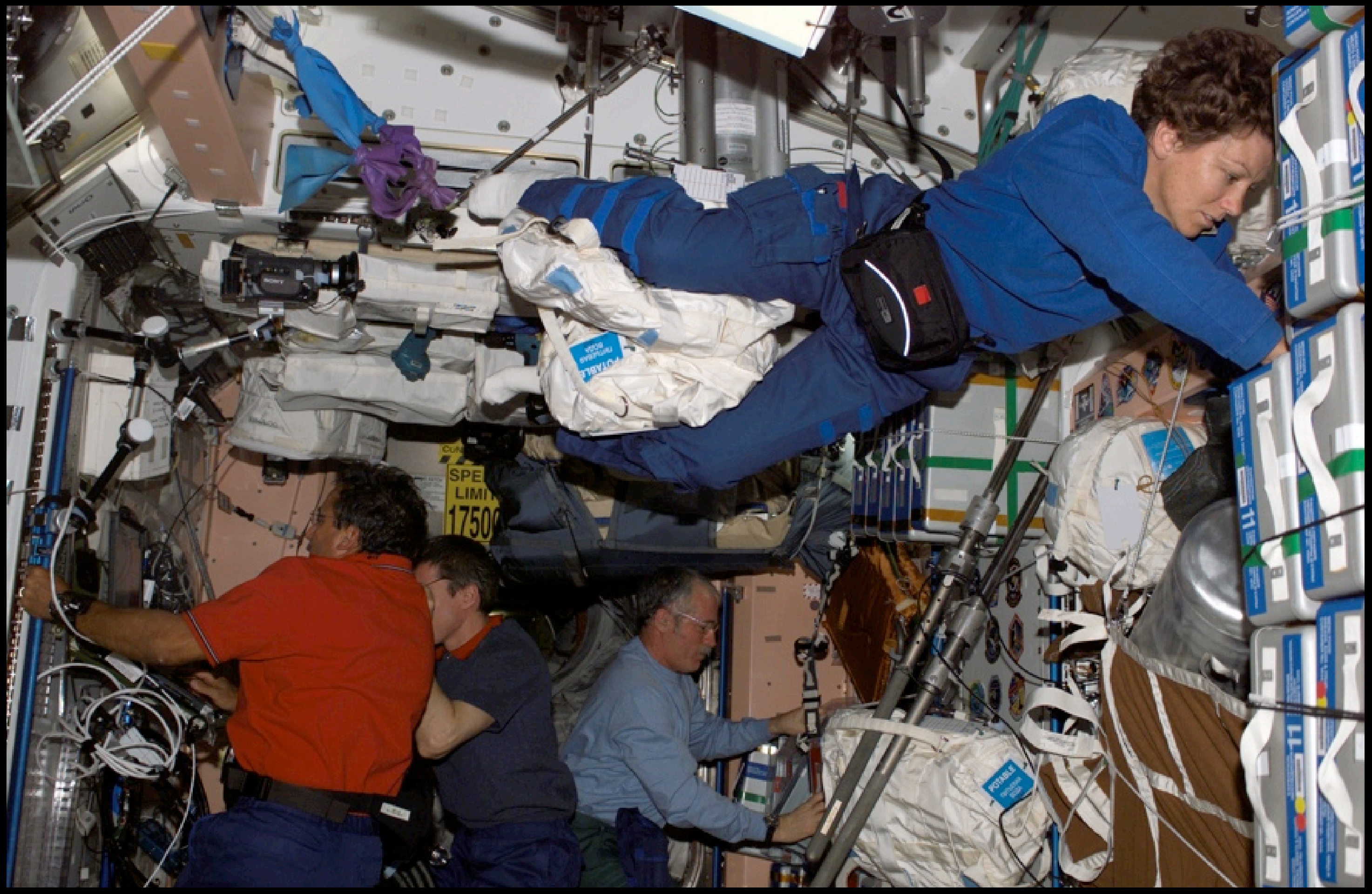
- Introduction
- Recurrence plots & quantification
- Spatial extension
- Application
- Conclusions

Introduction

Bone Loss in Space

- bone loss in space: 1.5% per month
- 2nd important problem after radiation

> monitoring bone alterations during space flights

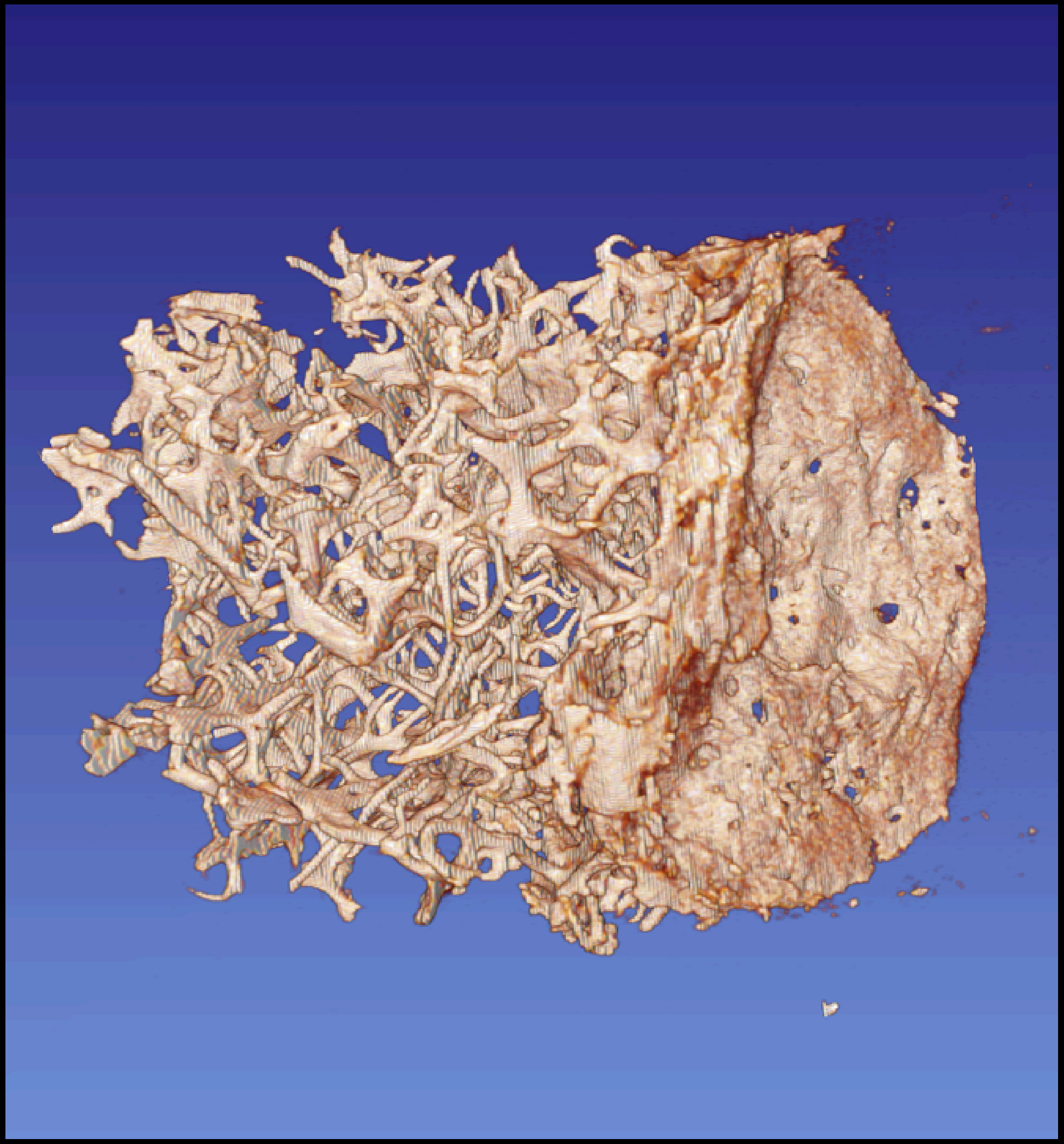


Trabecular Bone Structure

- plays important role for bone strength
- changes during development of osteoporosis or in micro-gravity

Purpose of this Study

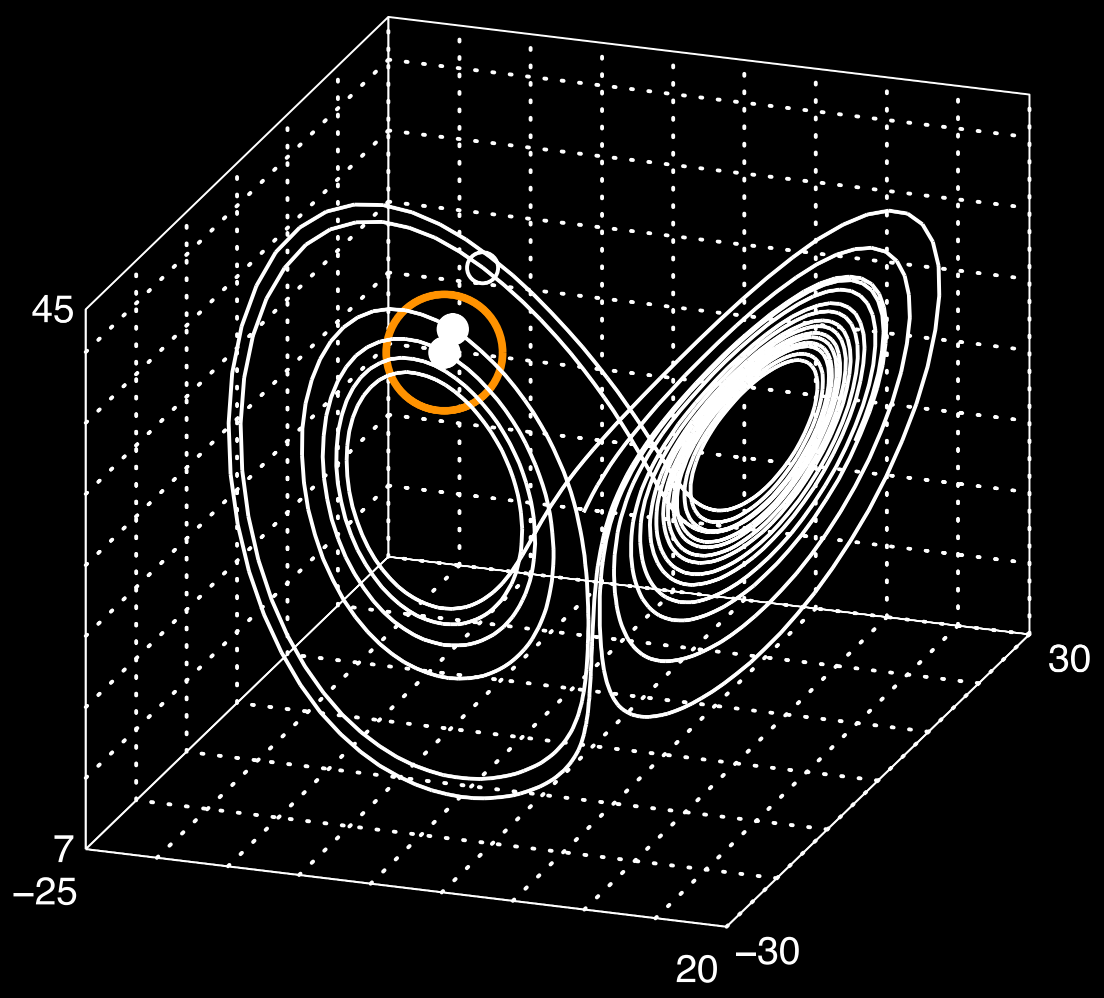
- define measures of complexity for 3D
- quantification of micro-architecture of trabecular bone (3D μCT)
- osteoporosis used as a model for bone loss in micro-gravity

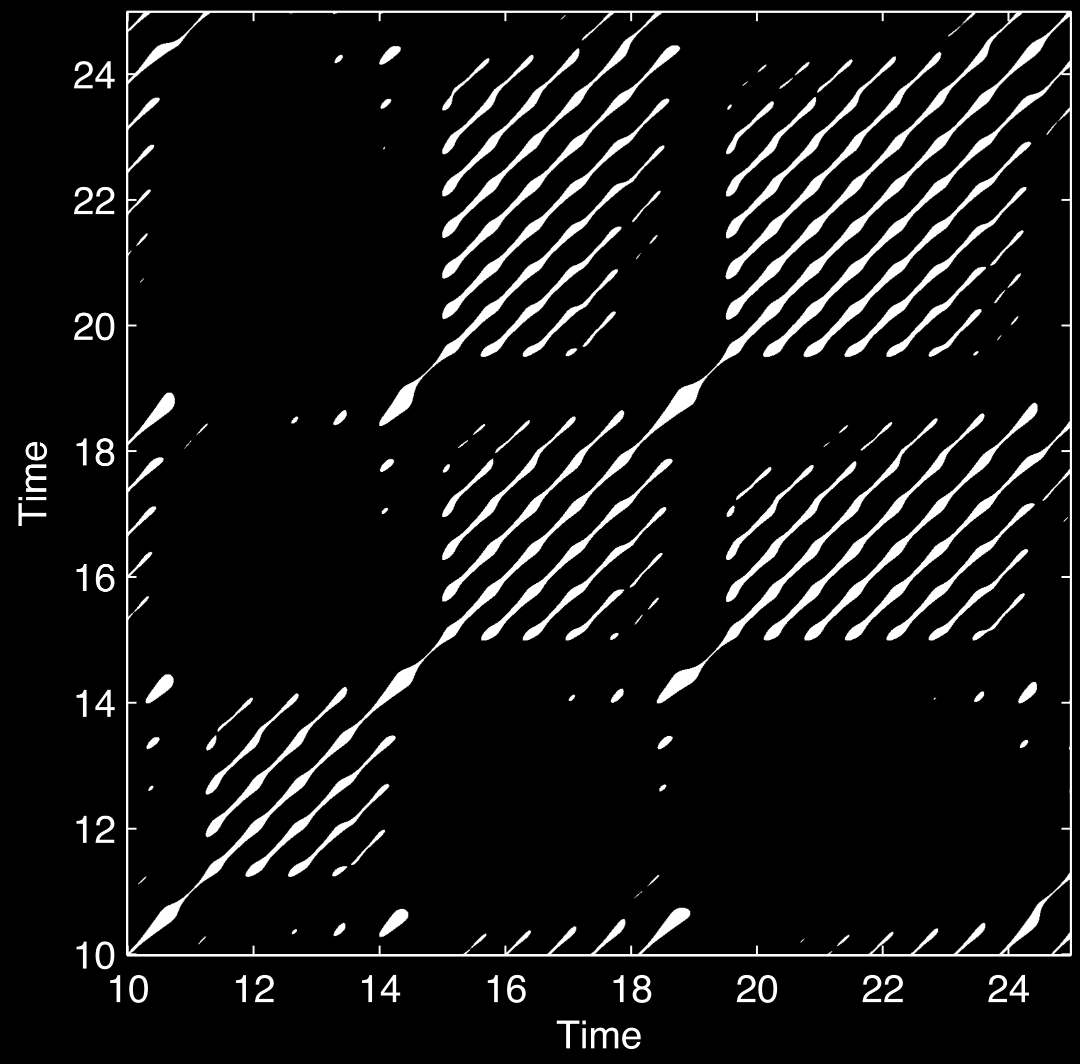


Recurrence Plots

Recurrence Plots

- visualisation of phase space (Eckmann et al, 1987)

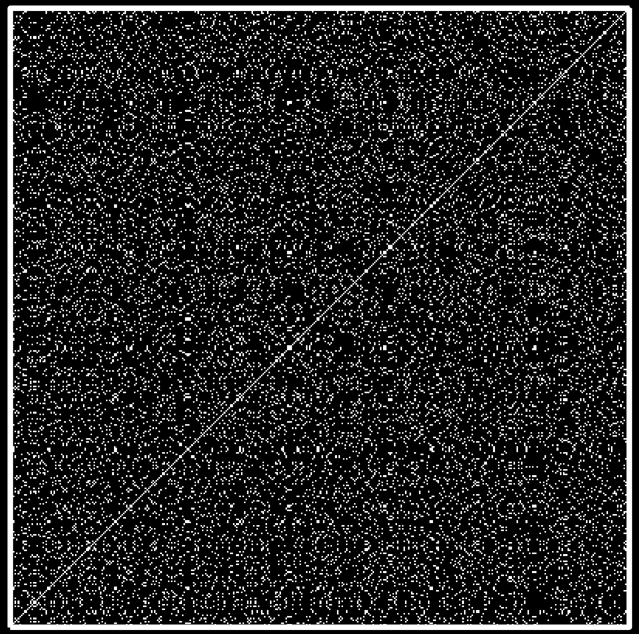




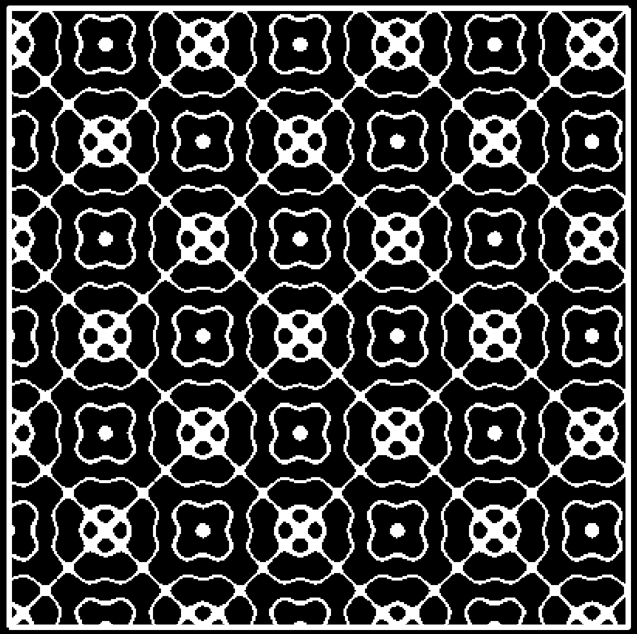
$$\mathbf{R}_{i,j} = \Theta\left(\varepsilon - \|\vec{x}_i - \vec{x}_j\|\right), \quad \vec{x}_i \in \mathbb{R}^m, \quad i,j \in \mathbb{Z}^1$$

Recurrence Plots

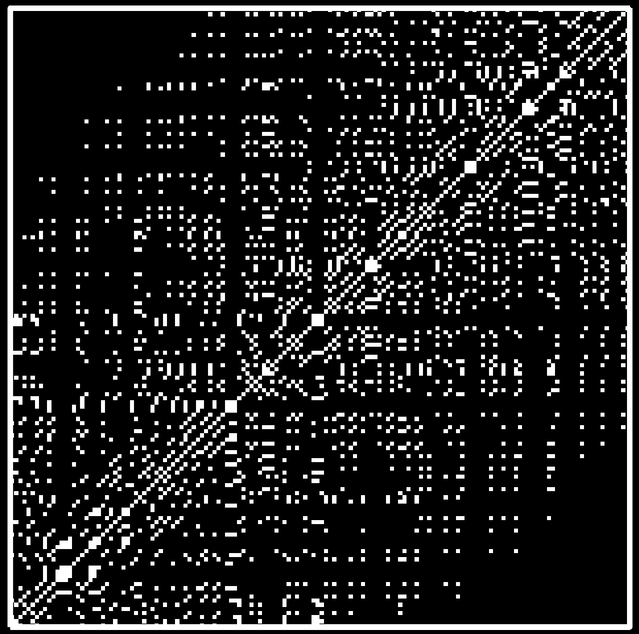
Noise



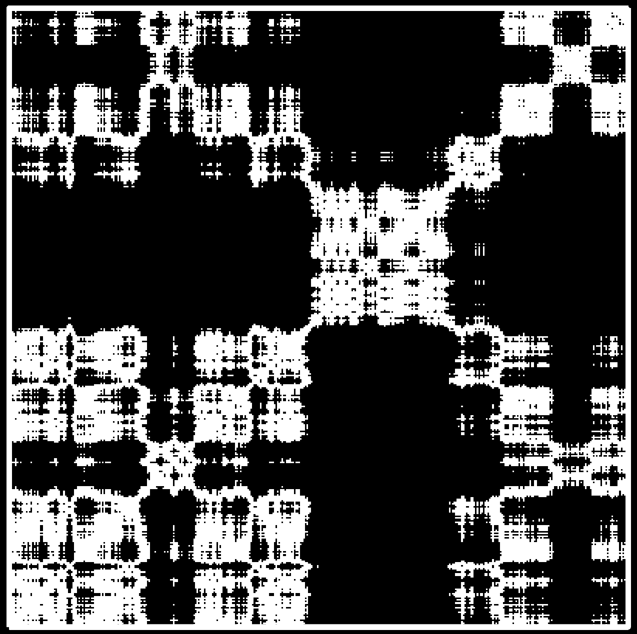
Periodic process



Chaos with drift



Auto-correlated process



Recurrence Quantification (RQA)

- diagonal structures of length l

$$\left(1 - \mathbf{R}_{i+l,j+l}\right) \prod_{\lambda=0}^{l} \mathbf{R}_{i+\lambda,j+\lambda} \equiv 1$$

distribution: $P(l)$

- vertical structures of length v

$$(1 - \mathbf{R}_{i,j+v}) \prod_{\varphi=0}^{v} \mathbf{R}_{i,j+\varphi} \equiv 1$$

distribution: $P(v)$

RQA Measures

- recurrence rate $$RR = \frac{\sum_{i,j} \mathbf{R}_{i,j}}{N^2}$$
- determinism $$DET = \frac{\sum_{l=l_{\min}}^{N} l\, P(l)}{\sum_{i,j} \mathbf{R}_{i,j}}$$
- laminarity $$LAM = \frac{\sum_{v=v_{\min}}^{N} v\, P(v)}{\sum_{i,j} \mathbf{R}_{i,j}}$$
- mean diagonal line length $$L = \frac{\sum_{l=l_{\min}}^{N} l\, P(l)}{\sum_{l=l_{\min}}^{N} P(l)}$$
- trapping time $$TT = \frac{\sum_{v=v_{\min}}^{N} v\, P(v)}{\sum_{v=v_{\min}}^{N} P(v)}$$

Webber & Zbilut, 1994; Marwan et al, 2002

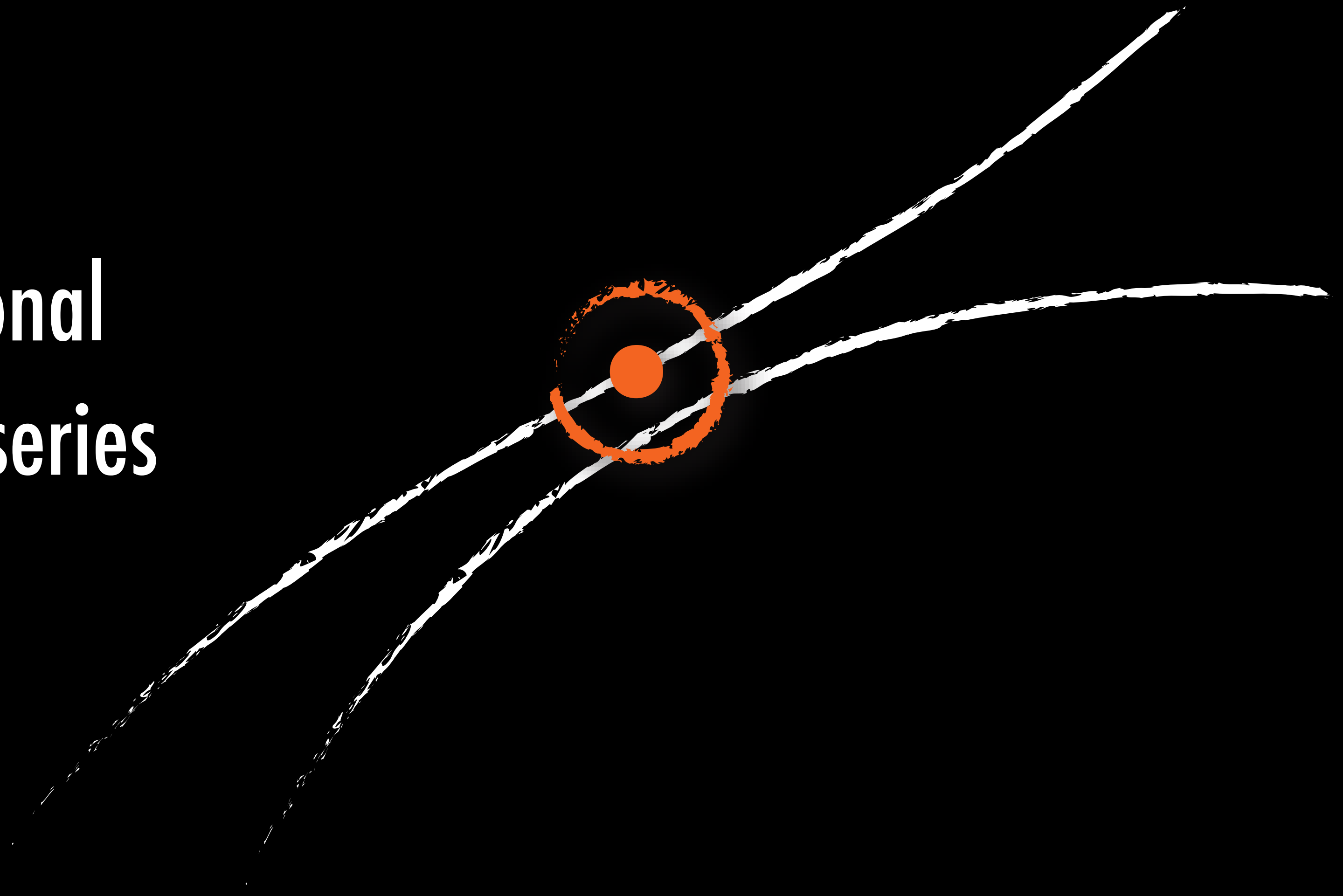
Further Reading

- Marwan, N.: Encounters With Neighbours – Current Developments Of Concepts Based On Recurrence Plots And Their Applications, Ph.D. thesis, University of Potsdam (2003)
- www.recurrence-plot.tk
- tocsy.agnld.uni-potsdam.de (CRP toolbox)

Spatial Extension

Recurrence Plots

- recurrences on one-dimensional objects:
  - phase space trajectories
  - time series
  - one-dimensional spatial data series



Suggested Spatial Extension

- recurrence plot

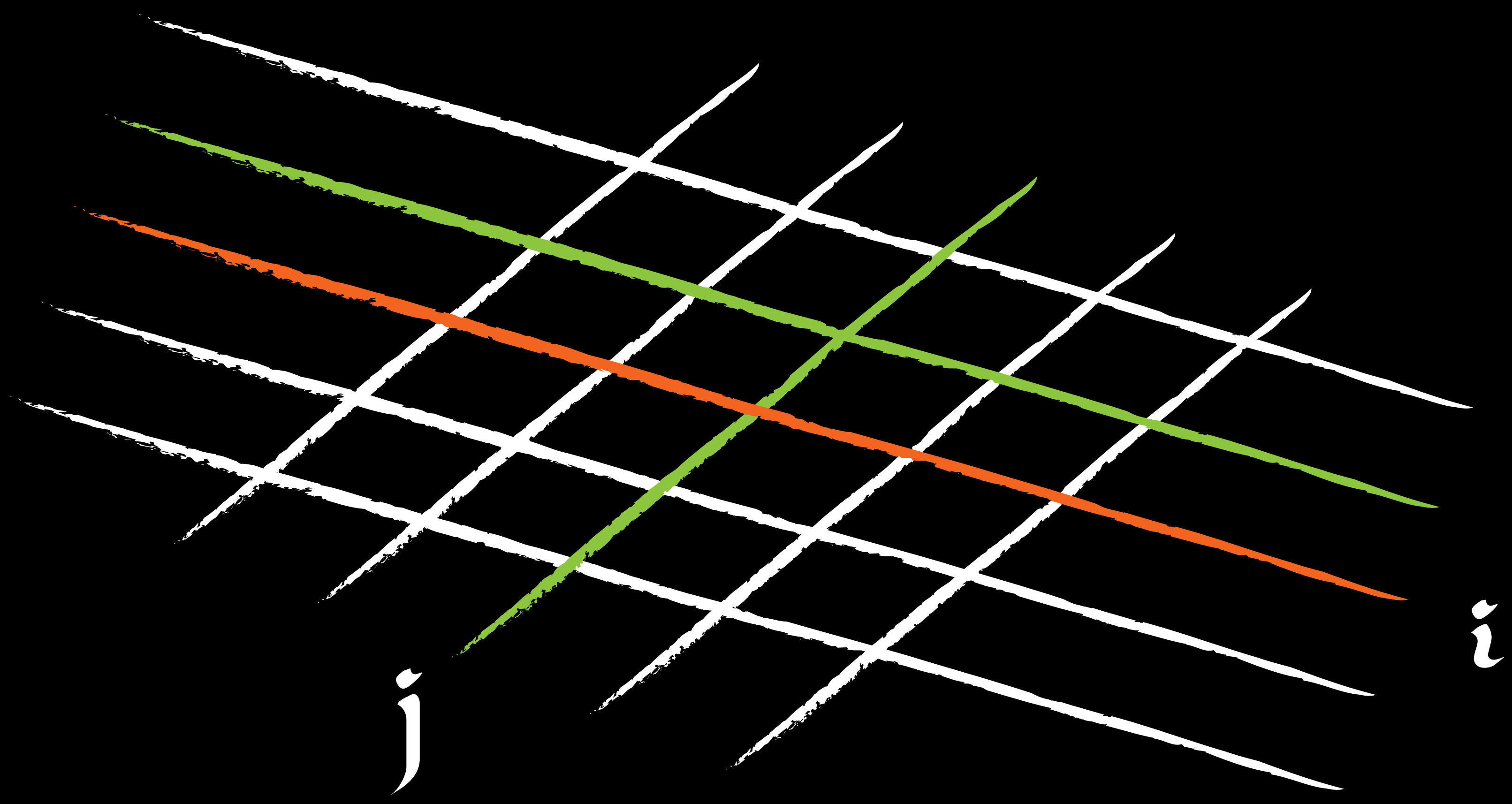
$$\mathbf{R}_{i,j} = \Theta\left(\varepsilon - \left\|\vec{x}_i - \vec{x}_j\right\|\right), \quad \vec{x}_i \in \mathbb{R}^m, \quad i, j \in \mathbb{Z}^1$$

- general extension to any dimension d

$$\mathbf{R}_{\vec{i},\vec{j}} = \Theta\left(\varepsilon - \left\|\vec{x}_{\vec{i}} - \vec{x}_{\vec{j}}\right\|\right), \quad \vec{x}_{\vec{i}} \in \mathbb{R}^m, \vec{i}, \vec{j} \in \mathbb{Z}^d$$

Suggested Spatial Extension

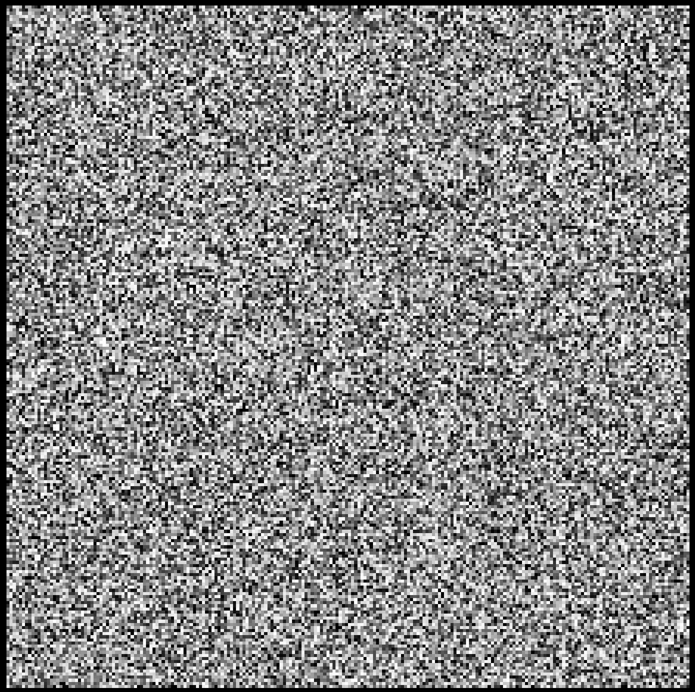
$$\mathbf{R}_{\vec{\imath},\vec{\jmath}} = \Theta\left(\varepsilon - \left\|\vec{x}_{\vec{\imath}} - \vec{x}_{\vec{\jmath}}\right\|\right), \quad \vec{x}_{\vec{\imath}} \in \mathbb{R}^m, \vec{\imath}, \vec{\jmath} \in \mathbb{Z}^d$$



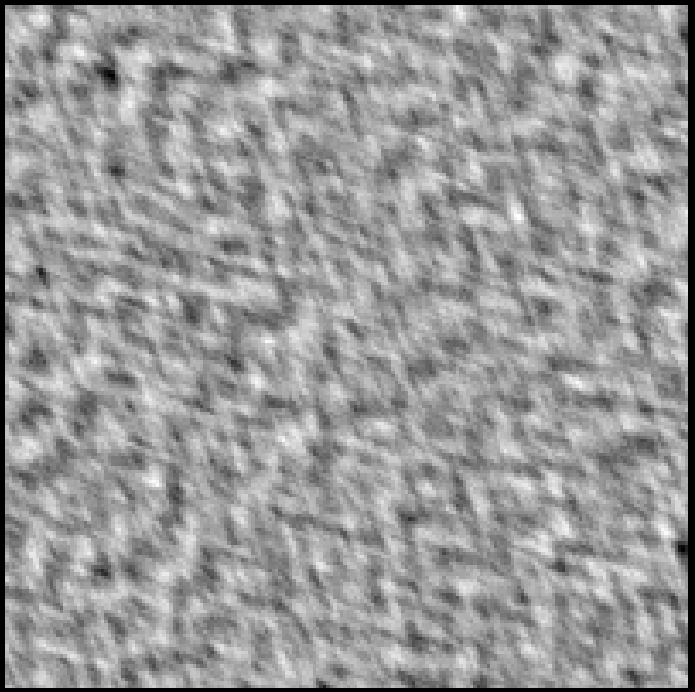
High-Dimensional Recurrence Plot (2 × d)

Examples (2D)

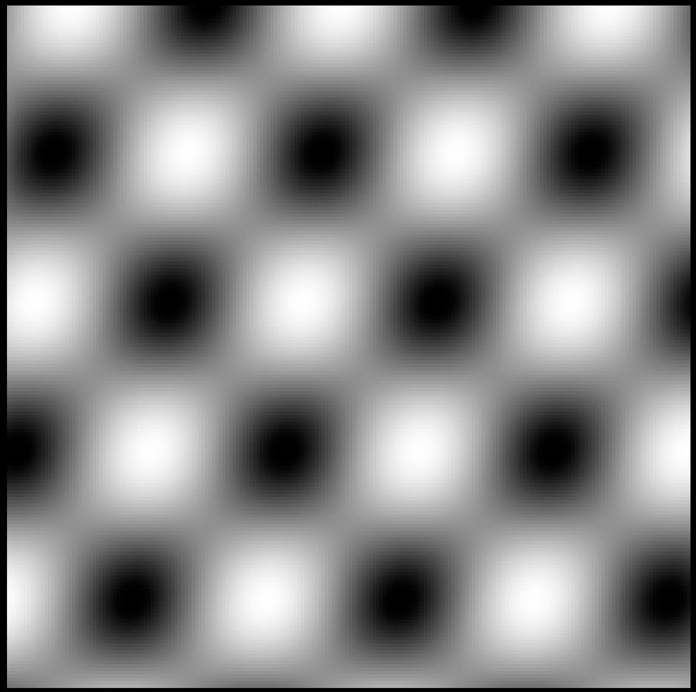
normally distributed noise



2D AR process (2nd order)

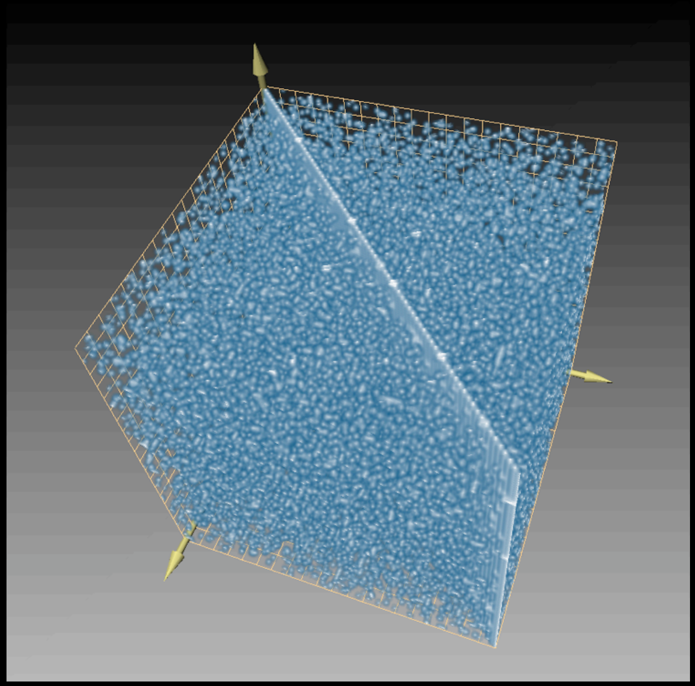


2D periodic image

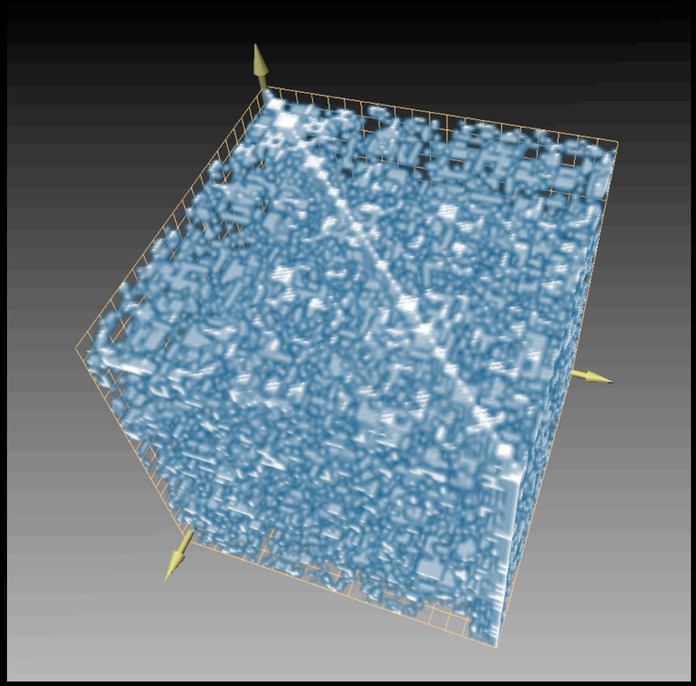


Recurrence Plots of Examples

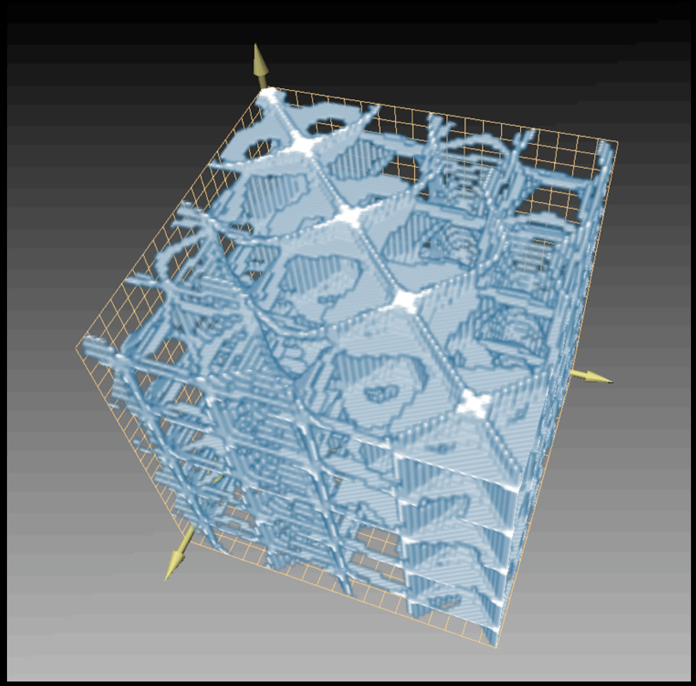
normally distributed noise



2D AR process (2^{nd} order)

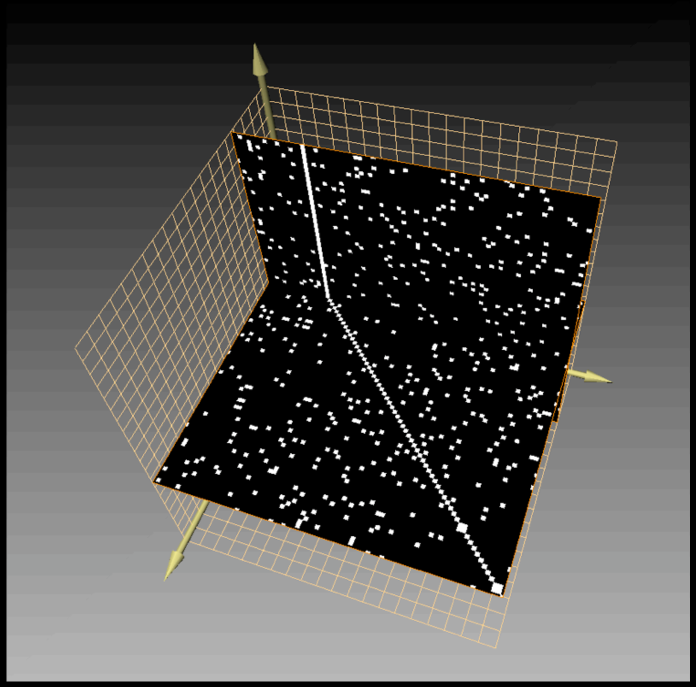


2D periodic image



Recurrence Plots of Examples

normally distributed noise



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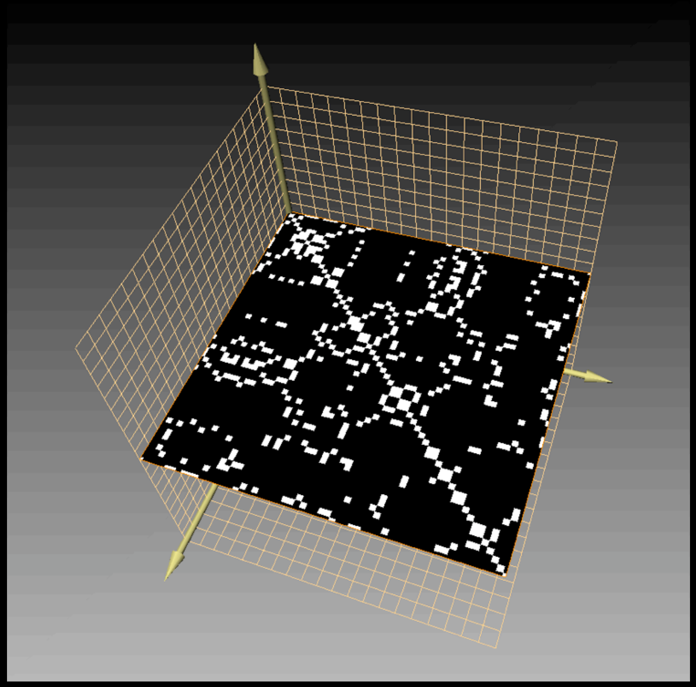
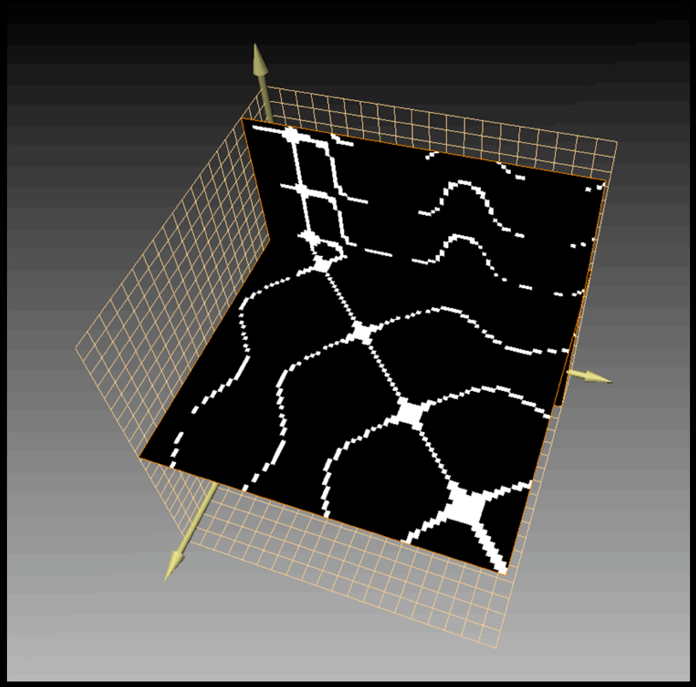


2D periodic image



Recurrence Quantification

- based on distributions of the lengths of diagonal and vertical lines

- RQA for spatial data:
  - definition of diagonal and vertical structures
  - algorithm for finding these structures
  - estimation of their sizes

Recurrence Quantification

- diagonal structures

$$\left(1 - \mathbf{R}_{i+l,j+l}\right) \prod_{\lambda=0}^{l} \mathbf{R}_{i+\lambda,j+\lambda} \equiv 1$$

$$\left(1 - \mathbf{R}_{\vec{\imath}+\vec{l},\vec{\jmath}+\vec{l}}\right) \prod_{\substack{\lambda_1,\lambda_2,\ldots,\\ \lambda_d=0}}^{l} \mathbf{R}_{\vec{\imath}+\vec{\lambda},\vec{\jmath}+\vec{\lambda}} \equiv 1$$

- vertical structures

$$\left(1 - \mathbf{R}_{i,j+v}\right) \prod_{\varphi=0}^{v} \mathbf{R}_{i,j+\varphi} \equiv 1$$

$$\left(1 - \mathbf{R}_{\vec{\imath},\vec{\jmath}+\vec{v}}\right) \prod_{\substack{\varphi_1,\varphi_2,\ldots,\\ \varphi_d=0}}^{v} \mathbf{R}_{\vec{\imath},\vec{\jmath}+\vec{\varphi}} \equiv 1$$

Recurrence Quantification

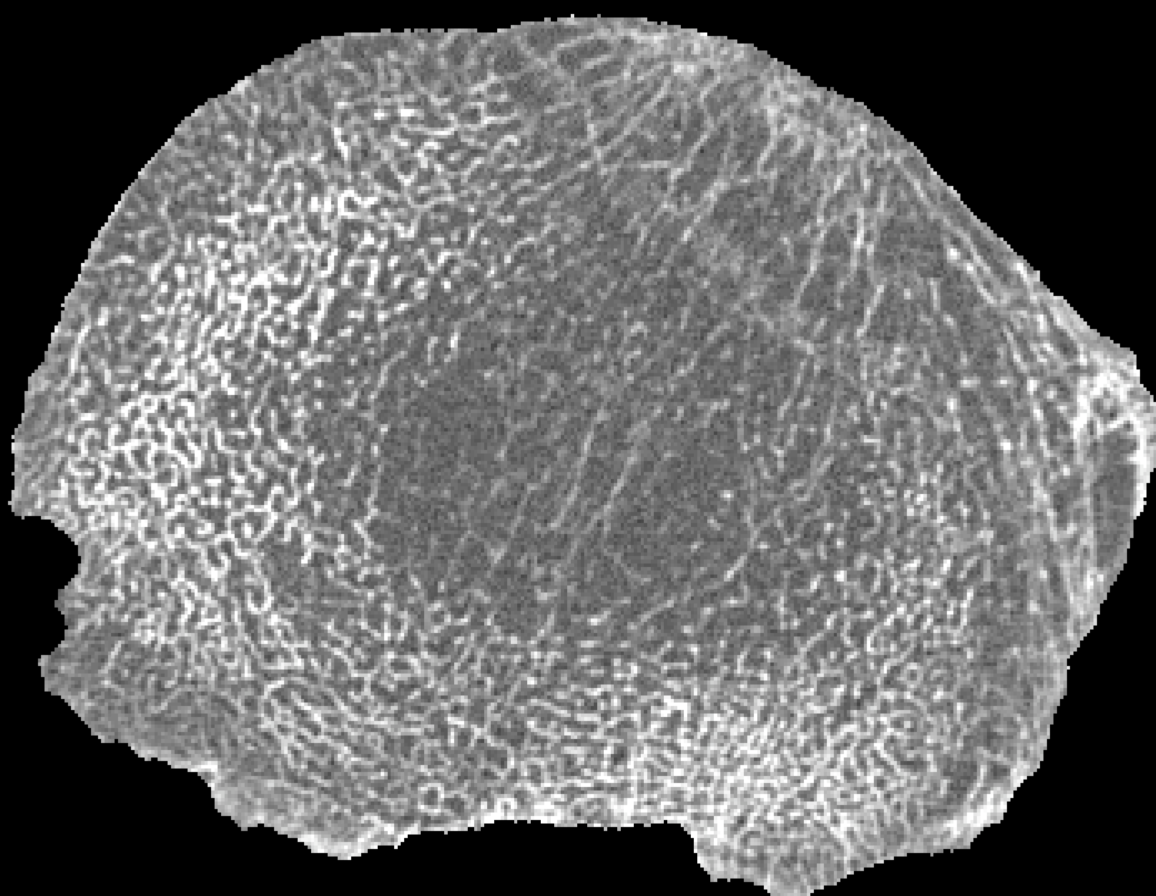
- estimation of the distributions as for common recurrence plots
- RQA measures as for common recurrence plots

RQA of Examples

Example	RR	DET	LAM	L	TT
noise	0.22	0.01	0.01	3.7	3.0
2D-AR2	0.22	0.03	0.07	3.1	3.1
periodic	0.20	0.32	0.31	5.8	5.6

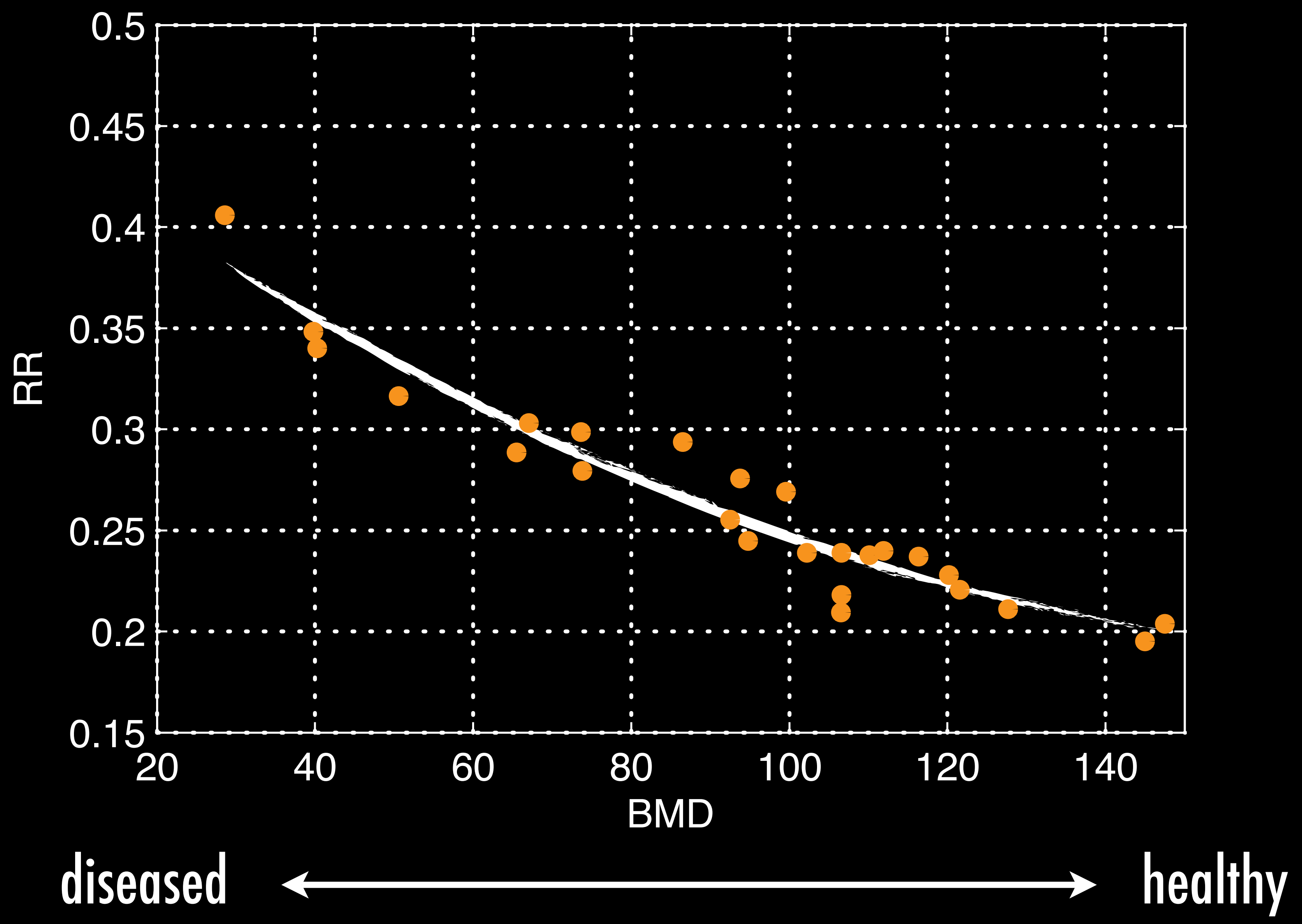
Application

Application on CT Images



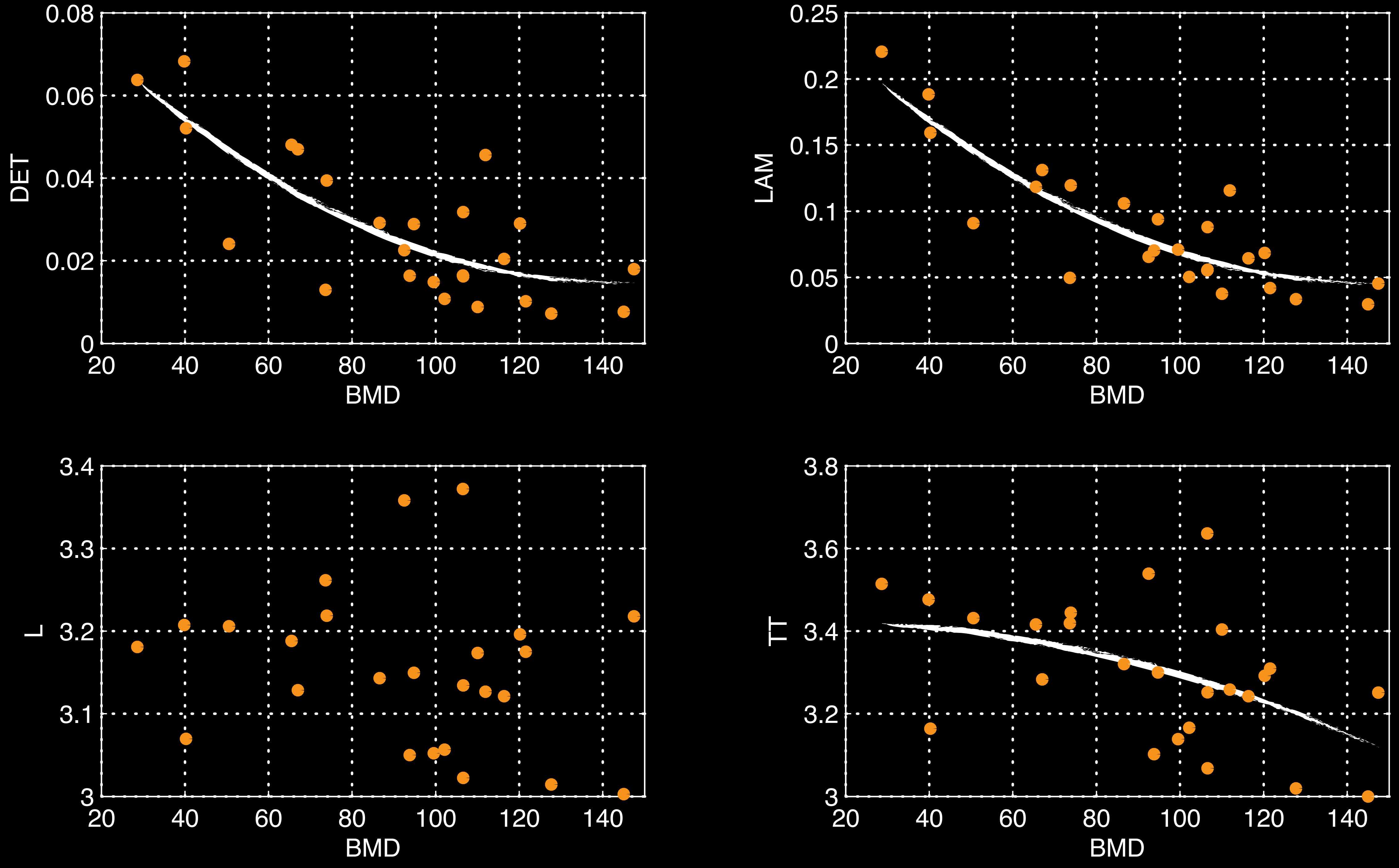
- pQCT scan of human proximal tibia
- 2D slice, 1 mm thick
- 200 × 200 μm pixel size
- 26 specimens

RQA of Proximal Tibia



- bone loss causes recurrent structures in CT (decreasing complexity)

RQA of Proximal Tibia



Conclusions

Conclusions

- extension to higher dimensional spatial data
- able to quantify recurrent structures in spatial data
- characterisation of the micro-architecture of bone
  - bone loss causes recurrent structures (more self-similar structures)
  - reduced complexity of bone micro-architecture

Outlook

- faster and more appropriate algorithm to find diagonal and vertical structures
- further measures of complexity for 3D image analysis
- base for diagnostic measures for structural alteration of bone due to osteoporosis or in micro-gravity